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Reliability-based design of rock slopes — A new perspective on design robustness

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ABSTRACT

Traditional reliability-based rock slope designs, in which the lowest-cost design is selected from all designs meeting target reliability requirements, are often sensitive to variations in noise factors such as rock shear properties. Consequently, a design that was initially judged acceptable may not satisfy reliability requirements if the variation of rock properties has been greatly underestimated. The authors present a Robust Geotechnical Design (RGD) approach for purposes of addressing this dilemma, by considering the robustness *explicitly* in the design process. In the context of rock slope design, this proposed RGD approach aims to make the response (i.e., failure probability) of a rock slope system insensitive to, or robust against, the variation of rock shear properties by adjusting design parameters (i.e., such as slope angle and height). Compared to traditional reliability-based design, the RGD approach adds design robustness as one of its design objectives. Thus, multi-objective optimization, considering both cost and robustness, is needed to select optimal designs in the acceptable design space where safety is guaranteed by a constraint on the reliability. In this paper, the concept of the Pareto Front, a collection of optimal designs that reflect the trade-off between cost and robustness, is implemented in the RGD approach. The proposed RGD approach is demonstrated with an example of a rock slope design.

1. Introduction

In recent years, risk-based approaches have been increasingly used in the analysis and design of rock slopes. This approach necessitates qualification of the probability of failure of the rock slope, which explicitly includes consideration for uncertainties in the underlying input parameters in a systematic manner (Vanmarcke, 1980; Oka and Wu, 1990; Christian et al., 1994; Duncan, 2000; Ching et al., 2009). Many investigators have contributed to the subject of reliability or failure probability of rock slopes (e.g., Low and Einstein, 1992; Low, 1997; Juang et al., 1998; Park and West, 2001; Duzgun et al., 2003; Park et al., 2005; Hoek, 2006; Low, 2007; Low, 2008; Duzgun and Bhasin, 2009; Li et al., 2011; Lee et al., 2012; Park et al., 2012). Traditional reliability-based rock slope designs, however, are often sensitive to variations in noise factors such as rock shear properties. In this paper, the authors propose a reliability-based robust design approach to achieve design robustness, while also meeting safety and cost requirements.

The shear strength of discontinuities, which is a key parameter that controls the stability of rock slopes, is often difficult to ascertain and has to be modeled as a random variable. A proper statistical characterization of this random variable often requires testing of a large number of samples extracted from a wide range of sites that contain the same roughness profile of the rock discontinuities in question. However, budgetary constraints for site investigation and field and laboratory tests often limit the amount of data available to an engineer. Thus, uncertainty is often inherent in the derived statistical parameters of rock properties because of a small sample size and imperfect knowledge. Unfortunately, traditional reliability-based designs are usually sensitive to the estimated variation of shear strength properties. As such, an initially acceptable reliability-based design may not satisfy the target reliability index or failure probability requirement if the variation of the rock properties is underestimated. The proposed reliability-based Robust Geotechnical Design (RGD) approach presented herein is aimed at addressing this dilemma by considering robustness *explicitly* in the design process. In the context of rock slope design, the proposed approach is aimed at making the response (i.e., failure probability) of a geotechnical system insensitive to, or robust against, the variation of noise factors (i.e., rock properties and their statistical characterization). Here, the design robustness is achieved by adjusting design parameters (i.e., slope angle and height, and protection measures) that can be controlled by the engineer.

The robust design concept was first proposed by Taguchi (1986) for improving product quality and reliability in industrial engineering, the early applications of which are closely related to the product design to avoid the effect of the uncertainty from environmental and operating conditions. More recent applications are found in various fields such as mechanical design, structural design, and aeronautical design (e.g., Chen et al., 1996; Chen and Lewis, 1999; Sandgren and Cameron, 2002; Lagaros and Papadrakakis, 2007; Jamali et al.,

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2010). Robust design methodology differs from the traditional design approaches primarily in the treatment of variation in the system response caused by noise factors.

In the proposed RGD framework, the computed failure probability of rock slope is modeled as the response of the system. The variation in the computed failure probability caused by the uncertainty in the estimated variation of rock properties is evaluated using statistical methods. The robustness for reliability-based design is achieved if the variation of the failure probability (i.e., the system response) can be minimized by manipulating design parameters of the rock slope. However, higher design robustness is often achieved at a higher cost. Thus, a multi-objective optimization considering cost and robustness is needed to select the optimal designs among those in the acceptable design space.

Multi-objective optimization does not usually produce a single best design with respect to all design objectives. Rather, the result is often expressed in a "Pareto Front" (Ghosh and Dehuri, 2004), which is a set of optimal designs that are "non-dominated" (Deb et al., 2002) by any other designs in all aspects. In our case, the points on the Pareto Front collectively express a trade-off relationship between robustness and cost. Thus, the Pareto Front may be used as a design aid for selecting the best design considering the desired cost range and/or target level of design robustness.

To illustrate the proposed RGD approach and its significance, the authors studied the Sau Mau Ping rock slope in Hong Kong, which is composed of unweathered granite with sheet joints (Hoek, 2006). By conducting and comparing reliability-based designs, both with and without consideration of robustness, the authors demonstrated the significance of the RGD approach.

2. Framework for reliability-based Robust Geotechnical Design (RGD)

The authors propose a methodology for the reliability-based robust geotechnical design (RGD) of a rock slope, which is a modification of the authors' recent work (Juang et al., 2012; Juang and Wang, 2013). In reference to Fig. 1, this design methodology consists of five steps:

(1) Establish the deterministic computational model for stability analysis of rock slope.
In a deterministic stability analysis of rock slope, design methods are often selected based upon the design situation

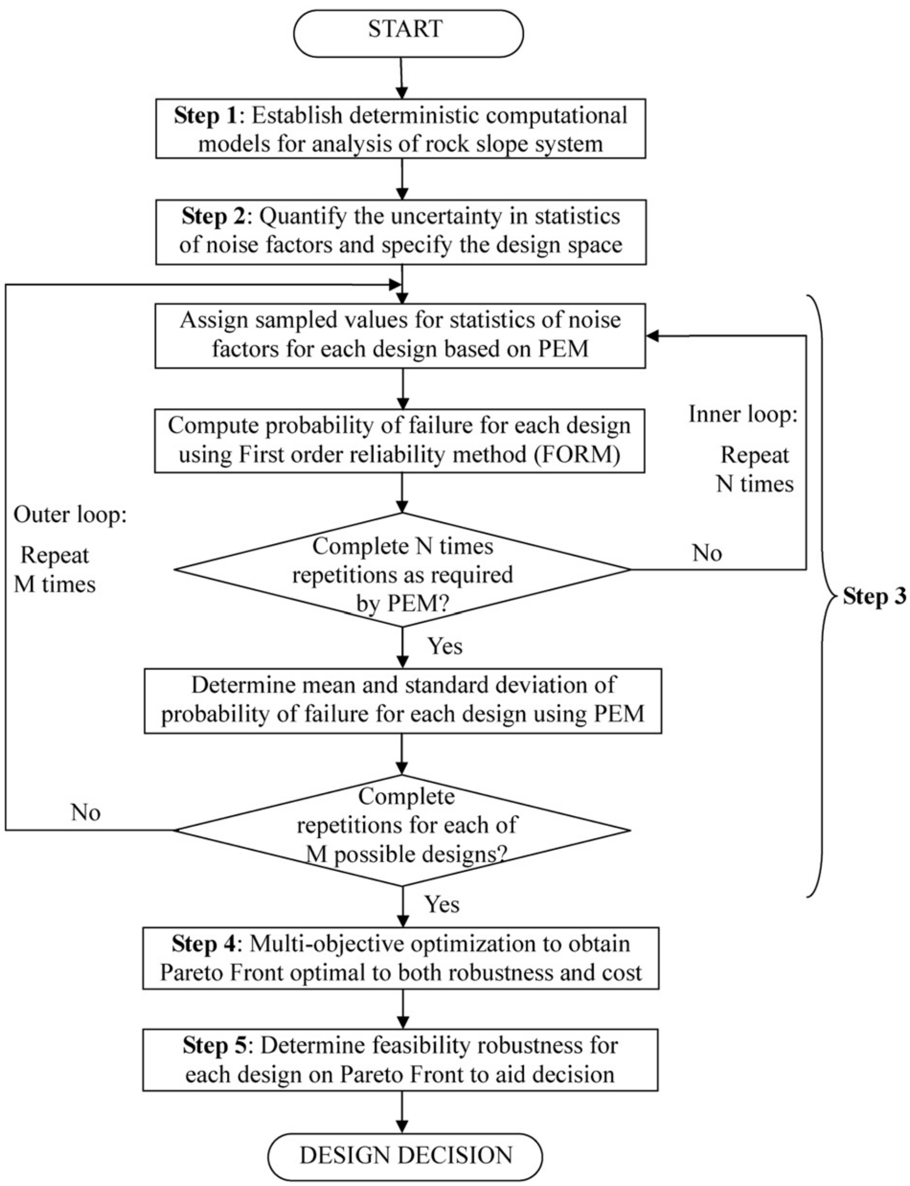

Fig. 1. Flowchart illustrating reliability-based robust geotechnical design of rock slopes (modified after Juang and Wang, 2013).

(e.g. the number of removable blocks). The removable blocks refer to the potential unstable blocks identified based on a proper characterization of rock mass structure. For illustration purposes, the deterministic model for single block analysis proposed by Hoek (2006) is used for the analysis of Sau Mau Ping slope.

(2) Characterize noise factors (i.e., uncertain rock properties) and design parameters of the rock slope (including an estimate of possible variation of the statistical parameters of noise factors). In the context of Robust Design, input parameters are divided into "easy-to-control" parameters (called design parameters), and "hard-to-control" parameters (called noise factors). The noise factors in the robust design refer to the parameters that are difficult to ascertain and/or impossible to manipulate in the design process. For the rock slope design, noise factors mainly refer to uncertain rock properties (e.g., shear strength of discontinuities), the statistics of which are difficult to ascertain due either to limited data availability or measurement error. If the statistics of rock properties are subjected to error, the result from a reliability-based design can be either under-designed or over-designed. The extent of the variation of the statistics of noise factors may be estimated based upon the computed statistical error and/or measurement error if the measured data of rock properties is available. If detailed data is unavailable or the sample size is too small, the extent of the variation of statistics of noise factors may be estimated based upon their typical ranges from published literature.

Design parameters refer to the parameters that can be specified freely and accurately by the designer. There are two approaches used for the design and remedial construction of a rock slope: geometry design, in which the slope height and slope face angle are reduced; and reinforcement of the rock slope, using rock bolts and cables. Stabilizing rock slopes by means of geometry design is generally more cost-efficient than the use of cables and bolts (Hoek, 2006). To demonstrate our proposed RGD methodology, we will focus on the geometrical design of a given rock slope, with slope height H and slope angle θ as our design parameters. The typical ranges for both slope height H and slope angle θ that are suitable for the particular design situation at hand are then specified. For construction practicality, these parameters are modeled as discrete variables in the design space.

(3) Evaluate the variation of the failure probability for robustness consideration.

As noted previously, a design is considered robust when the variation (in terms of standard deviation) of the probability of failure caused by variations of statistics of noise factors is small. When the statistics of noise factors (mainly rock properties) are fixed values, the predicted failure probability will also be a fixed value. When the statistics of noise factors are uncertain, however, the predicted failure probability will be a random variable. For the rock slope that is composed by a single block, the probability of failure can be determined using the traditional reliability method, such as the first-order reliability method (FORM; Ang and Tang, 1984).

To assess the robustness of the slope design, the standard deviation of the probability of failure caused by the variation in statistics of noise factors should be calculated. In this paper, the modified Point Estimate Method (PEM) developed by Zhao and Ono (2000) is used to compute this standard deviation of the probability of failure. The PEM first evaluates the probability of failure for each design at selected values of statistical parameters of noise factors, and then computes the mean and standard deviation of the failure probability. The reader is referred to Zhao and Ono (2000) for details. Here, the standard deviation of the probability of failure for each design in the design space is computed by PEM integrated with FORM procedure.

(4) Perform multi-objective optimization to establish the Pareto Front that is optimal for both robustness and cost.

The design of a rock slope can be thought of as a multi-criteria problem that involves the requirements of safety, cost and robustness. A target failure probability is set as the safety constraint and used to screen the unsatisfactory designs based on the computed mean failure probability by PEM. The robustness and cost are set as objectives for multi-objective optimization. Here, the standard deviation of the failure probability is used to gauge the robustness; a smaller standard deviation of the failure probability signals a higher robustness against noise factors. A simple cost estimation method is used in this paper to illustrate the RGD methodology, where the cost of a given rock slope design is approximated as the volume of rock mass that must be excavated (Duzgun et al., 1995). It should be noted that cost estimation is not the focus of this paper, and the proposed RGD method is not dependent on any particular cost estimation method. In fact, any reasonable cost estimation method can be used.

The multi-objective optimization of rock slope is realized in this paper using a fast elitist Non-dominant Sorting Genetic Algorithm version II (NSGA-II), developed by Deb et al. (2002). The multi-objective optimization may or may not lead to a single best design that is optimal in all aspects, depending upon whether the objectives are conflicting or cooperative. With conflicting objectives, it leads to a set of non-dominated solutions that collectively form a Pareto Front. For a minimization problem with two objectives, the solution P dominates solution Q when no objective value of Q is less than P, and at least one objective value of Q is strictly greater than P (Ghosh and Dehuri, 2004). By screening designs or solutions in the solution space and eliminating the solutions that are dominated by others, a set of solutions that are non-dominated by any other solution can be obtained, which collectively form a Pareto Front as shown in Fig. 2. Any solution (or design) on the Pareto Front cannot be improved in any one objective without worsening at least one other objective. The reader is referred to Deb et al. (2002) for details of the NSGA-II algorithm.

Generally speaking, any point (solution) on the obtained Pareto Front may be considered as an "optimum" solution. In practice, however, some designs on the Pareto Front may be more desirable than others depending upon the designer's preference. The Pareto Front establishes the trade-off relationship among conflicting objectives, which can be used as a design aid for decision-making. For the rock slope example, when a specific

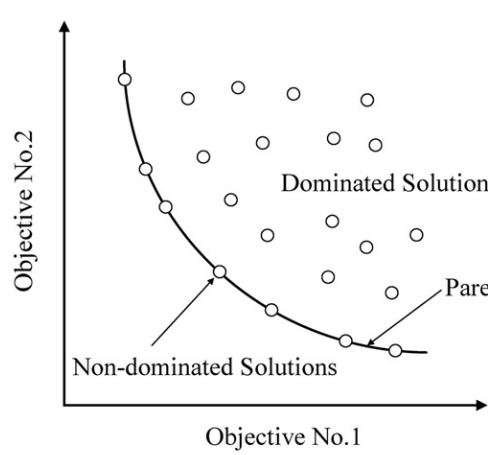

Fig. 2. Illustration of Pareto Front in a bi-objective design space.

maximum cost level is pre-defined, the most robust design can be easily identified, which would be the optimal design. Similarly, the least cost design can be easily identified on the Pareto Front if a target level of robustness is specified.

(5) Determine feasibility robustness for each design on the Pareto Front.
Although the trade-off relationship between cost and robustness on the Pareto Front itself is a valuable decision-making aid, further refinement is possible for a more informed and easier decision-making process. According to Parkinson et al. (1993), a system is considered to have "feasibility robustness," if it can remain feasible relative to the nominal constraints, even when the system undergoes variation. In this paper, the feasibility robustness is defined as the confidence probability that the target failure probability (i.e., the constraint) can still be satisfied even with uncertainty in the statistics of noise factors. Symbolically, this concept is formulated as follows (Juang et al., 2012):

$$P\left[p_f - p_T \leq 0\right] > P_0 \tag{1}$$

where p_f is the computed failure probability of the rock slope system, which is a random variable affected by the uncertainty in the statistics of noise factors; p_T is the target failure probability; $P[p_f - p_T \leq 0]$ is the confidence probability that the constraint of a target failure probability is satisfied; and P_0 is an acceptable confidence probability representing the feasibility robustness level pre-defined by a designer. In this paper, the target failure probability is set at $p_T = 0.0062$, which corresponds to a target reliability index of $\beta_T = 2.5$ (Low, 2008).

The computation of $P[p_f - p_T \leq 0]$ requires knowledge of the distribution of p_f, which is difficult to ascertain. A Monte Carlo simulation of selected designs shows that the histogram of the reliability index β (corresponding to p_f) can be approximated well with a normal distribution. Thus, an equivalent counterpart in the form of $P[(\beta - \beta_T) \geq 0]$, where $\beta_T = 2.5$, is employed for the feasibility robustness assessment. With the PEM procedure described previously, the mean and standard deviation of β, denoted as μ_β and σ_β can be obtained. Then, Eq. (1) becomes:

$$P[(\beta - \beta_T) \geq 0] = \Phi\left(\beta_\beta\right) \geq P_0 \tag{2}$$

where Φ is the cumulative standard normal distribution function, and β_β is defined as:

$$\beta_\beta = \frac{\mu_\beta - 2.5}{\sigma_\beta} \tag{3}$$

Similar to the acceptance probability P_0, β_β may also be used as an index for feasibility robustness. When the target feasibility-robustness index β_β^T is specified by the designer, the least-cost design on the Pareto Front can be readily identified. The approach of measuring robustness with a feasibility robustness index (β_β) allows for a more informed and effective design decision.

3. Case history of Sau Mau Ping rock slope

The case history of the Sau Mau Ping rock slope in Hong Kong is used to illustrate the reliability-based robust design of rock slopes. The rock mass of the Sau Mau Ping slope is unweathered granite with sheet joints, which are formed by the exfoliation processes during the cooling of granite. An initial study by Hoek (2006) led to a simplification of the Sau Mau Ping slope as that composed by a single unstable block with a water-filled tension crack involving only a single failure mode. Fig. 3 shows an illustration of the geometry of the Sau Mau Ping slope. Following Hoek (2006), the slope before remediation has a height H of 60 m and a slope angle θ of 50°. The potential failure plane is inclined at 35° ($\psi = 35°$), and the unit weight of rock (γ) is 2.6 ton/m^3, which is assumed to be a fixed value.

A deterministic model with single failure mode developed by Hoek and Bray (1981) is employed for our case study of the Sau Mau Ping slope. The model is a two-dimensional limit equilibrium model of the plane failure, in which the rock slope is assumed with a 1-meter thick slice through the slope. The factor of safety is computed by resolving all forces acting on the slope into components that are parallel and normal to the sliding surface. The vector sum of the block weight and water force acting on the plane is termed the driving force. The product of normal forces and the tangent of friction angle, plus the cohesion force, is the resisting force. The factor of safety is expressed as the ratio of the sum of resisting forces to the sum of driving forces (Hoek and Bray, 1981; Hoek, 2006). The detailed formulation for stability analysis of Sau Mau Ping slope is summarized in Appendix A.

4. Traditional reliability-based design and potential drawbacks

As summarized by Hoek (2006), five random variables should be considered in the reliability analysis of the Sau Mau Ping slope, which include cohesion of rock discontinuities c, friction angle of rock discontinuities ϕ, tension crack depth z, percentage of the tension crack depth filled with water i_w, and gravitational acceleration coefficient α. Following the previous study by Hoek (2006) and Low (2007), c, ϕ and z are assumed to be normal distributions; i_w and α are assumed to be truncated exponential distributions with mean and upper and lower bounds listed in Table 1. For the truncated exponential distribution with mean $1/\lambda$ and truncated to the range [a, b], the probability density and cumulative distribution function are arranged as follows (Low, 2007):

$$f(x) = \frac{1}{e^{-\lambda a} - e^{-\lambda b}} \lambda e^{-\lambda x} \quad a \leq x \leq b \tag{4}$$

$$F(x) = \frac{\left(e^{-\lambda a} - e^{-\lambda x}\right)}{e^{-\lambda a} - e^{-\lambda b}} \quad a \leq x \leq b \tag{5}$$

Furthermore, c and ϕ are likely to be negatively correlated with a correlation coefficient of −0.5; z and i_w are also likely to have negative correlation with a correlation coefficient of −0.5 (Low, 2007). Detailed statistics of the five input random variables are listed in Table 1.

The design space should be specified first, before performing the reliability-based design. For the two design parameters of the rock slope, slope height H may typically range from 50 m to 60 m, and slope angle θ may typically range from 44° to 50°. It should be noted that the upper bounds of these two parameters are selected based on the initial slope condition and the lower bounds based on the geometry requirement of the deterministic model. For convenience of construction, slope height H may be rounded to the nearest 0.2 m and slope angle θ may be rounded to the nearest 0.2°. Thus, H may assume 51 discrete values and θ may assume 31 discrete values in the possible ranges yielding a total of 1581 possible designs in the design space.

When the distributions of input parameters can be perfectly characterized with the statistics presented previously, the FORM procedure is readily employed to evaluate the probability of failure for all designs in the design space. Fig. 4 shows the probability of failure for selected designs with slope height $H = 50$ m, 52 m, …, 60 m. As expected, the probability of failure increases with the increase of both the slope height and the slope angle.

With the probabilities of failure obtained for all designs in the design space, reliability-based designs are then obtained by minimizing

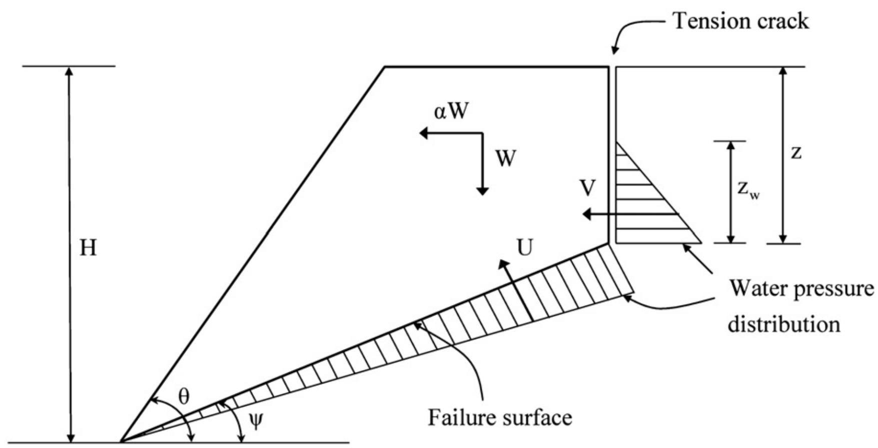

Fig. 3. Illustration of geometry for plane sliding of Sau Mau Ping rock slope (modified after Low, 2007).

the cost while satisfying the reliability constraint (Zhang et al., 2011). For demonstration purposes, $p_f < p_T = 0.0062$ as suggested by Low (2008) is used as the reliability constraint in this paper. By checking all designs that satisfy the reliability constraint, the least cost design can be chosen, which yields $H = 54.8$ m and $\theta = 50°$ with a design cost of 91.1 units [note: 1 unit = unit cost ($) of excavated volume of rock mass in m^3/m].

In a routine geotechnical investigation, only a small sample of data will be available for determining the shear properties of the rock discontinuities. The mean values of geotechnical properties can usually be adequately estimated even with a small sample size (Wu et al., 1989). However, the coefficient of variation (COV) and the correlation of coefficient (ρ) of geotechnical properties are difficult to ascertain. In past studies (e.g., Hoek, 2006; Low, 2007), COV and ρ of the input parameters were estimated primarily by engineering judgment or based on published literature.

According to a survey of literature by Lee et al. (2012) for rock slopes, the COV of cohesion c, denoted as $COV[c]$, typically ranges from 10% to 30%; the COV of friction angle ϕ, denoted as $COV[\phi]$, typically varies in the range from 10% to 20%. Furthermore, the coefficient of correlation for c and ϕ, denoted as $\rho_{c,\phi}$, is likely to be negative varying from −0.2 to −0.8 (Low, 2008); and the coefficient of correlation for z and i_w, denoted as ρ_{z,i_w} may vary from −0.2 to −0.8 (Low, 2008).

Table 2 lists results of traditional reliability-based designs for various parameter uncertainty levels, each representing a combination of variations in c and ϕ. Here, the COVs of c and ϕ are assumed to vary within the typical COV ranges for these two parameters. It is clear that from these results, the least-cost design obtained from traditional reliability-based design methods is very sensitive to the assumed COVs of rock properties. Under the lowest uncertainty level of rock properties, as shown in Table 2, the least-cost design costs only 39.3 units; whereas under the highest uncertainty level, the least-cost design costs 225.2 units. Thus, the traditional reliability-based design using least-cost criterion is meaningful only if the statistical parameters of rock properties can be precisely defined. If the COVs of rock properties are underestimated, an initially acceptable design may no longer be satisfactory. Similarly, if the COVs are overestimated, the traditional design may not be cost-effective. This is a significant drawback of the traditional reliability-based design as it places a burden on the user to estimate the statistics of uncertain parameters, which is often difficult to ascertain because of insufficient data availability due to budgetary constraints.

This finding is further demonstrated in Table 3. Here, an initially acceptable design ($H = 54.8$ m and $\theta = 50°$) based on an assumption of $COV[c] = 0.20$ and $COV[\phi] = 0.14$ and a target failure probability of $p_T = 0.0062$, is re-analyzed with various levels of $COV[c]$ and $COV[\phi]$. As shown in Table 3, if the design is implemented in a site with higher variability, the reliability requirement will no longer be satisfied, as the failure probability will be greater than $p_T = 0.0062$. Even under the highest uncertainty level among all scenarios listed in Table 2 ($COV[c] = 0.30$ and $COV[\phi] = 0.20$), however, an acceptable design (for example, $H = 50$ m and $\theta = 49.2°$) can still be found that satisfies the constraint of $p_f < p_T = 0.0062$. Thus, some designs, albeit at higher costs, can be chosen to ensure robustness against variation of the rock properties.

5. Reliability-based Robust Geotechnical Design of the Sau Mau Ping slope

5.1. Characterization of uncertain statistical parameters

For a robust design of the Sau Mau Ping slope, the noise factors mainly refer to the geotechnical parameters including cohesion c and friction angle ϕ of rock discontinuities. With the RGD approach,

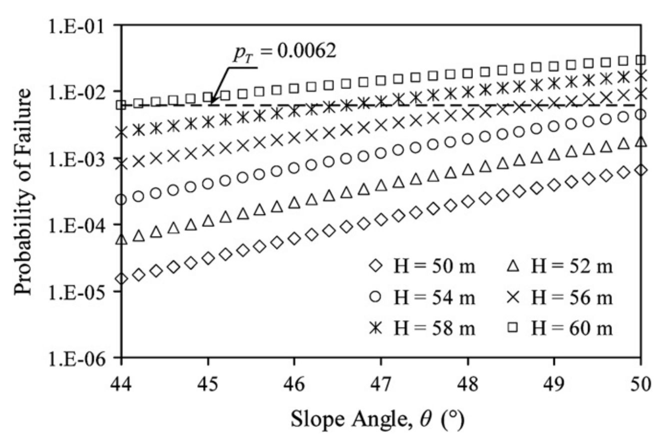

Fig. 4. Probabilities of failure of rock slope at various slope heights assumed with exactly characterized distributions.

Table 1
Statistics of random variables for Sau Mau Ping rock slope (modified after Hoek, 2006).

Random variables	Probability distribution	Mean	Std. dev.
c	Normal	10 ton/m²	2 ton/m²
ϕ	Normal	35°	5°
z	Normal	14 m	3 m
i_w	Exponential with mean 0.5, truncated to [0, 1]		
α	Exponential with mean 0.08, truncated to [0, 0.16]		

Table 2
Least-cost designs under various COV assumptions of rock properties.

$COV[c]$	$COV[\phi]$	H (m)	θ (°)	Cost (units)
0.10	0.10	57.6	50.0	39.3
0.10	0.14	56.0	50.0	68.1
0.10	0.20	52.8	50.0	132.0
0.20	0.10	55.8	50.0	71.8
0.20	0.14	54.8	50.0	91.1
0.20	0.20	52.2	50.0	145.0
0.30	0.10	51.6	50.0	158.3
0.30	0.14	51.0	50.0	171.8
0.30	0.20	50.0	49.2	225.2

the COVs of c and ϕ and the correlation coefficient (ρ) of c and ϕ are considered as random variables. Furthermore, as the water filling in tension cracks is mainly from direct surface run-off of heavy rain (Low, 2007), the tension crack with a shallow depth tends to have a higher percentage of its depth filled with water. Therefore, z and i_w are likely to have a negative correlation. As the extent of correlation between z and i_w is quite uncertain, however, the correlation of z and i_w is also considered as a random variable. Thus, $COV[c]$, $COV[\phi]$, $\rho_{c,\phi}$ and ρ_{z,i_w} are considered as random variables herein for reliability-based robust geotechnical design of the Sau Mau Ping slope.

For *illustration* purposes, the mean of $COV[c]$, denoted as $\mu_{COV[c]}$, is assumed to be 0.20; and the coefficient of variation of $COV[c]$, denoted as $\delta_{COV[c]}$, is assumed to be 17% (roughly to cover the typical range of $COV[c]$). Similarly, the mean of $COV[\phi]$, denoted as $\mu_{COV[\phi]}$, is assumed to be 0.14; and the coefficient of variation of $COV[\phi]$, denoted as $\mu_{COV[\phi]}$, is assumed to be 12%. The mean of $\rho_{c,\phi}$, denoted as $\mu_{\rho_{c,\phi}}$, is assumed to be −0.50; and the coefficient of variation of $\rho_{c,\phi}$, denoted as $\delta_{\rho_{c,\phi}}$, is assumed to be 25%. Finally, the mean of ρ_{z,i_w}, denoted as $\mu_{\rho_{z,i_w}}$, is assumed to be −0.50; and the coefficient of variation of ρ_{z,i_w}, denoted as $\delta_{\rho_{z,i_w}}$, is assumed to be 25%.

It should be noted that in a robust design, these parameter statistics should be estimated as accurately as possible based on available data and/or published statistics; however, the effect of the uncertainty of these estimates can be minimized in the design process using the RGD approach.

5.2. Robust geotechnical design using NSGA-II

To demonstrate the significance of incorporating robustness in the reliability-based design, the Sau Mau Ping slope is re-analyzed considering possible variations in the estimated statistics of noise factors $\left(COV[c], COV[\phi], \rho_{c,\phi}\, and\, \rho_{b,i_w}\right)$ as noted previously. Following the RGD procedure outlined previously, the mean and standard deviation of the computed failure probability, denoted as μ_p and σ_p, can be obtained for all designs in the design space using PEM. For illustration purposes, the mean and standard deviation of the failure probability are only presented for the designs with H = 50 m, 52 m, …, 60 m as shown in Figs. 5 and 6, respectively.

Table 3
Failure probability of a given design (H = 54.8 m, θ = 50.0°) under various COV assumptions of rock properties.

$COV[c]$	$COV[\phi]$	H (m)	θ (°)	p_f
0.10	0.10	54.8	50.0	8.68E-04
0.10	0.14	54.8	50.0	3.23E-03
0.10	0.20	54.8	50.0	1.23E-02
0.20	0.10	54.8	50.0	3.70E-03
0.20	0.14	54.8	50.0	6.04E-03
0.20	0.20	54.8	50.0	1.36E-02
0.30	0.10	54.8	50.0	1.33E-02
0.30	0.14	54.8	50.0	1.52E-02
0.30	0.20	54.8	50.0	2.22E-02

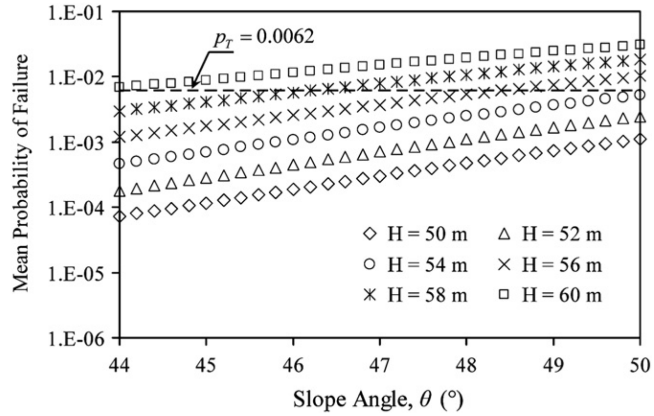

Fig. 5. Mean failure probabilities for selected designs obtained using PEM.

A multi-objective optimization can then be used to select the designs optimal to both cost and robustness while satisfying the safety constraint. The multi-objective optimization may be created accordingly:

Find $\mathbf{d} = [H, \theta]$
Subjected to : $H \in \{50\text{m}, 50.2\text{m}, 50.4\text{m}, \ldots, 60\text{m}\}$ and

$$\theta \in \{44°, 44.2°, 44.6°, \ldots, 50°\}\ \mu_p < p_T = 0.0062$$

Objectives : Minimizing the standard deviation failure probability $\left(\sigma_p\right)$,

NSGA-II (Deb et al., 2002) is used to perform the multi-objective optimization, in which the designs optimal to both objectives (cost and robustness) are searched iteratively in the discrete design space. With this algorithm, 89 non-dominated, optimal designs from the design space of 1581 possible designs are selected into the Pareto Front, as shown in Fig. 7. As can be seen, an apparent trade-off relationship exists for selecting a design from the Pareto Front. For example, in this case, greater design robustness can only be attained at the expense of a higher cost.

The Pareto Front is useful as a design aid for decision making especially when an acceptable cost or robustness level is specified. For example, when the maximum acceptable cost is set at 200 units, thirty-eight designs from the Pareto Front are deemed acceptable (see Figure 7). It should be noted that each point in Fig. 7 represents a unique design (in this example, it is defined by a pair of slope height and slope angle); for each design, however, only cost and standard deviation of the failure probability (σ_p) are shown in Fig. 7. The design with the lowest σ_p value in this cost range is the best design based on the robustness criterion, and in this scenario, it is a design

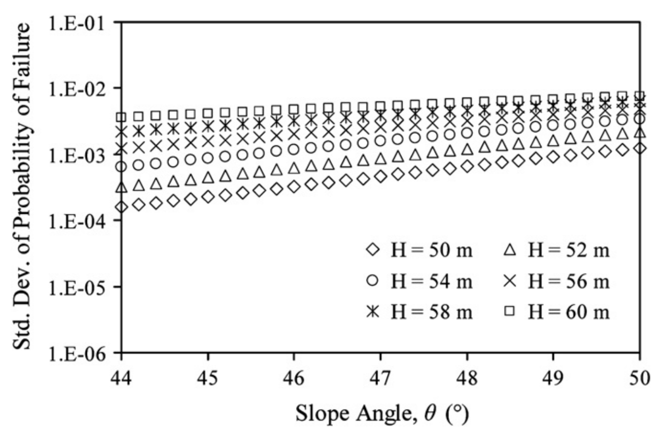

Fig. 6. Standard deviation of failure probabilities for selected designs using PEM.

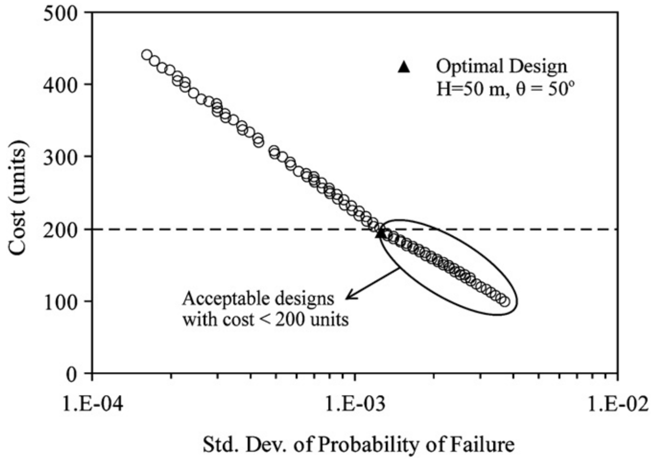

Fig. 7. Pareto Front obtained by NSGA-II and selection of final optimal design based on the acceptable cost level.

with $H = 50$ m and $\theta = 50°$. The optimal design of $H = 50$ m and $\theta = 50°$ has a cost of 195.1 units and a standard deviation of failure probability of 1.25×10^{-3}, which is denoted in Fig. 7 with the symbol of the filled triangle.

Furthermore, the feasibility robustness can also be used as a decision-making aid. The feasibility robustness index β_β computed for each of the 89 designs on the Pareto Front is shown in Fig. 8. As in Fig. 7, each point in Fig. 8 is a unique design, in which only cost and robustness are shown. As expected, the designs with higher feasibility robustness (β_β) cost more. When a target feasibility robustness level (β_β^T) is selected, the least-cost design that satisfies the feasibility robustness requirement can easily be identified, which renders a final design using the RGD approach. As an example, the final designs using the RGD approach are listed in Table 4 for the selected levels of target feasibility robustness. The feasibility robustness provides an easy-to-use quantitative measure for selecting the best design from the Pareto Front.

Finally, it should be noted that the outcome of a traditional reliability-based design, as listed in Table 2, is meaningful only if the statistical parameters of rock properties can be precisely defined. The dilemma of having to assume higher COVs of rock properties to ensure safety and having to reduce cost in a design is greatly eased using the RGD approach, in which the final design as those listed in Table 4, is systematically selected based on the feasibility robustness that has considered the cost, reliability, and robustness.

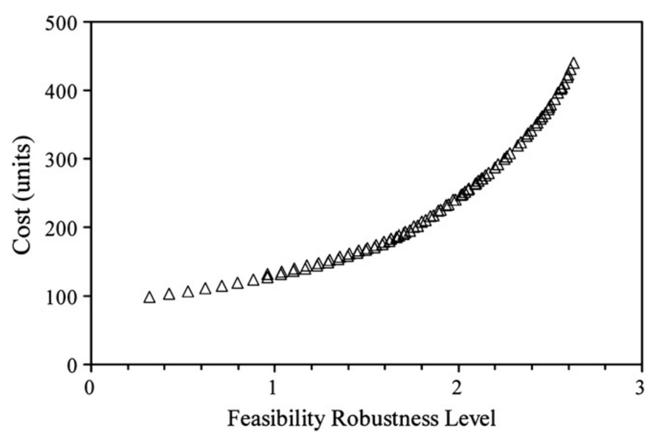

Fig. 8. Cost versus feasibility robustness for all designs on the Pareto Front.

Table 4
Selected final designs at various target feasibility robustness levels.

β_β^T	P_0	H (m)	θ (°)	Cost (units)
0.5	69.15%	54.0	50.0	107.0
1.0	84.13%	52.8	50.0	132.0
1.5	93.32%	51.4	49.8	170.6
2.0	97.72%	50.4	48.4	247.6
2.5	99.38%	50.0	45.4	378.9

6. Concluding remarks

In this paper, the authors present a reliability-based Robust Geotechnical Design (RGD) approach and demonstrate that approach using a rock slope design example. Rather than seeking to reduce the variation of noise factors, the RGD approach focuses on achieving an optimal design that is robust against variations in these noise factors.

The results of the example show that without considering design robustness, traditional reliability-based design methods may produce a least-cost design that was initially shown as adequate by meeting the failure probability requirement but later found inadequate because of an underestimation of the variation of noise factors. Since an underestimation of the variation of noise factors is not uncommon and the safety requirement cannot be comprised, it is desirable to have a design aid to assist in making a decision between the trade-offs of cost and robustness [recalling that higher robustness implies a greater certainty that the design remains adequate in the face of uncertainty].

By considering robustness as one additional design objective, the proposed RGD approach can be efficiently implemented as a multi-objective optimization problem. The results of the example show that the RGD approach can produce a Pareto Front, a set of non-dominated optimal designs that satisfy the safety (reliability) requirement. The results also show that a trade-off relationship between cost and feasibility robustness can be established from the Pareto Front for the design of rock slopes, which can then be used as a design aid in selecting the most suitable design.

The proposed reliability-based RGD approach is not a methodology to compete with traditional design approaches, but rather a complementary design strategy (Juang et al., 2012). With the RGD approach, the focus involves satisfying three design requirements, namely safety, cost, and robustness. In the example presented, safety is of paramount importance and treated as a constraint, while cost and robustness are the two objectives that are optimized in the optimization process. This paper is a first step in developing the RGD methodology. Further investigations by interested parties are encouraged to advance this design methodology.

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Appendix A. Deterministic model for rock slope analysis

The two-dimensional limit equilibrium model proposed by Hoek and Bray (1981) is adopted herein as the deterministic model for rock slope analysis with single failure mode. In reference to Fig. 3, the factor of safety is expressed as the ratio of the sum of all resisting forces to the sum of all driving forces (Hoek and Bray, 1981; Hoek, 2006):

$$FS = \frac{cA + [W(\cos\psi - \alpha \sin\psi) - U - V \sin\psi] \tan\phi}{W(\sin\psi + \alpha \cos\psi) + V \cos\psi} \quad (A-1)$$

where *FS* denotes the factor of safety for rock slope against sliding along slip surface; c is the cohesion of the rock discontinuity (or joint surface) (ton/m^2); A is the area of contact with slip surface (m^2); W is the weight of rock wedge resting on the failure surface (ton); ψ is the angle of failure surface measured from the horizontal plane (°); α is the gravitational acceleration coefficient defined by the ratio of horizontal to gravitational acceleration; U is the uplift force of water on the slip surface (ton); V is the horizontal force of water in tension crack (ton); ϕ is the friction angle of the joint surface (°).

The following intermediate terms for computing *FS* are obtained from basic slope geometry and rock properties (Hoek, 2006):

$$A = (H - z)/\sin\psi \quad (A-2)$$

$$W = 0.5\gamma H^2 \left\{ \left[1 - (z/H)^2\right] \cot\psi - \cot\theta \right\} \quad (A-3)$$

$$U = 0.5\gamma_w z_w A \quad (A-4)$$

$$V = 0.5\gamma_w z_w^2 \quad (A-5)$$

$$i_w = z_w/z \quad (A-6)$$

where H is the height of the overall slope or of each bench (m); z is the depth of tension crack (m); z_w is depth of water in the tension crack (m); θ is the overall slope angle measured from horizontal level (°); γ is the unit weight of rock (ton/m^3); γ_w is the unit weight of water (ton/m^3); and i_w is percentage of the tension crack depth filled with water.

References

Ang, A.H.-S., Tang, W.H., 1984. Probability concepts in engineering planning and design: decision, risk, and reliability, vol. 2. Wiley, New York.

Chen, W., Lewis, K., 1999. A robust design approach for achieving flexibility in multidisciplinary design. AIAA Journal 37 (8), 982–990.

Chen, W., Allen, J.K., Mistree, F., Tsui, K.-L., 1996. A procedure for robust design: minimizing variations caused by noise factors and control factors. Journal of Mechanical Design 118 (4), 478–485.

Ching, J., Phoon, K.K., Hu, Y.G., 2009. Efficient evaluation of reliability for slopes with circular slip surfaces using importance sampling. Journal of Geotechnical and Geoenvironmental Engineering 135 (6), 768–777.

Christian, J.T., Ladd, C.C., Baecher, G.B., 1994. Reliability applied to slope stability analysis. Journal of Geotechnical Engineering 120 (12), 2180–2207.

Deb, K., Pratap, A., Agarwal, S., Meyarivan, T., 2002. A fast and elitist multi-objective genetic algorithm: NSGA-II. IEEE Transactions on Evolutionary Computation 6 (2), 182–197.

Duncan, J.M., 2000. Factors of safety and reliability in geotechnical engineering. Journal of Geotechnical and Geoenvironmental Engineering 126 (4), 307–316.

Duzgun, H.S.B., Bhasin, R.K., 2009. Probabilistic stability evaluation of Oppstadhornet Rock Slope Norway. Rock Mechanics and Rock Engineering 42 (5), 729–749.

Duzgun, H.S.B., Pasamehmetoglu, A.G., Yucemen, M.S., 1995. Plane failure analysis of rock slopes: a reliability approach. International Journal of Surface Mining, Reclamation, and Environment 9 (1), 1–6.

Duzgun, H.S.B., Yucemen, M.S., Karpuz, C., 2003. A methodology for reliability-based design of rock slopes. Rock Mechanics and Rock Engineering 36 (2), 95–120.

Ghosh, A., Dehuri, S., 2004. Evolutionary algorithms for multi-criterion optimization: a survey. International Journal of Computing and Information Sciences 2 (1), 38–57.

Hoek, E., 2006. Practical rock engineering. Chapter 7: A Slope Stability Problem in Hong Kong; and Chapter 8: Factor or Safety and Probability of Failure. (In: http://www.rocscience.com/hoek/PracticalRockEngineering.asp.).

Hoek, E., Bray, J., 1981. Rock Slope Engineering, 3rd ed. Institution of Mining and Metallurgy, London.

Jamali, A., Hajiloo, A., Nariman-Zadeh, N., 2010. Reliability based robust Pareto design of linear state feedback controllers using a multi-objective uniform-diversity genetic algorithm (MUGA). Expert Systems with Applications 37 (1), 401–413.

Juang, C.H., Jhi, Y.Y., Lee, D.H., 1998. Stability analysis of existing slopes considering uncertainty. Engineering Geology 49 (2), 111–122.

Juang, C.H., Wang, L., 2013. Reliability-based robust geotechnical design of spread foundations using multi-objective genetic algorithm. Computers and Geotechnics 48, 96–106.

Juang, C.H., Wang, L., Atamturktur, S., Luo, Z., 2012. Reliability-based robust and optimal design of shallow foundations in cohensionless soil in the face of uncertainty. Journal of GeoEngineering 7 (3), 43–55.

Lagaros, N.D., Papadrakakis, M., 2007. Robust seismic design optimization of steel structures. Structural and Multidisciplinary Optimization 33 (6), 457–469.

Lee, Y.F., Chi, Y.Y., Juang, C.H., Lee, D.H., 2012. Reliability analysis of rock wedge stability — a Knowledge-based Clustered Partitioning (KCP) approach. Journal of Geotechnical and Geoenvironmental Engineering 138 (6), 700–708.

Li, D.Q., Chen, Y.F., Lu, W.B., Zhou, C.B., 2011. Stochastic response surface method for reliability analysis of rock slopes involving correlated non-normal variables. Computers and Geotechnics 38 (1), 58–68.

Low, B.K., 1997. Reliability analysis of rock wedges. Journal of Geotechnical and Geoenvironmental Engineering 123 (6), 498–505.

Low, B.K., 2007. Reliability analysis of rock slopes involving correlated nonnormals. International Journal of Rock Mechanics and Mining Sciences 44 (6), 922–935.

Low, B.K., 2008. Efficient probabilistic algorithm illustrated for a rock slope. Rock Mechanics and Rock Engineering 41 (5), 715–734.

Low, B.K., Einstein, H.H., 1992. Simplified reliability analysis for wedge mechanisms in rock slopes. Proc., 6th Int. Symp. on Landslides. Balkema, Rotterdam, The Netherlands, pp. 499–507.

Oka, Y., Wu, T.H., 1990. System reliability on slope stability. Journal of Geotechnical Engineering 116 (8), 1185–1189.

Park, H.J., West, T.R., 2001. Development of a probabilistic approach for rock wedge failure. Engineering Geology 59 (3–4), 233–251.

Park, H.J., West, T.R., Woo, I., 2005. Probabilistic analysis of rock slope stability and random properties of discontinuity parameters, Interstate Highway 40. Engineering Geology 79 (3–4), 230–250.

Park, H.J., Um, J.G., Woo, I., Kim, J.W., 2012. Application of fuzzy set theory to evaluate the probability of failure in rock slopes. Engineering Geology 125, 92–101.

Parkinson, A., Sorensen, C., Pourhassan, N., 1993. A general approach for robust optimal design. Journal of Mechanical Design 115 (1), 74–80.

Sandgren, E., Cameron, T.M., 2002. Robust design optimization of structures through consideration of variation. Computers and Structures 80 (20–21), 1605–1613.

Taguchi, G., 1986. Introduction to quality engineering: designing quality into products and processes. Quality Resources, New York.

Vanmarcke, E.H., 1980. Probabilistic stability analysis of earth slopes. Engineering Geology 16 (1–2), 29–50.

Wu, T.H., Tang, W.H., Sangrey, D.A., Baecher, G.B., 1989. Reliability of offshore foundations—State-of-the-art. Journal of Geotechnical Engineering 115 (2), 157–178.

Zhang, J., Zhang, L.M., Tang, W.H., 2011. Reliability-based optimization of geotechnical systems. Journal of Geotechnical and Geoenvironmental Engineering 137 (12), 1211–1221.

Zhao, Y.G., Ono, T., 2000. New point estimates for probability moments. Journal of Engineering Mechanics 126 (4), 433–436.