Bootstrapping for Characterizing the Effect of Uncertainty in Sample Statistics for Braced Excavations

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Abstract: A simple procedure for assessing the probability of serviceability failure in a braced excavation involving bootstrapping to characterize the effect of uncertainty in sample statistics is presented. Here, the failure is defined when an excavation system's response in terms of the maximum wall deflection or ground settlement that exceeds the limiting value specified by the client or in an applicable code. The analysis for the probability of failure (or probability of exceedance) necessitates an evaluation of the means and SDs of critical soil parameters. In geotechnical practice, these means and SDs are often estimated from a very limited data set, which can lead to uncertainty in the derived sample statistics. Thus, in this study bootstrapping is used to characterize the uncertainty or variation of sample statistics and its effect on the failure probability. Through the bootstrapping analysis, the probability of exceedance can be presented as a confidence interval instead of a single, fixed probability. The information gained should enable the engineer to make a more rational assessment of the risk of serviceability failure in a braced excavation. The study points to the potential of the bootstrap method in coping with the problem of having to evaluate failure probability with uncertain sample statistics. **DOI: 10.1061/(ASCE)GT.1943-5606.0000734.** © *2013 American Society of Civil Engineers.*

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Introduction

One of the main concerns in a braced excavation in an urban area is the risk of damage to adjacent infrastructures caused by the excavation-induced wall deflections and ground movements. Damage to the adjacent infrastructures caused by ground movements in a braced excavation is referred to herein as serviceability failure. In many excavation projects, the owners or regulatory agencies establish the limiting wall and/or ground responses as a means of preventing excavation failure and damage to adjacent infrastructures. Table 1 shows an example of such limiting response criteria from China [Professional Standards Compilation Group (PSCG) 2000]. Similar criteria are also reported for excavation projects in Taiwan and Japan (e.g., Yen and Chang 1991; Ou 2006). However, from the design point of view, a binary assessment of whether the response will exceed the specified limiting value is often not practical for two reasons. First, it is generally difficult to compute accurately the excavation-induced ground responses (Kung et al. 2007). In other words, model uncertainty (or bias) exists in the methods used for predicting these responses. Second, uncertainty often exists in the input parameters, which can reduce confidence in the computed responses. Thus, it is more meaningful to assess the serviceability failure in terms of probability, taking into account both model uncertainty and parameter uncertainty. In the context of braced excavation, this probability is referred to as the probability of exceedance (i.e., the probability of exceeding a specified limiting deformation value). This probability may be used to assess the risk of serviceability failure, and through which a satisfactory braced excavation design may be achieved. The key in this design process is a procedure to accurately compute the probability of exceedance under the influence of both the model and parameter uncertainties.

An accurate calculation of the probability of exceedance requires an accurate analysis model for the wall and ground responses (which can lead to a well-characterized and proper limit state or performance function) and an accurate statistical characterization of the input soil parameters. In this paper, the Kung-Juang-Hsiao-Hashash (KJHH) model (Kung et al. 2007), a semiempirical model that was generated with hundreds of finite-element (FE) simulations and validated with well-documented case histories, is used to compute the excavation-induced wall and ground responses in an excavation in clays. The KJHH model is well characterized, and thus the focus of this paper is on parameter uncertainty and its effect on the computed probability of exceedance.

The sources of parameter uncertainty include inadequate site investigation, measurement errors, as well as inherent and spatial variability of soil. Statistical methods have long been used for characterization of parameter uncertainty in geotechnical engineering (e.g., Harr 1987; Ang and Tang 2007; Fenton and Griffiths 2008). Reliability analysis offers a means to explicitly account for the uncertainty in soil parameters (Harr 1987; Ang and Tang 2007). Previous studies on reliability analysis of excavation-induced deformation showed that uncertainties in soil parameters can have a significant effect on the probability of serviceability failure in a braced excavation (e.g., Hsiao et al. 2008).

Many reliability analyses are based on sample statistics of soil parameters that are derived from very limited data. These sample

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Table 1. Criteria for Excavation Protection Levels in Shanghai, China (Source Data from PSCG 2000)

Excavation protection level	Infrastructures and facilities to be protected at the site	Requirement: Maximum wall deflection	Requirement: Maximum ground-surface settlement
I	Metro lines and important facilities such as gas mains and water drains exist within a distance of $0.7H_f$ from the excavation; safety has to be ensured.	$\leq 0.14\% H_f$	$\leq 0.1\% H_f$
II	Important infrastructures or facilities such as gas mains and water drains exist within a distance of $(1-2)H_f$ from the excavation.	$\leq 0.3\% H_f$	$\leq 0.2\% H_f$
III	No important infrastructures or facilities exist within a distance of $2H_f$ from the excavation.	$\leq 0.7\% H_f$	$\leq 0.5\% H_f$

Note: H_f = final excavation depth.

statistics are often assumed, out of necessity, to be the population statistics. Thus, the accuracy of a reliability analysis is affected by (1) the accuracy of the sample statistics (including the mean and SD in most applications) of the uncertain soil parameters, and also (2) the type of probability distribution of these parameters, which often has to be assumed (Schweiger and Peschl 2005). However, it is noted that most soil parameters can be adequately modeled with a lognormal distribution (Phoon and Kulhawy 1999) or truncated normal distribution (Most and Knabe 2010). Thus, the focus of this paper is to examine the effect of the uncertain sample statistics on the result of the reliability analysis.

Because of budget constraints, the geotechnical engineer often has to derive sample statistics from a small sample (i.e., a small data set), which can lead to uncertainty in these statistics. Thus, the effect of this uncertainty on the probability of failure should be examined. The effect of uncertain statistics of model input parameters on the failure probability has been reported in the literature (e.g., Ang and De Leon 2005; Most and Knabe 2010). Several approaches are available to consider the effect of uncertainty in the statistics of these input parameters on the failure probability. One approach is to model the statistical moments as random variables (Ditlevsen and Madsen 1996). The uncertain statistical moments may also be modeled with fuzzy numbers (Möller and Beer 2004). Other approaches include the resampling techniques such as jackknifing (Quenouille 1956) and bootstrapping (Efron 1979). Among the available approaches, the bootstrap method exhibits simplicity in engineering applications. The bootstrap method is especially advantageous for the situation when the distributions of statistics of interest are unknown and/or the sample size is insufficient (Adèr et al. 2008). Bootstrapping has also shown its versatility to combine with other approaches such as the maximum entropy method (Most 2011).

In this paper, this effect using the bootstrapping technique is investigated (Efron 1979). To demonstrate this technique, a case study investigating the effect of uncertain sample statistics of soil parameters on the computed probability of serviceability failure in a braced excavation is presented. Unlike traditional reliability analysis, where a single fixed probability is obtained with a set of fixed sample statistics, reliability analysis with the bootstrap method explicitly considers the uncertainty in the derived sample statistics. The latter approach enables an interval estimate of the failure probability at a specified confidence level. The information gained through this approach is in the form of a confidence interval and can enable the engineer to make a more informed design decision.

Performance Function for Probability of Exceedance

As noted previously, the KJHH model (Kung et al. 2007) is employed herein to compute the maximum wall deflection (δ_{hm}) and maximum ground-surface settlement (δ_{vm}) in a braced excavation in clays. This model was derived based on multivariate nonlinear regression analysis with data derived from 33 excavation histories and hundreds of numerical simulations using the FE method (FEM). This model consists of a set of equations that collectively can be used to compute δ_{hm} and δ_{vm} based on the following parameters: excavation depth (H); excavation width (B); the system stiffness [$S = EI/\gamma_w h_{\text{avg}}^4$ as defined in Clough and O'Rourke (1990), where E is the Young's modulus of the wall material, I is the moment of inertia of the wall section, γ_w is the unit weight of water, and h_{avg} is the average support spacing]; the normalized clay layer thickness [$\Sigma H_{\text{clay}}/H_{\text{wall}}$, where H_{wall} is the wall length and ΣH_{clay} is the total thickness of all clay layers within the wall length (in a clay-only deposit, this ratio is equal to 1)]; the normalized undrained shear strength (s_u/σ'_v); and the normalized initial tangent modulus (E_i/σ'_v). Here, σ'_v denotes the vertical effective stress.

The maximum lateral wall deflection (δ_{hm}) is determined as

$$\delta_{hm}(mm) = a_0 + a_1Z_1 + a_2Z_2 + a_3Z_3 + a_4Z_4 + a_5Z_5 + a_6Z_1Z_2 + a_7Z_1Z_3 + a_8Z_1Z_5 \quad (1)$$

where $Z_1 = t(H)$, $Z_2 = t[\ln(EI/\gamma_w h_{\text{avg}}^4)]$, $Z_3 = t(B/2)$, $Z_4 = t(s_u/\sigma'_v)$, and $Z_5 = t(E_i/\sigma'_v)$. The mean and SD of the model uncertainty or bias factor (BF) of this model are estimated to be at 1.0 and 0.25, respectively (Kung et al. 2007). The coefficients for Eq. (1) determined through the least-square regression are as follows: $a_0 = -13.41973$, $a_1 = -0.49351$, $a_2 = -0.09872$, $a_3 = .06025$, $a_4 = 0.23766$, $a_5 = -0.15406$, $a_6 = 0.00093$, $a_7 = 0.00285$, and $a_8 = 0.00198$. Variables $Z_i(i = 1, 5)$ are obtained through the following transformation:

$$Z_i = t(z_i) = b_1z_i^2 + b_2z_i + b_3 \quad (2)$$

where $z_i(i = 1, 5)$ = corresponding input parameter. The coefficients for the transformations in this model are summarized in Table 2.

To compute the maximum ground-surface settlement (δ_{vm}), the KJHH model employs a deformation ratio R defined as follows:

$$R = c_0 + c_1Y_1 + c_2Y_2 + c_3Y_3 + c_4Y_1Y_2 + c_5Y_1Y_3 + c_6Y_2Y_3 + c_7Y_3^3 + c_8Y_1Y_2Y_3 \quad (3)$$

where $Y_1 = \sum H_{\text{clay}}/H_{\text{wall}}$, $Y_2 = s_u/\sigma'_v$, and $Y_3 = E_i/1000\sigma'_v$, and the coefficients for Eq. (3) determined through the least-square regression are as follows: $c_0 = 4.55622$; $c_1 = -3.40151$; $c_2 = -7.37697$; $c_3 = -4.99407$; $c_4 = 7.14106$; $c_5 = 4.60055$; $c_6 = 8.74863$; $c_7 = 0.38092$, and $c_8 = -10.58958$. Then, the maximum ground-surface settlement (δ_{vm}) is obtained as

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Table 2. Coefficients for Transformation of Input Variables (Kung et al. 2007)

	Coefficients in Eq. (2)		
Variables x	b_1	b_2	b_3
H (m)	−0.4	24	−50
$\ln(EI/\gamma_w h_{\mathrm{avg}}^4)$	11.5	−295	2,000
$B/2$(m)	−0.04	4	90
s_u/σ_v'	3,225	−2,882	730
E_i/σ_v'	0.00041	−1	500

$$\delta_{vm} = R \cdot \delta_{hm} \tag{4}$$

The mean and SD of the model uncertainty or BF of this model are estimated to be at 1.0 and 0.34, respectively (Kung et al. 2007).

With the previous set of equations [Eqs. (1)–(4)], the maximum wall deflection (δ_{hm}) and maximum ground-surface settlement (δ_{vm}) in an excavation in clays can be computed. These equations provide a fairly accurate estimate of the wall and ground responses. If so desired, more sophisticated methods such as well-calibrated FEM models can be used for determining the wall and ground responses. Nevertheless, the simplified approach using the previous set of equations allows a performance function for evaluating the probability of exceeding the specified limiting wall and ground responses to be easily set up as follows:

$$G(x) = y_{\mathrm{lim}} - y = 0 \tag{5}$$

where $G(x)$ = limit state or performance function, x = vector of parameters, y = response (the maximum wall deflection or ground settlement) computed with the KJHH model, and y_{lim} = specified limiting response.

Point Estimate Method for Uncertainty Propagation and Probability of Exceedance

For a given braced excavation in clay, Hsiao et al. (2008) found that among all input parameters of the KJHH model, the wall and ground responses are most sensitive to the variations in s_u/σ_v' and E_i/σ_v'. Thus, for a given design of a braced excavation, all parameters except s_u/σ_v' and E_i/σ_v' may be considered as fixed variables, and as such the wall and ground responses become only a function of these two soil parameters and the BF. For simplicity, the two random variables, s_u/σ_v' and E_i/σ_v', are denoted as x_1 and x_2, respectively; meanwhile, the model BF of the KJHH model is denoted as x_3. Thus, response y, either as maximum wall deflection δ_{hm} or maximum ground settlement δ_{vm}, can be written as $y = f(x_1, x_2, x_3)$. Many different methods may be used to compute the mean and SD of response y. In this paper, the point estimate method (PEM) (see Rosenblueth 1975; Harr 1987; Christian and Baecher 1999) is chosen for this task because of its simplicity and ease of implementation in *Excel*. With the PEM, the mth moment for y can be readily expressed as (Harr 1987)

$$\begin{aligned} E[y^m] = {} & p_{+++}y_{+++}^m + p_{++-}y_{++-}^m + p_{+-+}y_{+-+}^m \\ & + p_{+--}y_{+--}^m + p_{-++}y_{-++}^m + p_{-+-}y_{-+-}^m \\ & + p_{--+}y_{--+}^m + p_{---}y_{---}^m \end{aligned} \tag{6}$$

where $y_{\pm\pm\pm}$ = values of δ_{hm} (or δ_{vm}) evaluated at points $x_{\pm\pm\pm}$, and can be computed with the following equation:

$$y_{\pm\pm\pm} = f\left(\bar{x}_1 \pm \sigma[x_1], \bar{x}_2 \pm \sigma[x_2], \bar{x}_3 \pm \sigma[x_3]\right) \tag{7}$$

For instance, y_{++-} is the value of δ_{hm} (or δ_{vm}) evaluated at a point x_{++-}, and thus $y_{++-} = f(\bar{x}_1 + \sigma[x_1], \bar{x}_2 + \sigma[x_2], \bar{x}_3 - \sigma[x_3])$ in which $\bar{x}_1$, $\bar{x}_2$, and $\bar{x}_3$ = mean values of random variables x_1, x_2, and x_3, respectively; and $\sigma[x_1]$, $\sigma[x_2]$, and $\sigma[x_3]$ = SDs of random variables x_1, x_2, and x_3, respectively. The terms $p_{\pm\pm\pm}$ are the weighting factors and can be computed with the following equation:

$$p_{\pm\pm\pm} = \frac{1}{8}\left(1 \pm \rho_{x_1,x_2} \pm \rho_{x_1,x_3} \pm \rho_{x_2,x_3}\right) \tag{8}$$

where $\rho_{i,j}$ = correlation coefficient between random variables i and j, and the sign preceding $\rho_{i,j}$ is determined by the sign of the multiplication of i and j. In this paper, no correlation between the BF and each soil parameter is assumed, while soil parameters s_u/σ_v' and E_i/σ_v' are positively correlated (Hsiao et al. 2008).

Once the first and second moments are obtained, mean μ_y and SD σ_y of response y can be computed as follows (Ang and Tang 2007):

$$\mu_y = E[y] \tag{9}$$

$$\sigma_y = \sqrt{E[y^2] - (E[y])^2} \tag{10}$$

Although both y and y_{lim} in Eq. (5) can be treated as random variables, in this paper y_{lim} is treated as a constant because it is almost always specified as a constant in an applicable design code (for example, see Table 1). Thus, the reliability index, β, can be computed as follows:

$$\beta = \frac{y_{\mathrm{lim}} - \mu_y}{\sigma_y} \tag{11}$$

Assuming that y is normally distributed (the effect of this assumption is examined subsequently), the probability of exceeding the limiting response (p_f) can be computed as

$$p_f = P[y > y_{\mathrm{lim}}] = 1 - \Phi(\beta) \tag{12}$$

where Φ = cumulative standard normal distribution, and in *Excel* it is implemented with a built-in function, NORMSDIST.

Variation of Sample Statistics Determined by Bootstrapping

Because of the uncertainties in the adopted analysis model and the input parameters, the answer to the question of whether the maximum wall or ground response will exceed the specified limiting value in a given excavation design cannot be expressed with certainty as a simple yes or no. A logical, and more appropriate, answer would be a probability of exceedance, which gauges the likelihood of exceeding the specified limiting values. In other words, the uncertainty leads us to the use of probability as an answer. Of course, the probability is an estimate of the likelihood before the event; after the event, it can only be equal to either 0 or 1.

In the procedure and formulation presented previously, knowledge of the mean and SD of the two key soil parameters, s_u/σ_v' and E_i/σ_v', is needed. Because geotechnical parameters, such as s_u/σ_v' and E_i/σ_v', are typically evaluated with a small sample size, the derived sample statistics (such as the mean and SD) are subjected to error. Therefore, variation from the true mean and SD (i.e., those of

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the population) is expected. As a result of this inevitable uncertainty in sample statistics, the probability of exceedance evaluated with the previous procedure can no longer be adequately expressed as a single, fixed value. By considering the effect of the variation of the derived sample statistics (mean and SD), an estimate of the variation of the computed probability of exceedance (through an evaluation of reliability index), in the form of a confidence interval, can be made. To derive the confidence interval of the probability of exceedance, it is first necessary to estimate the variation of the derived sample statistics.

Parametric methods are available for assessing the uncertainties in the sample mean and sample statistics (e.g., Ang and Tang 2007). However, in this paper a nonparametric approach is employed to assess the effect of uncertain sample statistics on the computed probability of exceedance. The nonparametric method generally requires fewer assumptions than the parametric method (Corder and Foreman 2009). The nonparametric bootstrap method is a straightforward approach to estimating the standard error and the confidence interval, which is especially useful for model parameters with an unknown distribution and small sample size (for example, fewer than 30 data points). The bootstrap method is also useful for estimating the distribution of sample statistics, and the correlation between soil parameters can easily be handled.

Bootstrapping (Efron 1979) is a technique that can be used to estimate the variation of the sample statistics derived from a small sample. To begin with, the original set of observations (e.g., soil test data) are denoted as $X_1, X_2, \ldots, X_n$ and a bootstrap sample set B_j with n_b samples is denoted as $B_{1,j}, B_{2,j}, \ldots, B_{n_b,j}$. Generally, in the bootstrapping approach the size of a resample, n_b in a single resampling is set to the original sample size, n (Efron and Tibshirani 1993). If n_b is set to be smaller or greater than n, the sample statistics may be overestimated or underestimated (Johnson 2001). Then, a bootstrap sample set is constructed by random resampling with replacement from the original observations, as illustrated in Fig. 1.

Using the small-strain triaxial test data shown in Table 3 as an example, the bootstrapping procedure is further described as follows. Table 3 consists of a sample of 17 pairs of s_u/σ'_v and E_i/σ'_v, where each pair (data point) is denoted as $X_n = [(s_u/\sigma'_v)_n, (E_i/\sigma'_v)_n]$ where $n = 1, 17$. Bootstrapping begins with random sampling of a data point (each pair) with replacement as shown in Fig. 1 (where only five data points are shown to save space). The correlation structure is maintained because the sampling is carried out in pairs (i.e., each time a pair is sampled with replacement from a set of 17 pairs).

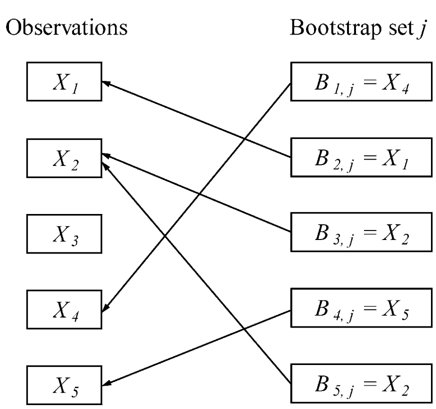

Fig. 1. Generation of one bootstrap sample from the original observations through random sampling with replacement (adapted from Most and Knabe 2010)

With the constructed resampling set, the sample statistics of concern (e.g., mean value and variance) can be obtained as follows:

$$\overline{B}_{n,j} = \frac{1}{n}\sum_{i=1}^{n} B_{i,j} \tag{13}$$

$$S_{n,j}^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(B_{i,j} - \overline{B}_{n,j}\right)^2 \tag{14}$$

The procedure for resampling described previously is repeated many times and the estimated mean and variance are calculated for each bootstrap sample. To this end, each statistic investigated can be estimated using its mean value, variance, and histogram. Thus, the bootstrap mean and variance estimates of the sample mean value, M_n, can be expressed as follows:

$$M_{n,\text{mean}} \approx \frac{1}{n_s}\sum_{j=1}^{n_s} \overline{B}_{n,j} \tag{15}$$

$$\sigma_{M_n}^2 \approx \frac{1}{n_s - 1}\sum_{j=1}^{n_s}\left(\overline{B}_{n,j} - M_{n,\text{mean}}\right)^2 \tag{16}$$

Similarly, the bootstrap mean and variance estimates of the sample SD S_n can be expressed as follows:

$$S_{n,\text{mean}} \approx \frac{1}{n_s}\sum_{j=1}^{n_s} S_{n,j} \tag{17}$$

$$\sigma_{S_n}^2 \approx \frac{1}{n_s - 1}\sum_{j=1}^{n_s}\left(S_{n,j} - S_{n,\text{mean}}\right)^2 \tag{18}$$

in which n_s = number of bootstrap sets and is generally chosen to be very large (e.g., 10^4) to obtain converged results in the statistical analysis (Most and Knabe 2010). With the bootstrap method, variations of the sample statistics of the soil parameters can be estimated, and their effects on the computed probability of

Table 3. Small-Strain Triaxial Test Results for Taipei Clay (Source Data from Kung 2003)

Test number	Type	s_u/σ'_v	E_i/σ'_v
1	Reconstituted	0.30	735.0
2	Reconstituted	0.31	606.7
3	Reconstituted	0.30	531.0
4	Reconstituted	0.29	652.5
5	Reconstituted	0.31	573.5
6	Reconstituted	0.33	528.0
7	Reconstituted	0.28	638.1
8	Reconstituted	0.32	501.4
9	Reconstituted	0.31	542.5
10	Undisturbed	0.357	686.2
11	Undisturbed	0.23	404.6
12	Undisturbed	0.37	822.1
13	Undisturbed	0.31	617.2
14	Undisturbed	0.35	765.5
15	Undisturbed	0.35	512.4
16	Undisturbed	0.318	448.1
17	Undisturbed	0.235	324.8
Mean		0.31	581.7
SD		0.04	129.8

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exceedance can be determined and expressed in terms of confidence intervals.

Case Study: Taipei National Enterprise Center Excavation

The Taipei National Enterprise Center (TNEC) case is a well-documented excavation case history (Ou et al. 1998). This excavation in soft-to-medium clays in the Taipei basin was completed in seven stages with the support of steel struts and floor slabs. The excavation width was 41.2 m, the final excavation depth was 19.7 m, and the length of diaphragm wall was 35 m. The soil profile and excavation depth of each stage are depicted in Fig. 2.

Reliability Analysis Based on the KJHH Model

The procedure described previously, formulated with Eqs. (1)–(12), is readily applicable for computing the probability of serviceability failure (p_f), defined herein as the probability of exceeding a specified limiting wall deflection or ground settlement. This probability of exceedance can be obtained once the mean and SD of the response are determined (i.e., maximum wall deflection and ground settlement). Thus, the key step in this solution process is to determine the mean and SD of the response given the uncertain soil parameters. Assuming that the mean values of s_u/σ'_v and E_i/σ'_v, denoted as μ_S and μ_E, respectively, and the SD of s_u/σ'_v and E_i/σ'_v, denoted as σ_S and σ_E, respectively, are available, the mean value and SD of maximum wall deflection δ_{hm} (or maximum ground settlement δ_{vm}) can be determined with the PEM approach.

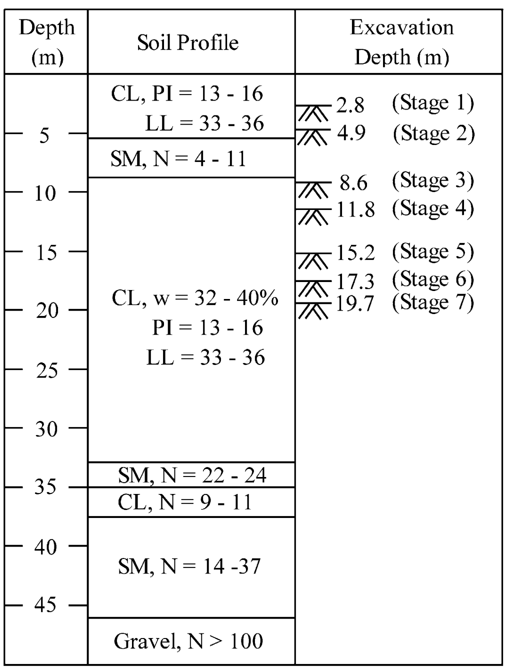

Fig. 2. Soil profile and excavation depth of the TNEC case (adapted from Kung et al. 2007)

For the TNEC case, the sample mean and sample SD of s_u/σ'_v and E_i/σ'_v are given in Table 3, based on the 17 small-strain triaxial test data on the reconstituted and undisturbed clay samples reported by Kung (2003). Taking $\mu_S = 0.31$, $\sigma_S = 0.04$, $\mu_E = 581.7$, and $\sigma_E = 129.8$, the mean of δ_{vm}, denoted as $\mu[\delta_{hm}]$, is computed to be $\mu[\delta_{hm}] = 108.8$ mm, and the SD of δ_{hm}, denoted as $\sigma[\delta_{hm}]$, is computed to be $\sigma[\delta_{hm}] = 38.5$ mm. If the limiting wall deflection is taken at $0.7\%H_f$ (PSCG 2000), where H_f is the final excavation depth ($H_f = 19.7$ m, in this case), the probability of exceeding this limiting wall deflection for the final excavation stage can be computed with Eq. (12), which yields $p_f = 0.22$. Similarly, the probability of exceeding the specified limiting ground settlement is determined to be $p_f = 0.24$. These results of the PEM analysis for the probability of exceedance are summarized in Table 4.

Bootstrapping to Consider the Effect of the Variation of Sample Statistics

As an example, the 17 pairs of small-strain triaxial test data listed in Table 3 are considered (an analysis based on fewer data points is presented subsequently). Here, the s_u/σ'_v values range from 0.23 to 0.37, and the E_i/σ'_v values range from 324.8 to 822.1. Fig. 3 shows the histograms of s_u/σ'_v and E_i/σ'_v, respectively. These histograms do not seem to suggest a normal or lognormal distribution. However, no conclusion can be made regarding the distribution type because the sample size is rather small. For the same reason, there is also uncertainty regarding the derived mean and SD of s_u/σ'_v and E_i/σ'_v.

Thus, the bootstrapping analysis was performed to better characterize these soil parameters and their sample statistics. Fig. 4 shows 10,000 bootstrap simulations of the small-strain triaxial test results following the procedure described previously. Here, each bootstrap simulation (sample) yields a mean and a SD. Because s_u/σ'_v and E_i/σ'_v are determined from the same soil sample, they are treated as a pair of data when they are resampled. Fig. 5 shows the probability density of the correlation coefficients between s_u/σ'_v and E_i/σ'_v for the 10,000 bootstrap samples. The distribution of the correlation coefficient is heavily left skewed and almost all the values are positive, which indicates a positive correlation between s_u/σ'_v and E_i/σ'_v.

The selection of 10,000 bootstrap simulations was based on a sensitivity analysis. The statistical fluctuation of the bootstrapped mean and SD of s_u/σ'_v and E_i/σ'_v with the number of bootstrap sets n_s is shown in Fig. 6. As seen, the converged results of the estimated means and SDs are observed at 10,000 bootstrap simulations. With the 10,000 sets of the mean (μ_S) and SD (σ_S) of s_u/σ'_v being secured through bootstrapping (Fig. 4), the histograms of μ_S and σ_S can be

Table 4. Probability of Exceedance in the TNEC Excavation Using PEM and the Bootstrapping Method with 17 Data Points

Ground or wall response	Probability of exceedance[a]	Probability of exceedance[b] Mean	SD	95% confidence interval
Maximum lateral wall deflection (δ_{hm})	0.22	0.22	0.07	0.08–0.36
Maximum ground-surface settlement (δ_{vm})	0.24	0.23	0.08	0.07–0.39

Note: Level III requirements (PSCG 2000) are employed in the probabilistic analyses: the limiting wall deflection is taken at $0.7\%H_f$ and the limiting ground-surface settlement is taken at $0.5\%H_f$, in which H_f = final excavation depth (19.7 m).
[a]PEM results with the mean and SD of the soil parameters derived from the 17 data points.
[b]Based on the bootstrapping analysis.

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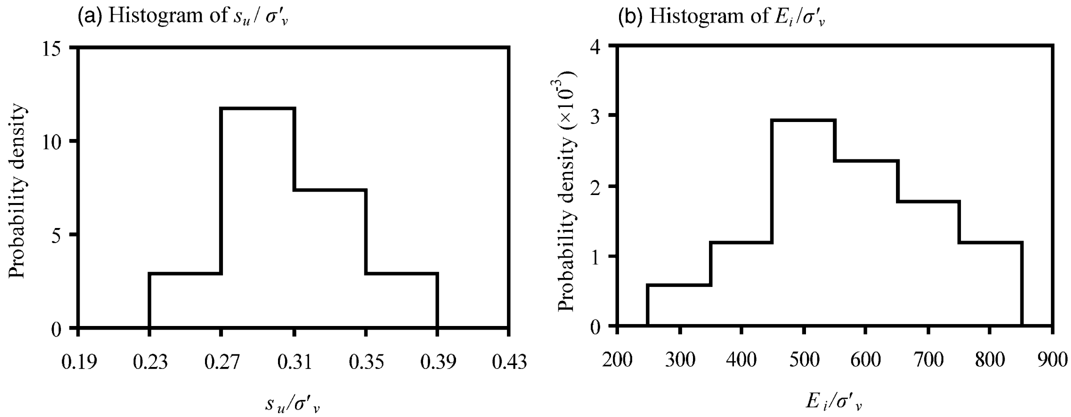

Fig. 3. Probability distribution of the small-strain triaxial test results listed in Table 2

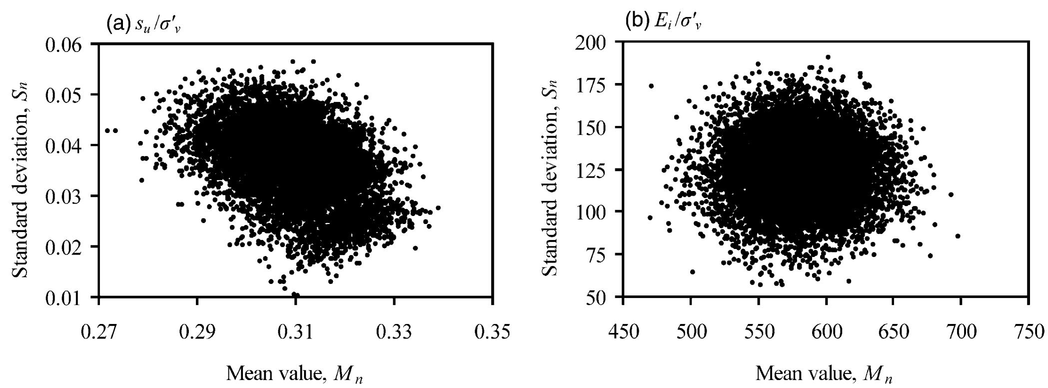

Fig. 4. Bootstrap samples generated from the original test data listed in Table 2

derived (Fig. 7). Both the mean (μ_S) and SD (σ_S) were found to essentially follow a normal distribution. The bootstrapped mean and SD are 0.310 and 0.036, respectively. These numbers match well with the sample mean and sample SD shown in Table 3, which indicates that the bootstrapped histograms reflect the main characteristics of the original data set (sample). Furthermore, additional knowledge was gained from bootstrapping; i.e., the SDs of the mean (μ_S) and SD (σ_S denoted as $\sigma[\mu_S]$ and $\sigma[\sigma_S]$, respectively, were obtained. Thus, an interval estimate of the mean and SD of this uncertain parameter at a specified confidence level (for example, 95%) is readily available. In other words, there is a 95% chance that the true values for the mean and SD will fall within the respective confidence intervals. Finally, an observation can be made regarding the uncertainty or variation of the mean (μ_S) and SD (σ_S) from Fig. 7. The variation [in terms of the coefficient of variation (COV)] of the mean (μ_S), as shown in Fig. 7(a), is much smaller than the variation of the SD (σ_S), as shown in Fig. 7(b). This confirms the expectation that the variation in the SD as a result of the small sample size is greater than the variation in the mean.

Similarly, Fig. 8 shows the histograms of the mean (μ_E) and SD (σ_E) of E_i/σ'_v, respectively; both the mean (μ_E) and SD (σ_E) essentially follow a normal distribution. The bootstrap mean and SD of E_i/σ'_v are 581.9 and 124.3, respectively, which match well with the sample statistics shown in Table 3. Again, this indicates that the bootstrapped histograms reflect the main characteristics of the original data set. Furthermore, additional knowledge was gained from bootstrapping; i.e., the SDs of the mean (μ_E) and SD (σ_E) denoted as $\sigma[\mu_E]$ and $\sigma[\sigma_E]$, respectively, were obtained. Similarly,

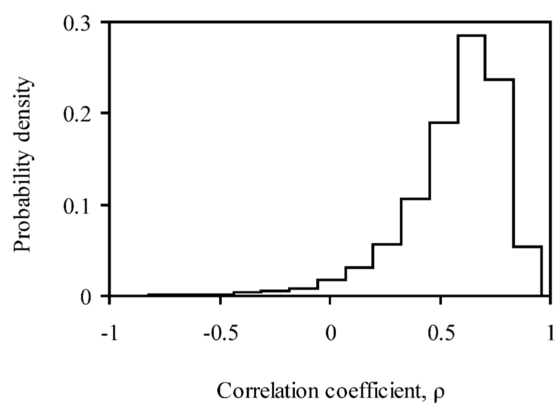

Fig. 5. Probability distribution of the correlation coefficients for the generated bootstrap samples in Fig. 4

the variation in terms of the COV of the mean (μ_E) [as shown in Fig. 8(a)] was much smaller than the variation of the SD (σ_E) [as shown in Fig. 8(b)]. Again, this confirms the expectation that the variation in the SD as a result of the small sample size is greater than the variation in the mean.

The effect of the uncertainty or variation of the sample statistics on the computed probability of exceedance was investigated next. For each bootstrapped sample, the mean and SD of the two critical soil parameters, s_u/σ'_v and E_i/σ'_v, were determined. With the known

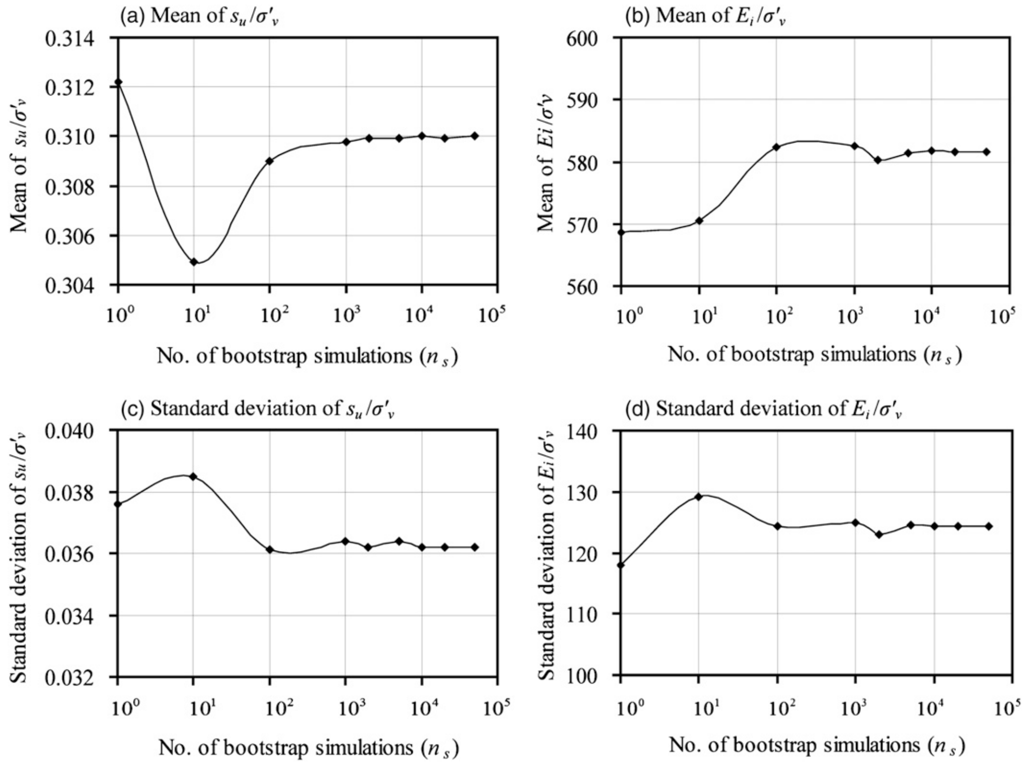

Fig. 6. Bootstrap mean and SD of s_u/σ'_v and E_i/σ'_v with respect to the number of bootstrap simulations

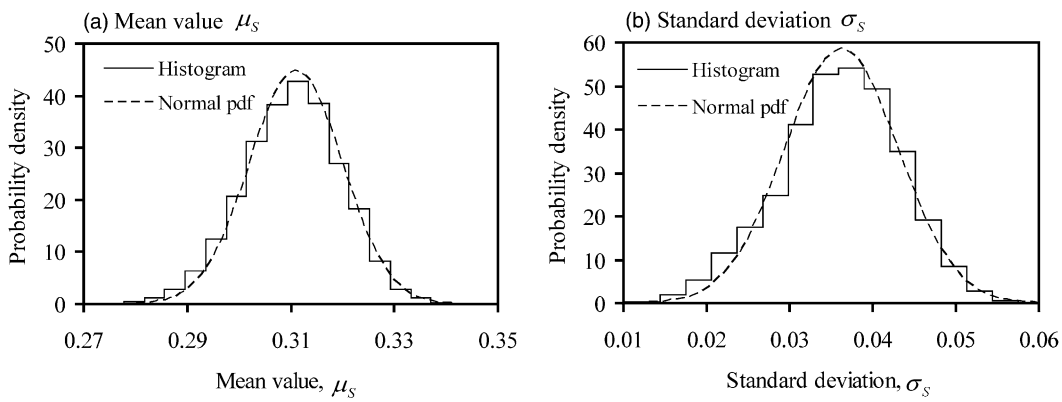

Fig. 7. Probability distribution of the mean value and SD of s_u/σ'_v

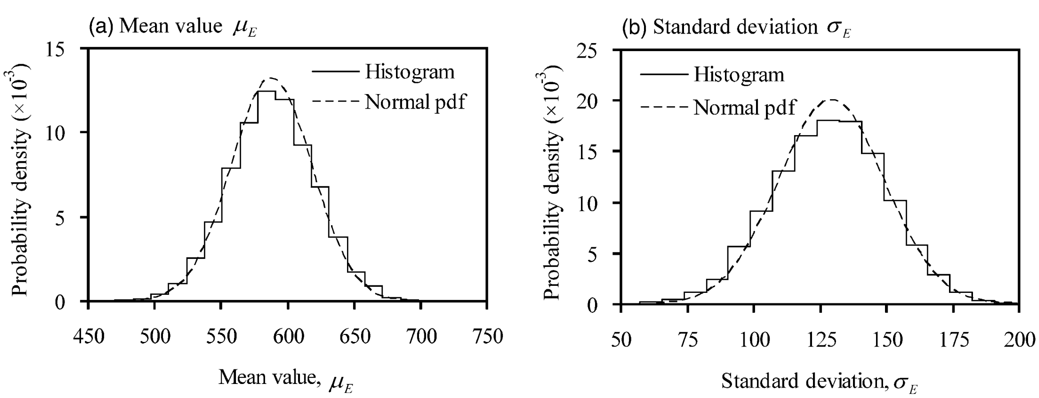

Fig. 8. Probability distribution of the mean value and SD of E_i/σ'_v

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means (μ_S and μ_E) and SDs (σ_S and σ_E), the PEM analysis can be conducted to determine the mean and SD of response y based on the KJHH model. This follows that the reliability index (β) and the probability of exceedance (p_f) can be determined with Eq. (11) and (12), respectively. For demonstration purposes, the limiting wall deflection and ground settlement were set at 0.7 and 0.5%H_f, respectively (where H_f is the final excavation depth). These limiting values are specified for Excavation Protection Level III in the design code adopted in Shanghai, China (PSCG 2000). Other criteria may be adopted depending on the local design codes or requirements of the client. Furthermore, also for demonstration purposes, the analysis was carried out at the final excavation stage in the TNEC case because this stage is often the most critical. For each sample, a reliability index (and thus a probability of exceedance) was obtained.

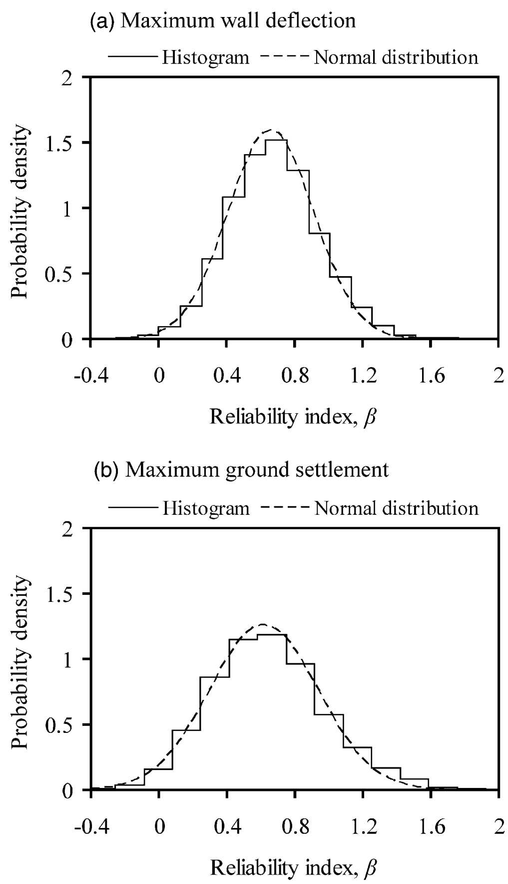

Fig. 9. Distribution of the reliability indices computed with the specified limiting: (a) wall deflection; (b) ground settlement (at the final excavation stage)

After repeating the analysis with all 10,000 bootstrapped samples, the same number of β was obtained. Fig. 9(a) shows a distribution of β when the limiting wall deflection is set at 0.7%H_f; and Fig. 9(b) shows a similar plot of the distribution of β when the limiting ground settlement is set at 0.5%H_f. The variation in the computed reliability index as a result of the variation in the sample statistics can be observed. Assuming a normal distribution for the computed β, a confidence interval at, for example, the 95% level can be derived; this follows that the probability of exceedance can be expressed in terms of a confidence interval at the 95% level. The results of the probability of wall deflection exceeding 0.7%H_f in the TNEC case ranging from 0.08 to 0.36 at the 95% confidence level and the probability of ground-surface settlement exceeding 0.5%H_f in the TNEC case ranging from 0.07 to 0.39 are also shown in Table 4. The knowledge of the variation of the sample statistics, which is derived through bootstrapping, allows inferring the probability of exceedance in terms of a confidence interval, instead of a single, fixed probability.

Further Discussions

Effectiveness of the Bootstrapping Method

The analyses presented previously were based on a sample of 17 small-strain triaxial test data. In many instances, the available data may be fewer than this number. To obtain a preliminary estimate of this effect and to examine the effectiveness of the bootstrapping method, all of the previous analyses were repeated with only eight data points, taking only those data from the undisturbed samples as shown in Table 3. Based on the eight data points of the small-strain triaxial tests on the undisturbed clays, the sample statistics were determined as follows: $\mu_S = 0.32$, $\sigma_S = 0.055$, $\mu_E = 572.6$, and $\sigma_E = 178.5$. The mean values were found to be comparable to those computed with the 17 data points, while the SDs were found to be greater than those computed with the 17 data. Using the new data, the mean of maximum wall deflection δ_{hm}, denoted as $\mu[\delta_{hm}]$, was computed to be $\mu[\delta_{hm}] = 107.3$ mm, and the SD of δ_{hm}, denoted as $\sigma[\delta_{hm}]$, was computed to be $\sigma[\delta_{hm}] = 37.8$ mm. If the limiting wall deflection is taken at 0.7%H_f (PSCG 2000), where H_f is the final excavation depth ($H_f = 19.7$ m), the probability of exceeding this limiting wall deflection at the final excavation stage will be $p_f = 0.21$. Similarly, the probability of exceeding the specified limiting ground settlement is determined to be $p_E = 0.21$, as listed in Table 5.

To consider the variation of the sample statistics as a result of the use of a small sample (in this case, a sample consisting of only eight data points), bootstrapping was again applied and all the previous analyses were repeated. The new results are summarized in Table 5. The probability of the maximum wall deflection exceeding 0.7%H_f in the TNEC case ranged from 0.06 to 0.34 at the 95% confidence level and the probability of the maximum ground

Table 5. Probability of Exceedance in the TNEC Excavation Using PEM and the Bootstrapping Method with Only Eight Data Points

Ground or wall response	Probability of exceedance[a]	Probability of exceedance[b] Mean	SD	95% confidence interval
Maximum lateral wall deflection (δ_{hm})	0.21	0.20	0.07	0.06–0.34
Maximum ground-surface settlement (δ_{vm})	0.21	0.20	0.08	0.04–0.36

Note: Level III requirements (PSCG 2000) are employed in the probabilistic analyses: the limiting wall deflection is taken at 0.7%H_f and the limiting ground-surface settlement is taken at 0.5%H_f, in which H_f = final excavation depth (19.7 m).
[a]PEM results with the mean and SD of the soil parameters derived from the eight data points.
[b]Based on the bootstrapping analysis.

settlement exceeding $0.5\%H_f$ in the TNEC case ranged from 0.04 to 0.36 at the 95% confidence level. The new results (Table 5), which relied on the eight data points, were approximately the same as the previous results (Table 4), which relied on the 17 data points. Thus, the confidence interval of the failure probability depends on the variation and range of the sample, not just the size of the sample. Because the variation and range of the two samples in this case happen to be approximately the same, it was no surprise to see that the confidence intervals of the failure probability obtained with the two samples were comparable.

The implication is that the bootstrap method can be an effective tool in geotechnical engineering to capture the variation of sample statistics as a result of the use of a small sample. Considering that it is a norm in geotechnical practice to have limited data availability, it is essential to estimate the variation of the derived sample statistics and to assess the effect of this variation. In this regard, the bootstrap method has the potential to be an indispensible tool in geotechnical engineering. The case study presented in this paper shows that additional information (such as the confidence interval of the probability of serviceability failure as opposed to a single, fixed probability) can be gained through bootstrapping analysis. The gained information enables the engineer to make a more informed decision.

Predicting Performance of an Excavation Design

In this section, the predicted performance of the previously discussed excavation design for the TNEC case is reassessed considering the Level I and II design requirements for urban excavation protection in Shanghai, China (PSCG 2000). The probabilities of exceedance presented previously were computed for the Level III scenario, where no important infrastructures or facilities existed within a distance of $2H_f$ from the excavation. An excavation with the Level III scenario requires that $\delta_{hm} \le 0.7\%H_f$ and $\delta_{vm} \le 0.5\%H_f$. Herein, the probabilities of exceedance are also analyzed for the Level II scenario, where important infrastructures or facilities such as gas mains and water drains exist within a distance of $1 - 2H_f$ from the excavation, and for the Level I scenario, where metro lines and important facilities such as gas mains and water drains exist within a distance of $0.7H_f$ from the excavation. As summarized in Table 1, an excavation with the Level II scenario requires that $\delta_{hm} \le 0.3\%H_f$ and $\delta_{vm} \le 0.2\%H_f$, and an excavation with the Level I scenario requires that $\delta_{hm} \le 0.14\%H_f$ and $\delta_{vm} \le 0.1\%H_f$ The computed probabilities of exceedance in the TNEC case for all three protection levels are summarized in Table 6.

For the TNEC excavation case, maximum wall deflection δ_{hm} predicted using the deterministic KJHH model was 107.3 mm, which is less than $0.7\%H_f$ (137.9 mm) and is greater than $0.3\%H_f$ (59.1 mm) or $0.14\%H_f$ (27.6 mm). Thus, in terms of the maximum wall deflection, the excavation design would be satisfactory under a Level III protection scenario ($\delta_{hm} \le 0.7\%H_f$) and unsatisfactory under a Level II scenario ($\delta_{hm} \le 0.3\%H_f$) or a Level I scenario ($\delta_{hm} \le 0.14\%H_f$). Similar assessment and conclusion can be reached when the TNEC case is evaluated based on maximum ground settlement δ_{vm}.

Although the deterministic solutions appear to be able to offer a clear-cut answer to the question of whether the excavation design would satisfy the design code requirements, as shown in the previous analysis, the uncertainties in the analysis model (in this case, the KJHH model) and the input parameters (in this case, s_u/σ'_v and E_i/σ'_v) can hinder such an ability. In many cases, it would be difficult to judge whether the limiting wall deflection or ground settlement would be exceeded because of the uncertainty in the computed wall and ground responses. Facing such uncertainties, the probabilities of exceedance (or the probability of failure to meet the criterion of a limiting maximum wall deflection or ground settlement) offer a complementary tool to assess the likelihood of unsatisfactory design. As shown in Table 6, the confidence intervals of the probability of exceedance at the 95% confidence level are obtained for all three excavation protection levels. If an excavation design with the probability of exceedance of less than 0.35 is considered acceptable (where the acceptable probability should be determined based on an additional study of risk and should be agreeable among the parties involved), the TNEC case under the Level III scenario would be satisfactory as far as the maximum wall deflection or ground settlement is concerned. However, under the Level II and I scenarios, the TNEC case would be unsatisfactory because of the high probability of exceedance even at the lower end of the confidence interval.

The gained knowledge (i.e., the confidence interval of the probability of exceedance in this case) enables the engineer to make a more informed decision. Furthermore, this knowledge may be carried over to the task of evaluating the risk of damage to the adjacent infrastructures and facilities. However, the latter task is beyond the scope of this paper, which focuses on the estimate of confidence intervals of the probability of exceedance using the bootstrapping method.

Finally, an interesting observation is made from Table 6, in which the range of the confidence intervals can be relatively small or large, depending on the value of the chosen limiting wall deflection or ground settlement relative to the mean of the computed response. To further elaborate this observation, the probability of exceedance in the TNEC excavation was computed for a series of limiting wall deflection and ground settlement values. Fig. 10 shows the confidence intervals of the probability of exceedance obtained for various limiting wall deflection values [Fig. 10(a)] and ground settlement values [Fig. 10(b)]. When the limiting wall deflection or ground settlement values are either very small or very large (relative to the mean of the computed responses), the confidence interval of the computed probability of exceedance is very narrow. This phenomenon is easily understood because when the chosen limiting response approaches to the left tail or the right tail of the probability distribution of the computed response, the variation of the probability of exceedance caused by the uncertainties in the mean and SD of the computed response is naturally small. On the contrary, the

Table 6. Probability of Exceedance in the TNEC Excavation under Three Excavation Protection Scenarios

Probability of exceedance limiting criterion	Excavation protection level (PSCG 2000)					
	Level I		Level II		Level III	
	$\delta_{hm} \le 0.14\%H_f$	$\delta_{vm} \le 0.1\%H_f$	$\delta_{hm} \le 0.3\%H_f$	$\delta_{vm} \le 0.2\%H_f$	$\delta_{hm} \le 0.7\%H_f$	$\delta_{vm} \le 0.5\%H_f$
Mean	0.98	0.91	0.90	0.79	0.20	0.20
SD	0.007	0.02	0.02	0.04	0.07	0.08
95% confidence interval	0.97–0.99	0.87–0.95	0.86–0.94	0.71–0.87	0.06–0.34	0.04–0.36

Note: H_f = final excavation depth = 19.7 m.

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range of the confidence interval increases as the chosen limiting response approaches to the mean of the computed response. For example, when the limiting wall deflection is at approximately 110 mm, which is close to the mean of the computed wall deflection (107.3 mm in TNEC), the range of the confidence interval is the widest, as shown in Fig. 10(a), indicating the greatest variation in the computed probability of exceedance. A similar observation can be made from Fig. 10(b) regarding the confidence interval of the probability of exceedance with respect to the computed maximum ground settlement.

Fig. 10 can also be treated as a case-specific design curve. For a given excavation design at a given site (with a limited number of data on the key soil parameters), the curve presents the confidence intervals (at 95% level) of the probability of exceedance at a range of limiting wall deflection or ground settlement. This curve enables the designer to make a more informed decision. For example, the designer can easily see the change in the probability of exceedance when a different limiting response value is selected. This is significant because in an assessment of the risk of damage to adjacent infrastructures, the engineer would want to explore the probability of exceedance for a range (however narrow it should be) of limiting responses, rather than for a fixed level of responses.

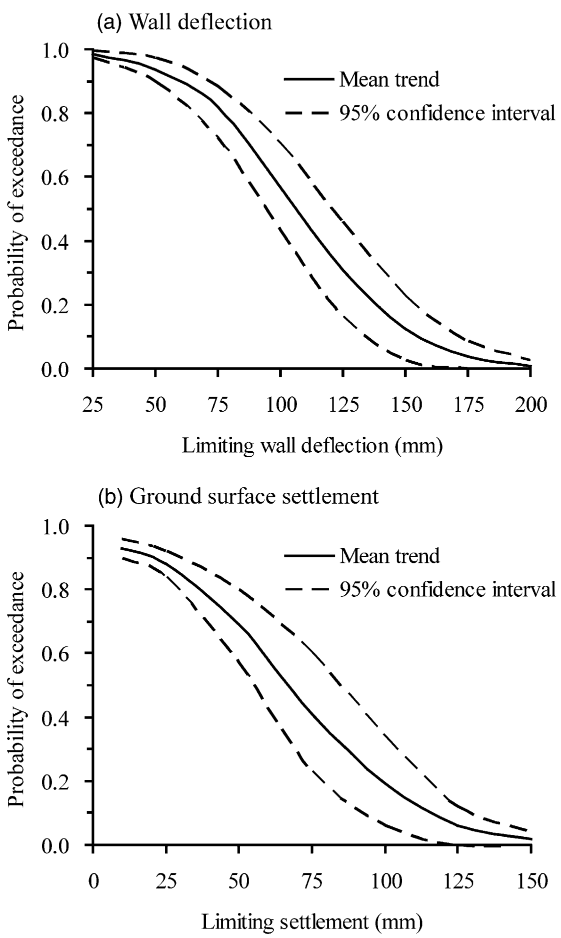

Fig. 10. Confidence intervals for probability of exceedance in the TNEC excavation computed with various levels of limiting wall deflection and ground-surface settlement

Effect of Assumed Distribution of Computed Wall and Ground Responses

In Eq. (11), wall and ground response y is assumed to follow a normal distribution, and the probability of exceedance is computed accordingly. It would be of interest to examine the effect of this assumption, for example, by repeating the analysis with an assumption of a lognormal distribution. The results are compared in Table 7 for a selected protection level (Level III in Table 1). Comparable results were obtained regardless of whether a normal or lognormal distribution was assumed.

Summary and Concluding Remarks

A simple procedure for assessing the probability of serviceability failure in a braced excavation, where the failure is defined as the state in which the response of an excavation system in terms of the maximum wall deflection or ground settlement exceeds the specified limiting value, is presented and demonstrated with a case study. Because of the uncertainties in the analysis model and the input parameters, the question of whether the response of an excavation system will exceed the specified limiting value cannot be answered with certainty. Thus, it is useful to evaluate the probability of failure (or, in this paper, the probability of exceedance) taking into account these uncertainties explicitly. The probability of exceedance allows for a more informed decision to be made regarding the risk of serviceability failure in a braced excavation. However, use of reliability analysis for the probability of exceedance necessitates an evaluation of the means and SDs of the critical soil parameters. In geotechnical practice, these means and SDs are often estimated from limited data, which leads to uncertainty in the derived sample statistics. Thus, there is a need to characterize the uncertainty in the sample statistics derived from a small sample, and to determine the effect of such uncertainty.

Through the case study presented, this paper demonstrates that the bootstrap method is an effective tool for characterizing the uncertainty (or variation) in the sample statistics derived from a small sample, and that additional information (such as the confidence interval of the probability of serviceability failure, as opposed to a single, fixed probability) can be gained through bootstrapping analysis. The gained information enables the engineer to more rationally assess the probability serviceability failure (and the associated risk) in a braced excavation. The case study shows the potential of the bootstrap method in coping with the problem of having to evaluate failure probability with uncertain sample statistics.

Table 7. Probability of Exceedance in the TNEC Excavation for Various Distribution Assumptions

Probability of exceedance limiting criterion[a]	Assuming normal distribution		Assuming lognormal distribution	
	$\delta_{hm} \leq 0.7\%H_f$	$\delta_{vm} \leq 0.5\%H_f$	$\delta_{hm} \leq 0.7\%H_f$	$\delta_{vm} \leq 0.5\%H_f$
Mean	0.20	0.20	0.20	0.19
SD	0.07	0.08	0.05	0.06
95% confidence interval	0.06–0.34	0.04–0.36	0.10–0.30	0.07–0.31

Note: H_f = final excavation depth = 19.7 m.
[a]For Protection Level III in Table 1.

Finally, a few words of caution are in order regarding the bootstrapping method as applied to geotechnical engineering problems. The accuracy of the computed confidence interval of the probability of exceedance is certainly affected by the quality and quantity of data in the original sample. The bootstrapping method is useful when the sample size is small [for example, $n < 30$ as in Wisz et al. (2008)]; for a larger sample size, the classical statistical methods may be preferred. In statistical methods, the required minimum sample size depends on the tolerable error and the desired confidence level. However, the required minimum sample size based on statistical theory is usually much larger than what is typically available in geotechnical practice. Thus, there will always be a trade-off between cost and accuracy in the determination of the minimum sample size in geotechnical practice. The issue of sample size can be further complicated by the nature of soil deposits, including spatial variability, at the project site.

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