Effect of Spatial Variability on Probability-Based Design of Excavations against Basal-Heave

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ABSTRACT

This paper examines the effect of spatial soil variability, modeled as a two-dimensional (2-D) random field, on the computed probability of basal-heave failure in a braced excavation in clay. The random field modeling (RFM) of clay is realized using a framework that combines *Cholesky* decomposition method and Monte Carlo simulation (MCS). Considering that the horizontal and vertical scales of fluctuation are generally unequal for clay, various combinations of the horizontal and vertical scales of fluctuation are examined within this framework. The effect of 2-D RFM of undrained shear strength of clay in a braced excavation is demonstrated with a case study that focuses on a simple probability-based design. The results show that negligence of the 2-D spatial random effect could yield a basal-heave design that is unduly over-conservative: at an acceptable or target failure probability level, the required factor of safety in a limit equilibrium analysis is unreasonably high.

INTRODUCTION

Conventionally, the basal-heave stability in a braced excavation in clay is designed with deterministic methods (e.g., Terzaghi 1943; Bjerrum and Eide 1956; Chang 2000), which requires a design factor of safety (*FS*) that satisfies the minimum *FS* requirement in a design code (e.g., JSA 1988). Because of soil parameters uncertainties, design of braced excavations against basal heave based on the failure probability (p_f) may be preferred, which generally necessitates the use of probabilistic approaches. Alternatively, the deterministic method can still be used if the required minimum *FS* can be calibrated for a target p_f. The probability-based design referred to in this paper concerns the second approach, in which the design seeks to satisfy a minimum *FS* that corresponds to a target p_f against basal-heave. Examples of the probability-based design of basal-heave stability in a braced excavation can be found in Goh et al. (2008) and Wu et al. (2010), in which design charts that relate the probability of basal-heave failure (p_f) to *FS* are provided.

Uncertainty of soil parameters stems not only from inherent variability but also from spatial variability. Inherent variability of soil parameters is interpreted by their probability distributions or sample statistics (e.g., mean value and standard deviation). Spatial variability is generally described by the scale of fluctuation, which is the maximum distance within which the spatially random parameters are correlated (e.g., Vanmarcke, 1983). Spatial variability may be modeled with the random field theory (Vanmarcke, 1977). Recent studies of random field modeling (RFM) based on Monte Carlo simulation (MCS) by Griffiths and his colleagues (Griffiths et al., 2006; Griffiths and Fenton, 2009; Huang et al., 2010) demonstrate that spatial variability plays an important role in probability-based design in geotechnical engineering. Neglecting spatial soil variability in probabilistic analysis of geotechnical problems can lead to either an overestimation or underestimation of the failure probability in a given design, depending on the specified limiting *FS* and its location in the probability distribution of *FS* (e.g., Wang et al., 2011). A recent study on basal-heave stability using the slip circle method reported that the traditional probabilistic analysis that does not account for the effect of spatial variability tends to overestimate the failure probability (Wu et al., 2010).

In this paper, the effect of two-dimensional (2-D) spatial variability on the probability of basal-heave failure in a braced excavation in clay is presented. The random field modeling of clay is realized using a framework that combines *Cholesky* decomposition method and Monte Carlo simulation (MCS). Taking into account the unequal horizontal and vertical scales of fluctuation for clay, various combinations of the horizontal and vertical scales of fluctuation are examined within this framework. The effect of 2-D RFM of undrained shear strength of clay in a braced excavation is demonstrated with a case study that focuses on a simple probability-based design. The results show that negligence of the 2-D spatial random effect could lead to over-conservative basal-heave design: at an acceptable or target failure probability level, the required *FS* in a limit equilibrium analysis is unreasonably high.

SLIP CIRCLE METHOD FOR BASAL-HEAVE ANALYSIS

In this study, the slip circle method (JSA, 1988) for determining *FS* against basal-heave in soft to medium clay is adopted for its simplicity and suitability for modeling the random field of undrained shear strength. The *FS* in the slip circle method is defined as the ratio of resistance over load (Wu et al. 2010):

$$FS = \frac{M_R}{M_D} \tag{1}$$

where M_R and M_D are the resistance moment and the driving moment respectively. In slip circle method, the Arc *bcde* as shown in Figure 1 is first subdivided into a series of small arcs, and then the resistance moment from each small circle is computed as the product of the undrained shear strength (s_u) on that arc, arc length (ds) and the radius of the slip circle (r). The M_R is equal to the summation of the resistance moments along the Arc *bcde*. The driving moment M_D is caused by the weight of the soil in front of the vertical failure plane and above the excavation surface and the possible surcharge.

In the basal-heave analysis with the slip circle method, the undrained shear strength (s_u/σ'_v) of clay plays a critical role in the design of a braced excavation against basal-heave. In other words, *FS* is a function of s_u/σ'_v and other parameters.

The focus of this paper is the failure mode along the slip circle only. Other failure modes are beyond the scope of this study.

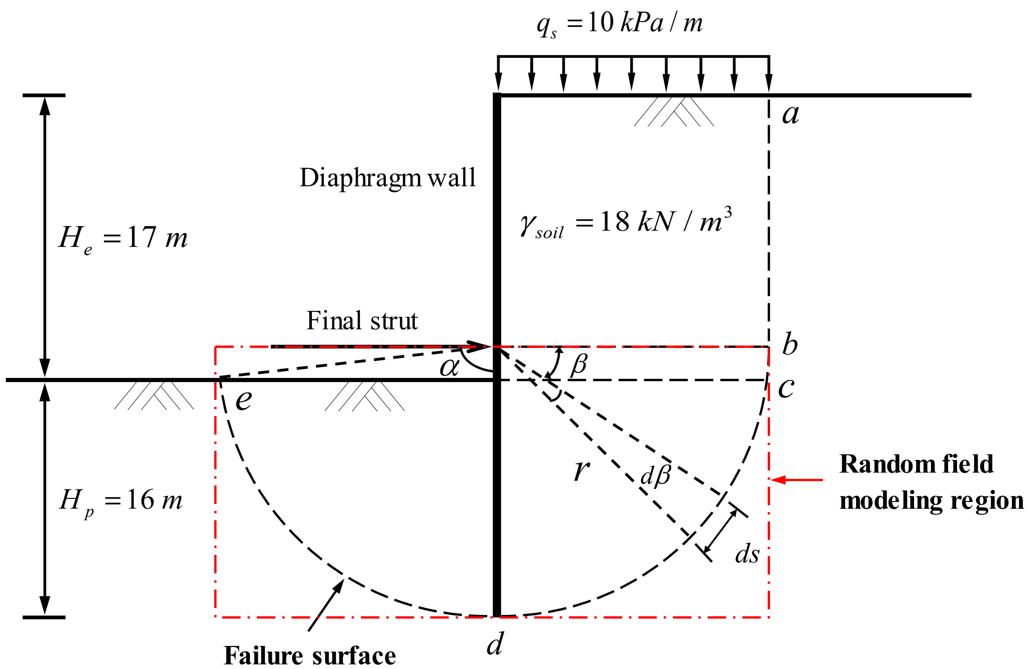

Figure 1. Geometry of slip circle method and 2-dimensional random field modeling for basal-heave stability (modified from Luo et al. 2012).

TWO-DIMENSIONAL RANDOM FIELD MODELING

The undrained shear strength generally increases with depth for most normally consolidated clay but the ratio of undrained shear strength over the effective overburden stress (s_u/σ'_v) remains roughly constant (e.g., Skempton, 1948). Thus, in this paper the parameter s_u/σ'_v is modeled using lognormal random field, and all other input parameters are modeled as spatially-constant lognormal variables or constants. The assumption of lognormal distribution for inherent soil variability assures positive soil parameter values and has been widely advocated by past studies (e.g., Phoon and Kulhawy, 1999). In RFM, the uncertainty of s_u/σ'_v is represented by its spatially-constant mean μ_{ln} and coefficient of variation COV_{ln} and its scale of fluctuation θ. Thus, the basal-heave problem here involves a stationary random field modeling of s_u/σ'_v. In this study, the exponential correlation function, which is commonly used in random field modeling (e.g., Luo et al., 2011), is selected. The correlation matrix built with the exponential correlation function is then decomposed by the *Cholesky* decomposition method which has been proved simple and effective (Fenton, 1997).

The normalized undrained shear strength s_u/σ'_v at each spatial position in the random field can be obtained for a specified mean, standard deviation, and scale of fluctuation using Monte Carlo simulation (Fenton et al., 2005). In each simulation, the same mean, standard deviation, and scales of fluctuation of s_u/σ'_v are used. The statistics of output such as *FS* [Eq. (1)] can be obtained after a sufficient number of simulations are carried out. The failure probability p_f is computed as the ratio of the number of simulations that yield failure ($FS < 1$) over the total number of simulations N. The number of MCS samples should be at least 10 times of the reciprocal of the target failure probability (Wang et al., 2011). In this study, the level of failure probability of interest is greater than 10^{-4}, therefore N is set at 10^5.

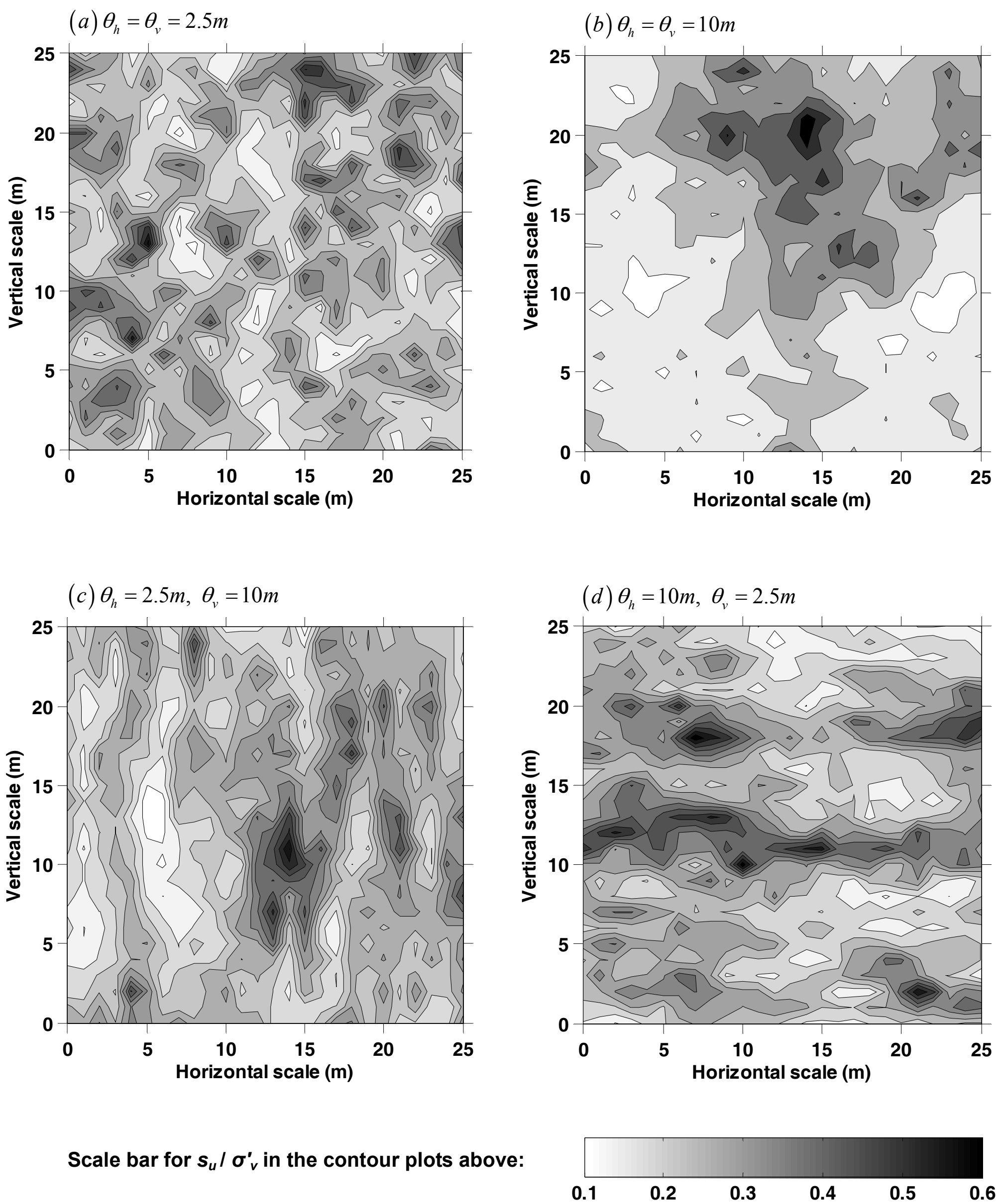

Figure 2. Influence of scale of fluctuation on the 2-D random field modeling of s_u/σ'_v at given mean of 0.3 and COV of 0.3.

Figure 2 shows the results of random field modeling at four combinations of θ_v and θ_h given as an example the mean of $s_u/\sigma'_v = 0.3$ and coefficient of variation (COV) = 0.3: (a) $\theta_h = \theta_v = 2.5$m; (b) $\theta_h = \theta_v = 10$m; (c) $\theta_h = 2.5$m, $\theta_v = 10$m; and (d)

θ_h = 10 m, θ_v = 2.5m. Considering that the aforementioned RFM procedure is defined at the point level, the local averaging is performed to obtain the locally averaged statistics. The local averaging is realized through multiplying a variance reduction factor to the variance of a normal variable. Then the statistics of the equivalent lognormal variable are computed (Griffiths and Fenton, 2004). As shown in Figure 2, the darker color represents higher s_u/σ'_v and lighter color represents smaller s_u/σ'_v. The effect of scales of fluctuation is apparent in the 2-D RFM: either in the vertical or horizontal direction, smaller θ (θ_v or θ_h) corresponds to more drastic variation of s_u/σ'_v in that direction of the random field; conversely, a larger θ corresponds to more uniform s_u/σ'_v in that direction of the random field. In either direction, the spatial variation in the case of smaller θ is much more significant than that for larger θ.

CASE STUDY

The probability of basal-heave failure in a braced excavation in clay is analyzed herein using the random field modeling with *Cholesky* decomposition method. The geometry and input data for the excavation case employed in this study is illustrated in Figure 1 and listed in Table 1, respectively. The undrained shear strength is modeled as a spatially random variable, and the unit weight of soil and surcharge are modeled as spatially-constant random variables. All other geotechnical and structural parameters are treated as constants for simplicity, since the uncertainties in these parameters are relatively negligible.

Table 1. Input parameters for a basal-heave stability problem shown in Figure 1.

Parameters*	Notations	Values	
		Mean	COV
Unit weight of soil	γ	18 kN/m^3	0.1
Surcharge	q_s	10 kPa/m	0.2
Depth of GWT	D	2m	N/A
Final excavation depth	H_e	17m	N/A
Final strut depth	H_s	15m	N/A
Penetration depth	H_p	16m	N/A

*In this paper, many basal-heave problems defined with this set of input parameters and geometry are analyzed. The difference in these problems is in the choice of the mean value of s_u/σ'_v, which results in different factors of safety (FS).

Typical *COV* of the undrained shear strength is about 0.3, although it could be as high as 0.8 (e.g., Phoon and Kulhawy, 1999). In this paper, the COV of s_u/σ'_v is first set at 0.3 and the effect of assuming higher *COV*s is examined later. Based on a statistical study by Phoon and Kulhawy (1999), the average vertical and horizontal scales of fluctuation for clay are 2.5m and 50.7m, respectively. In a 2-D RFM, both vertical and horizontal spatial variability are considered. It should be noted that for basal-heave analysis, only the random field shown in Figure 1 (the box region) needs to be modeled. Although a larger modeling region can be adopted, the region shown in Figure 1 is the minimum region that covers the slip circle where the resistance moment M_R is derived.

Effect of 1-D random field modeling of s_u/σ'_v

To provide a reference, the effect of vertical and horizontal scales of fluctuation is first examined separately (i.e., treating it like 1-D RFM). Thus, when vertical or horizontal spatial variability is considered, the other direction is assumed to be spatially constant. To study the effect of spatial variability, a series of scales of fluctuation (θ = 2.5m, 10m, 50m and 100m) for each direction is investigated.

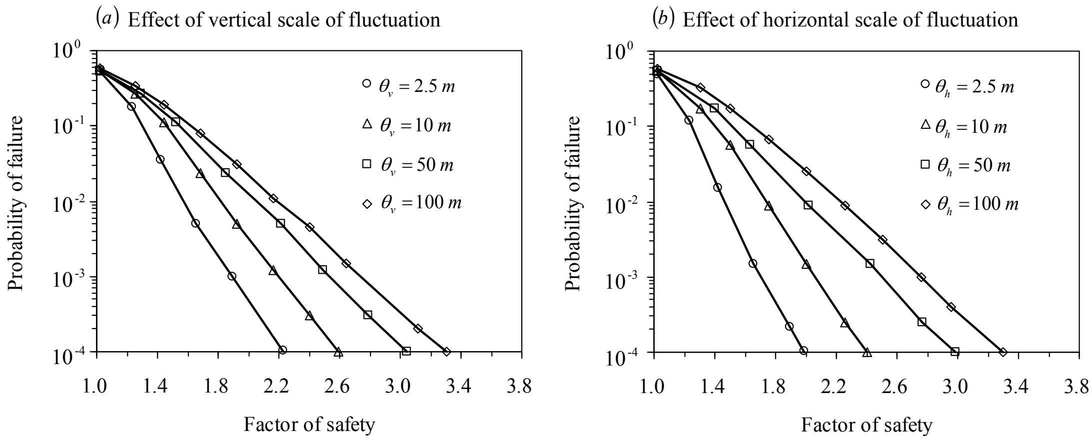

Figure 3. Effect of 1-D spatial variability on the relationship between target failure probability and safety factor.

Thus, given a set of input data for a braced excavation (Figure 1 and Table 1), the probability of basal-heave failure is computed for a design with a given *FS* [say, *FS* = 2.0 as per Eq. (1)] and a given scale of fluctuation that reflects the 1-D random field of s_u/σ'_v. This probability can be calculated using the MCS. The MCS is performed for each series of scale of fluctuation and each series of "designs" (signaled by a series of *FS* values, which was realized by assuming different mean s_u/σ'_v values while keeping the mean values of all other parameters the same). The results are shown in Figure 3(a) for vertical spatial variability and Figure 3(b) for horizontal spatial variability, respectively. The results presented in this figure provide the engineer a basis for selecting a factor of safety for design against basal-heave using the slip circle method. This basis is the target probability of failure that considers the spatial variability of soil parameters. The design, based on the target probability of failure is referred to herein as the probability-based design against basal-heave failure.

The effect of scales of fluctuation in a 1-D random field is quite obvious: a smaller scale of fluctuation results in a smaller p_f for a given *FS*. As shown in Figure 3(a), if the target p_f is set at 10^{-4}, the required FS is about 2.23 at θ_v = 2.5m (note: this value is the mean of the vertical scale of fluctuation for clay), and is about 3.31 at θ_v

= 100m (note: this value is close to a spatially-constant condition). Similar conclusions may be drawn from Figure 3(b) for the effect of horizontal spatial variability. The implication is that the required *FS* will be overestimated for a target p_f if the effect of spatial variability is ignored. Thus, traditional probabilistic analysis that considers variation of input soil parameters (for example, through *COV*) but not spatial variability exhibited in a random field can over-estimate the probability of failure for a given deterministic-based design (i.e., a given *FS*).

Effect of 2-D random field modeling of s_u/σ'_v

The effect of 2-D spatial variability is examined next. To begin with, three different horizontal scales of fluctuation (θ_h = 2.5m, 50m and 100m) are considered simultaneously with the average vertical scale of fluctuation (θ_v = 2.5m) to study the effect of θ_h at fixed θ_v. The results are shown in Figure 4(a). Afterwards, three different vertical scales of fluctuation (θ_v = 2.5m, 50m and 100m) are considered simultaneously with the average vertical scale of fluctuation ($\theta_h \approx$ 50m) to study the effect of θ_v at fixed θ_h. The results are shown in Figure 4(b). The effect of the scales of fluctuation in a 2-D random field is also obvious: at a fixed scale of fluctuation in one direction, a smaller scale of fluctuation in the other direction results in a smaller p_f for a given *FS*. Furthermore, the required *FS* will still be overestimated for a target p_f if the soil parameter is modeled with only a 1-D random field, as opposed to a 2-D random field. Therefore, it is essential to consider 2-D spatial variability in the probability-based design against basal-heave failure in a braced excavation.

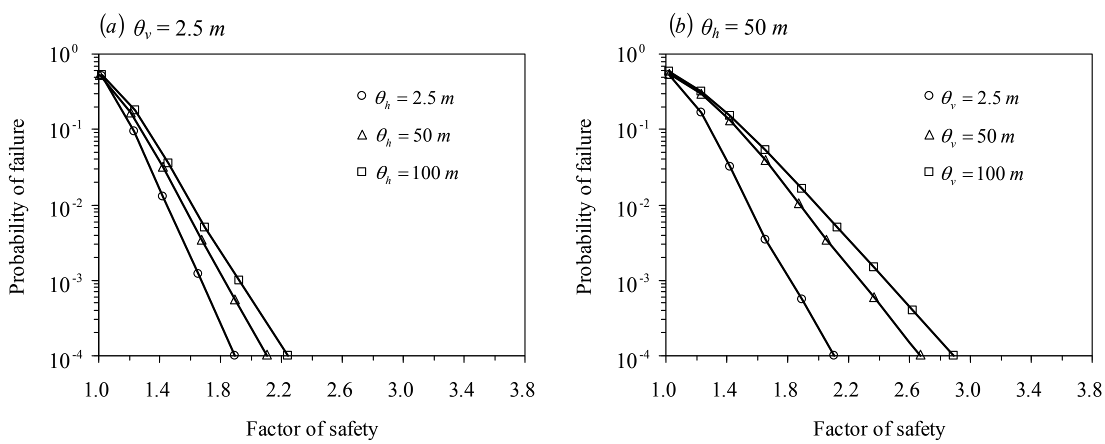

Figure 4. Effect of 2-D spatial variability on the relationship target failure probability and safety factor.

One concern with the traditional probability-based analysis in geotechnical practice in the past is that the computed failure probability is often high in a design that satisfies the minimum *FS* specified in the codes, but failure seldom occurs in such cases. For example, this concern has been reported by Wu et al. (2010) in their study of basal-heave stability in an excavation. As recently pointed out by Christian and Baecher (2011), one of the unresolved problems in the current geotechnical practice is that the failures are less frequent than the predictions made with probabilistic analysis. Overestimation of variation in soil parameters is often pointed out as a possible cause for having a higher computed probability of failure. Based on the results presented (Figures 3 and 4), it is evident that negligence in the effect of spatial variability of soil parameters may contribute to an overestimation of the failure probability. Thus, to apply the probability-based method for evaluating the failure probability, spatial variability must be considered in the analysis.

CONCLUDING REMARKS

In this paper, the effect of 2-D spatial variability on the probabilistic analysis of the basal-heave stability of a braced excavation in clay is examined. The 2-D spatial variability is modeled using *Cholesky* decomposition method with the aid of Monte Carlo simulation. After a sufficient number of Monte Carlo simulations, the probability of basal-heave failure may be estimated as the ratio of the number of simulations that yield failure ($FS < 1$) over the total number of simulations. Assuming various combinations of the vertical and horizontal scales of fluctuation, the relationship between the target failure probability and the required minimum factor of safety is obtained (Figures 3 and 4).

Based on the results presented, it is found that the traditional probabilistic analysis that considers the variation of soil parameters, but not spatial variability in a random field, can significantly overestimate the probability of basal-heave failure for a given design with a certain *FS*. Case study further shows that the required *FS* will still be overestimated for a target failure probability if the soil parameter is modeled with only a 1-D random field, as opposed to a 2-D random field. Therefore, it is essential to consider 2-D spatial variability in the probability-based design against basal-heave failure in a braced excavation.

The charts presented in Figure 4 provide a simple way to determine the required minimum FS for a target failure probability at a given scale of scale of fluctuation. Thus, the probability-based design can be achieved indirectly using the slip circle method with a factor of safety that corresponds to a target failure probability.

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