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# RELIABILITY-BASED ROBUST AND OPTIMAL DESIGN OF SHALLOW FOUNDATIONS IN COHESIONLESS SOIL IN THE FACE OF UNCERTAINTY

C. Hsein Juang [1], Lei Wang [2], Sez Atamturktur [3], and Zhe Luo [4]


## ABSTRACT

Quantification of uncertainties in soil parameters and geotechnical models is a prerequisite for a reliability-based design. If there is abundant amount of high quality data that can characterize the adopted geotechnical model and its parameters perfectly, the result of reliability analysis will be a certain value (a fixed reliability index or failure probability). Then, the reliability-based design will be a straightforward process and the least cost design that satisfies the constraint of a target failure probability can be selected as the final design. If uncertainty exists in the statistical characterization of the adopted geotechnical models and their parameters, as is usually encountered in geotechnical practice, then the computed failure probability will not be a fixed value and the design decision will not be as straightforward, as there will be uncertainty as to whether the design actually meets the failure probability requirement. To reduce the effect of uncertainty of the statistical characterization of the adopted geotechnical models and soil parameters, a new geotechnical design approach, called reliability-based robust geotechnical design (RGD) method, is developed. This new design methodology is aimed at achieving a certain level of design robustness, in addition to meeting safety and cost requirements. Here, a design is deemed "robust" if the predicted system response is "insensitive" to the uncertainty of the statistical characterization of soil parameters and model factors. A Pareto Front, which describes a trade-off relationship between cost and robustness at a given safety level, is established through a multi-objective optimization based on the RGD concept. The new design methodology is illustrated with an example of spread foundation design. The significance of this methodology is elaborated and demonstrated in this paper.


*Key words:* Reliability, optimization, robust design, shallow foundations.

## 1. INTRODUCTION

Uncertainties in geotechnical models and parameters and their effect have long been recognized (Lacasse and Nadim 1994; Gilbert and Tang 1995; Phoon and Kulhawy 1999; Whitman 2000; Juang *et al*. 2004; Schuster *et al*. 2008; Zhang *et al*. 2009; Juang *et al*. 2009; Zhang *et al*. 2012). To perform a geotechnical design using deterministic approach, "conservative" values of the uncertain soil parameters are often adopted along with an experience-calibrated factor of safety. While the deterministic approach has been successfully used for many decades, it lacks the capability to render a consistent measure of safety of the geotechnical system in the face of uncertainties. To obtain a more rational design, many investigators (*e.g.*, Wu *et al*. 1989; Christian *et al*. 1994; Whitman 2000; Phoon *et al*. 2003a,b; Fenton *et al*. 2005; Najjar and Gilbert 2009; Wang 2011; Zhang *et al*. 2011) have turned to a probabilistic approach.

Quantification of the uncertainties in soil parameters and geotechnical models is a prerequisite for probability or reliability-based design. If there is abundant amount of quality data that can characterize the statistics of the adopted geotechnical model and its parameters, the result of reliability analysis will be a certain value (a fixed reliability index or failure probability). Thus, the design meeting the target reliability (*i.e.*, safety) requirements with least cost would be the "best" choice, and the reliability-based design would be a straightforward process. However, the statistics of soil parameters and model factor (which quantifies the accuracy and precision of the adopted geotechnical model) are quite difficult to ascertain due to lack of data and/or incomplete knowledge. If the statistics of model factor and input parameters cannot be characterized with certainty, the computed failure probability will not be a fixed value. The design decision will not be straightforward with a variable failure probability. In such a scenario, a difficult trade-off decision may be required.

One way to reduce the effect of the uncertainties of statistical characterization of soil parameters and model factors is considering robustness of the system response (*e.g.*, failure probability of the designed geotechnical system) against these uncertainties. A design is deemed "robust" if the predicted system response is "insensitive" to the uncertainties of the statistical characterization of soil parameters and model factors. By considering robustness explicitly in the reliability-based design optimization,

Manuscript received July 17, 2012; revised September 12, 2012; accepted September 27, 2012.
[1] Glenn Professor (corresponding author), Glenn Department of Civil Engineering, Clemson University, Clemson, SC 29634, USA; also affiliated with National Central University, Jhongli City, Taoyuan County 32001, Taiwan (e-mail: hsein@clemson.edu).
[2] Research Assistant, Glenn Department of Civil Engineering, Clemson University, Clemson, SC 29634, USA.
[3] Asst. Professor, Glenn Department of Civil Engineering, Clemson University, Clemson, SC 29634, USA.
[4] Visiting Asst. Professor, Glenn Department of Civil Engineering, Clemson University, Clemson, SC 29634, USA.

as is shown later, a more informed design decision may be made.

Robust design concept, originally proposed by Taguchi (1986) for product quality control in manufacturing engineering, has been applied to many design fields including mechanical design, aeronautical design and structural design (*e.g.*, Chen *et al*. 1996; Tsui 1999; Lagaros and Fragiadakis 2007; Marano *et al*. 2008; Lee *et al*. 2010; Paiva 2010). From the perspective of a designer aiming to achieve a robust design, the input parameters for the design can be divided into two groups: Easy-to-control and hard-to-control parameters. In the context of robust design, the easy-to-control parameters such as dimension of a foundation are called design parameters, while the hard-to-control factors such as uncertain soil parameters and model factors are called noise factors. Assuming that the uncertainty of these noise factors cannot be eliminated (or further reduced because of inherent variability or lack of data), the aim is then to reduce the effects of the uncertainty of these noise factors on the response of the system. Thus, Robust Design aims to find a design (represented by a set of design parameters) that is robust against the uncertainty of these noise factors, thereby reducing the variability of the system response.

In this paper, a reliability-based robust geotechnical design (RGD) methodology is introduced. Here, the objective of RGD is to ensure the robustness of reliability-based design even if the statistics of noise factors are not precisely defined (meaning that uncertainty exists in the estimated statistical moments of these noise factors). When robustness is included in the design decision along with safety (reliability) and cost, the search for the "best" design becomes a multi-objective optimization problem. One possible approach is to treat the safety requirement as a constraint (for example, by requiring the failure probability of the design to be less than the acceptable target failure probability) in an optimization with respect to cost and robustness. Recall that in a traditional reliability-based design, the safety requirement is used as a constraint and the design is optimized with respect to one objective, cost. Thus, the new RGD approach is seen as an extension of the traditional reliability-based design.

To illustrate the RGD framework, the design of a shallow foundation in cohesionless soil is used as an example herein. The normalized load-settlement curve approach (Akbas and Kulhawy 2009a; Akbas and Kulhawy 2011), which ensures uniformity in the reliability analysis across both ultimate limit state (*ULS*) and serviceability limit state (*SLS*), is adopted for the design of shallow foundation. Through the examples presented, the effectiveness of the reliability-based RGD approach and the significance of considering robustness in the design process are clearly demonstrated.

## 2. DETERMINISTIC MODELS FOR *ULS* AND *SLS* CAPACITY OF SHALLOW FOUNDATION

The procedure for calculating the *ULS* capacity of shallow foundation in cohesionless soil under compressive loads proposed by Vesić (1975), with minor improvements by Kulhawy *et al*. (1983), is adopted in this paper. Based on the extensive database of field testing, Akbas and Kulhawy (2009b) demonstrated that the *ULS* capacity estimated by Vesić model as updated by Kulhawy *et al*. (1983) agreed well with the field testing results when the foundation width $B \geq 1$ m. The *ULS* capacity ($R_{ULS}$) of a shallow foundation with width $B$, length $L$, and embedment depth $D$ is calculated as follows (Vesić 1975; Akbas and Kulhawy 2009b):

$$R_{ULS} = \left[ (1/2)B\gamma' N_\gamma \ \xi_{\gamma s} \ \xi_{\gamma d} \ \zeta_{\gamma r} + q' N_q \ \xi_{qs} \ \xi_{qd} \ \zeta_{qr} \right](BL) \quad (1)$$

where $\gamma'$ = effective unit weight of soil below foundation; $q'$ = effective overburden stress at foundation level; and $N_\gamma$ and $N_\gamma$ are bearing capacity factors defined as (Vesić 1975):

$$N_\gamma \approx 2(N_q + 1)\tan\phi' \quad (2)$$

$$N_q = e^{\pi \tan\phi'} \tan^2(45 + \phi'/2) \quad (3)$$

And $\zeta_{\gamma s}$ and $\zeta_{qs}$ = shape correction factors; $\zeta_{\gamma d}$ and $\zeta_{qd}$ = depth correction factors; and $\zeta_{\gamma r}$ and $\zeta_{qr}$ = rigidity correction factors. Detailed formulations for these correction factors are documented in Kulhawy *et al*. (1983).

The *ULS* failure is checked by comparing the bearing capacity ($R_{ULS}$, as "resistance") with the applied loading $G + Q$, where $G$ is the permanent load and $Q$ is the transient load. The condition $R_{ULS} < G + Q$ denotes the *ULS* failure of shallow foundation.

For the *SLS* capacity ($R_{SLS}$) of shallow foundation, Akbas and Kulhawy (2011) derived the following equation based on the normalized load-settlement behavior of shallow foundation:

$$R_{SLS} = \frac{R_{ULS}(s_t / B)}{a(s_t / B) + b} \quad (4)$$

where $s_t$ is the allowable settlement limit (in this paper, 25 mm), $B$ is the width of the foundation, and the coefficients $a$ and $b$ are parameters of a hyperbolic model that fit the normalized load-settlement curve defined below (Akbas and Kulhawy 2009a):

$$\frac{G + Q}{R_{ULS}} = \frac{s/B}{a(s/B) + b} \quad (5)$$

where $(G + Q)/R_{ULS}$ is the normalized loading, and $s$ is the corresponding settlement. Based on data from 167 full-scale tests, the mean and coefficient of variation (COV) of $a$ and $b$ are $\mu_a = 0.70$ and $\delta_a = 22\%$, and $\mu_b = 1.77$and $\delta_b = 54\%$, respectively.

It is noted that the normalized load-settlement curve approach with Eqs. (4) and (5) provides a framework to correlate the *ULS* capacity with the *SLS* capacity, and thus, the two limit states can be treated uniformly. For a given design (with known $B$, $L$ and $D$), if the *ULS* bearing capacity ($R_{ULS}$) is less than the applied load $G + Q$, the *ULS* failure is said to occur. The *SLS* failure is said to occur if the bearing capacity at the allowable settlement limit ($R_{SLS}$) is less than the applied load $G + Q$.

## 3. ESTIMATION OF COST FOR SHALLOW FOUNDATIONS

The total cost for a shallow foundation is determined using the cost summation of five individual tasks in foundation construction (Wang and Kulhawy 2008):

$$Z = Q_e \, c_e + Q_f \, c_f + Q_c \, c_c + Q_r \, c_r + Q_b \, c_b \quad (6)$$

where $Q_e$, $Q_f$, $Q_c$, $Q_r$, $Q_b$ = quantities for excavation, formwork, concrete, reinforcement, and compacted backfill, respectively; $c_e$, $c_f$, $c_c$, $c_r$, $c_b$ = unit prices for excavation, formwork, concrete, reinforcement, and compacted backfill, respectively. Table 1 gives the U.S. average unit price for construction of shallow foundation compiled by Wang and Kulhawy (2008). The five quantities $Q_e$, $Q_f$, $Q_c$, $Q_r$, $Q_b$ depend on the design parameters, foundation width $B$, length $L$, and embedment depth $D$. The reader is referred to Wang and Kulhawy (2008) for details.

## 4. DESIGN EXAMPLE OF SHALLOW FOUNDATION

An example of shallow foundation is used to illustrate the proposed reliability-based robust geotechnical design (RGD) approach. A square foundation ($B = L$), as shown in Fig. 1, is to be designed to support vertical compressive loads with a permanent load component of $G$ = 2000 kN and a transient load component of $Q$ = 1000 kN. $G$ and $Q$ are assumed to follow lognormal distribution with a COV of $G$ of 10% and a COV of $Q$ of 18% (Zhang *et al*. 2011).

The soil profile at the site is assumed to follow the example presented by Orr and Farrel (1999), which consists of a homogeneous dry sand with a deterministic unit weight of $\gamma$ = 18.5 kN/m$^3$. Ten effective friction angles $\phi'$ (for dry sand, $c'$ = 0) are obtained from triaxial tests conducted on samples of this homogeneous sand and the results are listed in Table 2. The ground water is assumed to be well below any topsoil and disturbed ground such that it has negligible effects on the shallow foundation design. The maximum allowable settlement is set at 25 mm for this foundation design.

## 5. STATISTICAL CHARACTERIZATION OF UNCERTAINTY IN NOISE FACTORS

### 5.1 Bootstrapping for Characterizing Uncertainty in Sample Statistics

In geotechnical engineering practice, soil parameters are usually derived with a small sample, thus the derived sample statistics (such as mean and standard deviation) are often subjected to error. These derived sample statistics, which are required in reliability analysis and design, are often uncertain and should be modeled as random variables. To characterize the uncertainty in these sample statistics, non-parametric bootstrap method may be used (Luo *et al*. 2012b). Bootstrapping is a resampling technique that yields an estimate of the mean and standard deviation of the sample statistics.

In reference to Fig. 2, the procedure for bootstrapping is summarized below (Bourdeau and Amundaray 2005; Luo *et al*. 2012b):

1. Based on the original sample $A$ (with $k$ elements or data points), a large number ($N$) of re-samples, $A_j^*, j = 1, N$, are formed by "random sampling with replacement," which means that each element (for example, $a_{j,1}^*$) of $A_j^*$ can assume the value of any of the elements of $A$. In this study, $N$ = 10,000 is adopted.

**Table 1 Unit price for shallow foundation (data from Wang and Kulhawy 2008)**

| Work item | Unit | National average unit price in U.S. (USD) |
|---|---|---|
| Excavation | m$^3$ | 25.16 |
| Formwork | m$^3$ | 51.97 |
| Reinforcement | kg | 2.16 |
| Concrete | m$^3$ | 173.96 |
| Compacted backfill | m$^3$ | 3.97 |

**Table 2 Triaxial test results of effective friction angle (data from Orr and Farrell 1999)**

| Test No. | $\phi'$(°) |
|---|---|
| 1 | 33.0 |
| 2 | 35.0 |
| 3 | 33.5 |
| 4 | 32.5 |
| 5 | 37.5 |
| 6 | 34.5 |
| 7 | 36.0 |
| 8 | 31.5 |
| 9 | 37.0 |
| 10 | 33.5 |

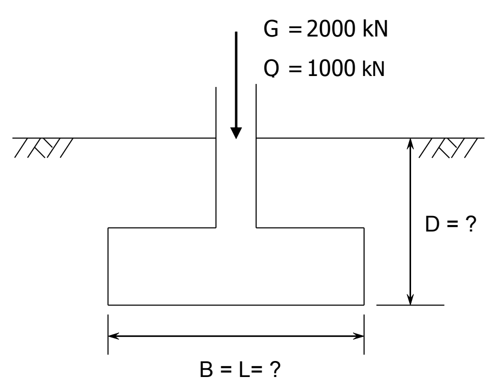


**Fig. 1 A square shallow foundation design example**

2. For each re-sample, $A_j^*$, the statistics of interested $X_i$ (*e.g.*, mean and standard deviation) are computed.
3. The mean ($\mu_{X_i}$) and standard deviation ($\sigma_{X_i}$) of statistics $X_i$ can be computed once Steps 2 has been repeated $N$ times.

With only 10 data of $\phi'$ listed in Table 2, there is uncertainty concerning the mean and standard deviation derived from this sample. Thus, bootstrapping method is applied to evaluate the uncertainty of the sample mean and standard deviation. While not shown here, it took less than 10,000 bootstrap samples to obtain converged results in this study. With $N$ = 10,000, the histograms of the mean ($\mu_S$) and standard deviation ($\sigma_S$) of $\phi'$ is obtained as shown in Fig. 3. Both $\mu_S$ and $\sigma_S$ can be approximated well with a normal distribution in this example. Table 3 shows the mean and standard deviation of both $\mu_S$ and $\sigma_S$. It can be found that the variation of sample mean $\mu_S$ is quite negligible (COV of $\mu_S \approx$ 1.7%), while the variation of sample standard deviation $\sigma_S$ is

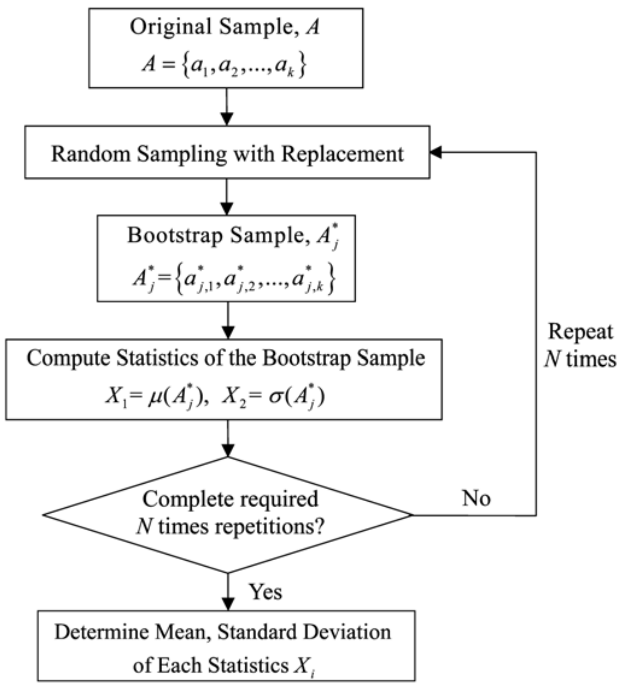


**Fig. 2 Illustration of bootstrap procedure for characterizing uncertainty in sample statistics**

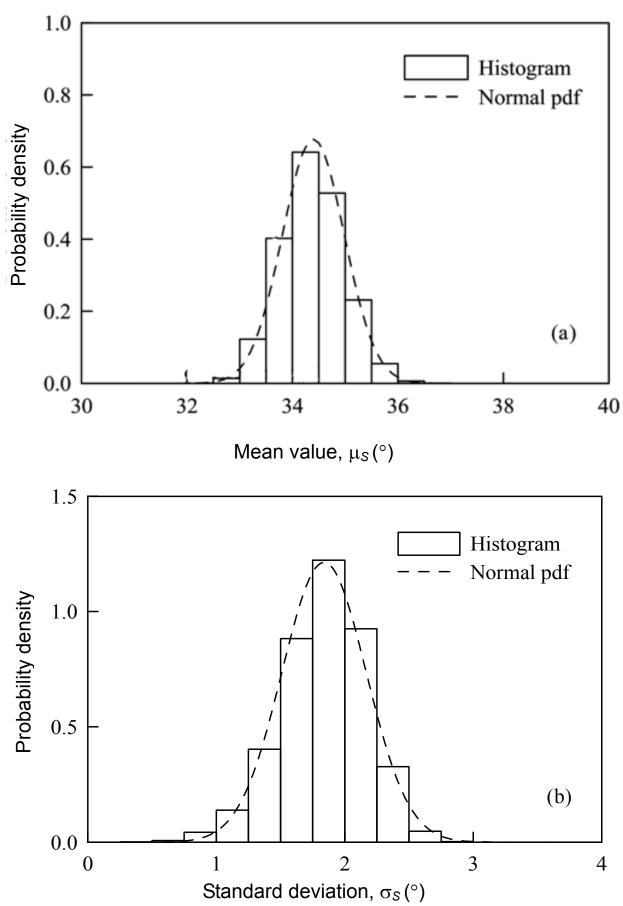


**Fig. 3 Probability distribution of sample statistics of $\phi'$: (a) mean; (b) standard deviation**

**Table 3 Sample statistics of effective friction angle $\phi'$ by bootstrapping method**

| Uncertain variables | $\mu_S$ (°) | $\sigma_S$ (°) |
|---|---|---|
| Mean | 34.40 | 1.84 |
| Std. dev. | 0.59 | 0.33 |

large (COV of $\sigma_S \approx$ 17.9%). This suggests that the standard deviation of soil parameters estimated from a small sample is usually not precise (*i.e.*, having a large variation), while the sample mean is generally quite precise, which is consistent with the statistical theory.

### 5.2 Statistical Characterization of Model Uncertainty

Model uncertainty is often significant in a geotechnical analysis. In fact, Zhang *et al*. (2009) has demonstrated that a geotechnical design that did not include model uncertainty in the analysis could be un-conservative even if parametric uncertainty was fully characterized. The model uncertainty is usually calibrated using statistical methods (Phoon and Kulhawy 2005; Dithinde *et al*. 2011) if data is available. For example, a multiplicative model is often employed to describe the model uncertainty using a model bias factor (or model factor):

$$BF_Q = \frac{\text{observed value}}{\text{predicted value}} = \frac{Q_o}{Q_P} \tag{7}$$

For the *ULS* capacity of shallow foundation, the predicted capacity is the calculated $R_{ULS}$, while the observed capacity is the "interpreted failure load" obtained from full-scale field load test. In this paper, the database of field load tests compiled by Akbas and Kulhawy (2009b) is used to compute the mean ($\mu_{BF}$) and standard deviation ($\sigma_{BF}$) of bias factor $BF_Q$. Then, the bootstrapping method is used to characterize the uncertainty in $\mu_{BF}$ and $\sigma_{BF}$. A summary of the statistical characterization of $\mu_{BF}$ and $\sigma_{BF}$ is provided in Table 4.

For the *SLS* failure, the model uncertainty parameters are reflected in parameters $a$ and $b$, in addition to the bias factor $BF_Q$. In this paper, the mean ($\mu_a$) and standard deviation ($\sigma_a$) of parameter $a$, the mean ($\mu_b$) and standard deviation ($\sigma_b$) of parameter $b$, and the correlation coefficient ($\rho_{ab}$) between $a$ and $b$ are calculated using the database compiled by Akbas and Kulhawy (2009a). To evaluate the possible variation in these statistical parameters, the bootstrapping method is employed, and the results are shown in Table 5.

## 6. RELIABILITY-BASED ROBUST GEOTECHNICAL DESIGN

An outline for reliability-based robust geotechnical design (RGD) is presented below, using shallow foundation design in cohesionless soil as an example. In reference to Fig. 4, the RGD approach is summarized in the following steps (with commentaries):

### 6.1 Step 1

Characterize the uncertainty in the sample statistics of noise factors (including both key soil parameters and model factors) and identify the design domain. This step is shown as the first two blocks in the left side of the flowchart shown in Fig. 4.

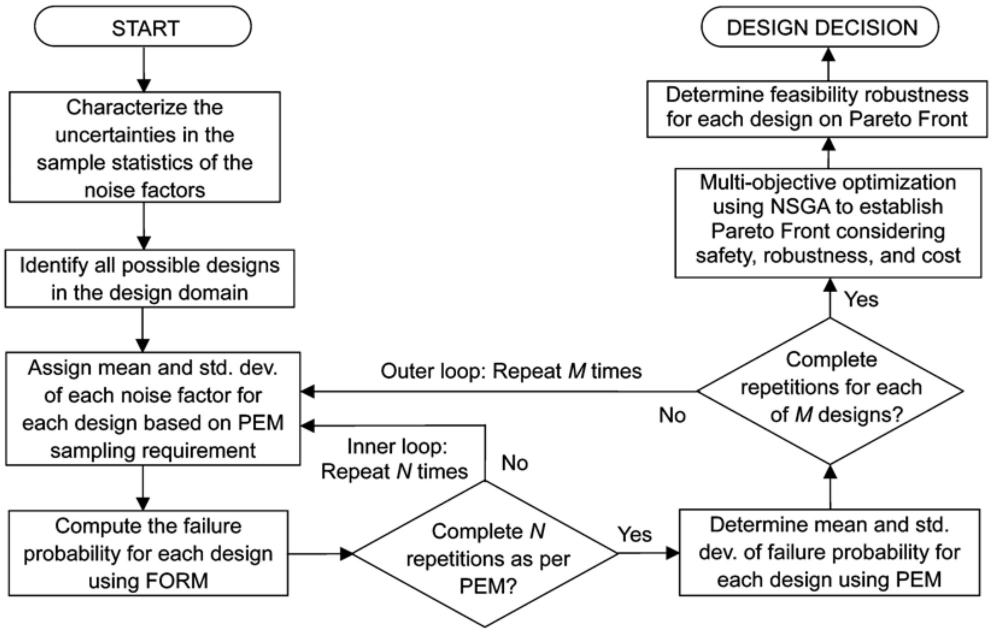


**Fig. 4 Flowchart illustrating robust geotechnical design of shallow foundation (Juang and Wang 2013)**

**Table 4 Sample statistics of model bias factor $BF_Q$ by bootstrapping method**

| Uncertain variables | $\mu_{BF}$ | $\sigma_{BF}$ |
|---|---|---|
| Mean | 1.010 | 0.203 |
| Std. dev. | 0.033 | 0.034 |

**Table 5 Results from bootstrapping method for estimating uncertainty in statistics of *a* and *b***

| Uncertain variables | $\mu_a$ | $\mu_b$ | $\sigma_a$ | $\sigma_b$ | $\rho_{ab}$ |
|---|---|---|---|---|---|
| Mean | 0.6992 | 1.7675 | 0.1549 | 0.9416 | –0.7177 |
| Std. dev | 0.0139 | 0.0845 | 0.0125 | 0.0794 | 0.0472 |

For the design of shallow foundation in cohesionless soils, soil parameter $\phi'$, the *ULS* model factor $BF_Q$ and the two curve fitting parameters *a* and *b* of the *SLS* model are identified as noise factors. The uncertainty in the statistics (mean and standard deviation) of each of the noise factors may be estimated with bootstrapping method.

In the geotechnical design of a square shallow foundation, the design parameters are the foundation width *B* and the embedment depth *D*. The design range for footing width *B* typically varies from a minimum of 1 m to a maximum value of 5 m (Akbas 2007; Akbas and Kulhawy 2011). The minimum foundation embedment depth *D* is set at 1 m based on the load level in this example (Coduto 2000), and the maximum depth is set at 2 m to minimize the disturbance to adjacent structures (Wang and Kulhawy 2008). For a shallow foundation, the ratio of embedment depth to foundation width (*D*/*B*) is generally kept below 4. Of course, the engineer may have to consider local design concerns such as expansive soils, collapsing soils, frost heave, or construction issues. Thus, different constraints may be adopted to identify the domain of design parameters.

For convenience of construction, the foundation dimensions are typically rounded to the nearest 0.1 m (Wang 2011). Thus, within the constraints of three geometric requirements, namely, $1 \le B \le 5$; $1 \le D \le 2$; $(D/B) < 4$, a finite number of designs (each represented by a pair of *B* and *D*) can be identified. For example, for the shallow foundation (Fig. 1) considered in this paper, the number of possible designs in the design domain is $M \approx 450$.

## 6.2 Step 2

For each design, determine the mean failure probability of the design and the standard deviation of the failure probability. This step is shown as the inner loop (Fig. 4) that ends in the bottom block in the left side of the flowchart. In this paper, the failure probability based on either ultimate limit state (*ULS*) or serviceability limit state (*SLS*) is used as a measure of system response. Recall that a design is considered robust if the variation of its system response caused by the uncertainty of noise factors is small. The variation of the failure probability is mainly caused by the variation of the derived statistics of the noise factors. Thus, in this step the mean and standard deviation (as a measure of robustness) of the failure probability are evaluated based on a modified point estimate method (PEM; Zhao and Ono 2000).

When mean ($\mu_S$) and standard deviation ($\sigma_S$) of $\phi'$, as well as mean ($\mu_{BF}$) and standard deviation ($\sigma_{BF}$) of model factor $BF_Q$. are fixed values, the traditional reliability analysis using, for example, first order reliability method (FORM; see Ang and Tang 1984) will yield a fixed value for *ULS* failure probability. Of course, the resulting *ULS* failure probability will no longer be a fixed value if uncertainties exist in $\mu_S$, $\sigma_S$, $\mu_{BF}$ and $\sigma_{BF}$, and they have to be treated as random variables. In such a scenario, the variation of the *ULS* failure probability can be obtained using PEM with 4 input random variables, $\mu_S$, $\sigma_S$, $\mu_{BF}$ and $\sigma_{BF}$. Detailed formulation for PEM with multiple input variables can be found in Zhao and Ono (2000). Similarly, the variation of the *SLS* failure probability is caused by uncertainty in the statistical moments of noise factors, and thus can be evaluated with 9 input random variables, including $\mu_S$, $\sigma_S$, $\mu_{BF}$, $\sigma_{BF}$, $\mu_a$, $\sigma_a$, $\mu_b$, $\sigma_b$ and $\rho_{ab}$. Again, the PEM procedure by Zhao and Ono (2000) can be used to evaluate the variation of the *SLS* failure probability.

The PEM approach requires an evaluation of the failure probability at each of a set of "estimating" points (or sampling points) of the input random variables. Thus, the computation of the failure probability needs to be repeated for a total of $N = 7 \times k$ times, where $k$ is the number of input random variables and the multiplier "7" represents the seven sampling points that are required in the seven-point PEM formulation by Zhao and Ono (2000). In each repetition, statistics of input random variables at each PEM estimating point must be assigned, and then the failure probability is evaluated using FORM. The resulting $N$ failure probabilities (at the completion of the inner loop shown in Fig. 4) are then used to compute the mean and standard deviation of the failure probability.

### 6.3 Step 3

Repeat Step 2 for each of the $M$ designs in the design domain. For each design, the mean and standard deviation of the failure probability are determined. This step is represented by the outer loop shown in Fig. 2.

### 6.4 Step 4

Perform a multi-objective optimization using non- dominated sorting genetic algorithm to establish a Pareto Front, followed by determination of feasibility robustness for choosing best design. This step is represents by the last two blocks (in the right side) of the flowchart shown in Fig. 4.

In the proposed RGD methodology, multi-objective optimization is required. In the illustrative example presented later, cost and design robustness are set as the objectives and safety (reliability) is achieved by means of a set of constraints. This is quite similar to the traditional reliability-based design except that the design robustness is explicitly considered as an additional objective. It is noted that the robustness in terms of standard deviation of the failure probability for each design is obtained in Step 3.

The concept of Pareto Front is briefly introduced with Fig. 5. When multiple objectives (in this case, two objectives) are enforced, it is likely that no single best design exists that is superior to all other designs in all objectives. However, a set of designs (such as $D_2$, $D_3$, and $D_4$ shown in Fig. 5) may exist that are superior to all other designs (such as $D_1$) in all objectives; but within the set, none of them is superior or inferior to others in all objectives. For example, $D_3$ is superior to $D_4$ in objective 1, but is inferior to $D_4$ in objective 2. This set of optimal designs constitutes a Pareto Front (Ghosh and Dehuri 2004).

Selection of a set of optimal designs (such as $D_2$, $D_3$, and $D_4$) that constitute Pareto Front is a multi-objective optimization problem. In this paper, the Non-dominated Sorting Genetic Algorithm version II (NSGA-II), developed by Deb *et al*. (2002), summarized later, is used for establishing the Pareto Front for its accuracy and efficiency.

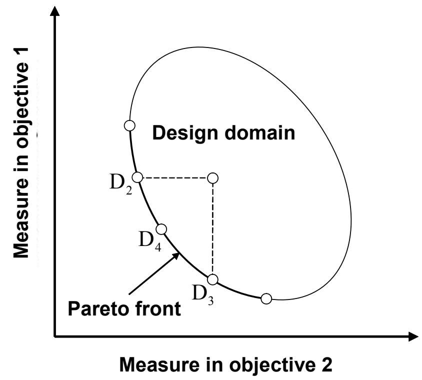


**Fig. 5 Illustration of Pareto Front constituted by non-dominated optimum designs**

## 7. TRADITIONAL RELIABILITY-BASED DESIGN OF SHALLOW FOUNDATION

The traditional reliability-based design of square shallow foundation is first presented herein to provide a reference. The spread foundation example is shown in Fig. 1 and statistics of uncertain parameters are assumed with a fixed value (that is, taking only mean values of these statistics in Tables 3, 4, and 5). The probability of *SLS* and *ULS* failure for each design for a combination of vertical permanent load component of $G$ and variable load of $Q$ is determined using FORM. This analysis is repeated for all possible designs in the design space. For illustration purpose, the results (*i.e.*, failure probabilities) are plotted only for designs with $D$ = 1.0 m, 1.5 m and 2.0 m, as shown in Fig. 6.

It can be seen from Fig. 6 that the probabilities of both *ULS* failure and *SLS* failure decrease with the increase of $B$ and $D$. The probability of failure for *ULS* and *SLS* is quite similar. As the *ULS* failure probability requirement is more stringent than the *SLS* failure probability requirement in this case, the former controls the design of shallow foundations, which is consistent with previous investigations (Wang and Kulhawy 2008; Wang 2011).

In a traditional reliability-based design, the reliability is used as a constraint to screen for acceptable designs, and then the best design is attained by selecting the least-cost design (Zhang *et al*. 2011). In this paper, the procedure for cost estimation by Wang and Kulhawy (2008), described previously, is adopted. It should be noted that cost estimation is not the focus of this paper, and that the proposed RGD approach is not dependent on any particular cost estimation method. In fact, any reasonable cost estimation methods can be used.

In the example discussed herein (Fig. 1), the reliability requirements defined in Eurocode 7 for foundation design, specifically, the target *ULS* reliability index $\beta_T^{ULS} = 3.8$ (corresponding to $p_T^{ULS} = 7.2 \times 10^{-5}$) and the target *SLS* reliability index $\beta_T^{SLS} = 1.5$ (corresponding to $p_T^{SLS} = 6.7 \times 10^{-2}$), are adopted (Wang 2011). If the minimum cost is the only criteria for selecting the "best" design after screening with reliability requirements, then the design with $B$ = 1.9 m and $D$ = 2.0 m will be selected.

The traditional reliability-based design is predicated on the accuracy of the estimated statistics of soil parameters and model

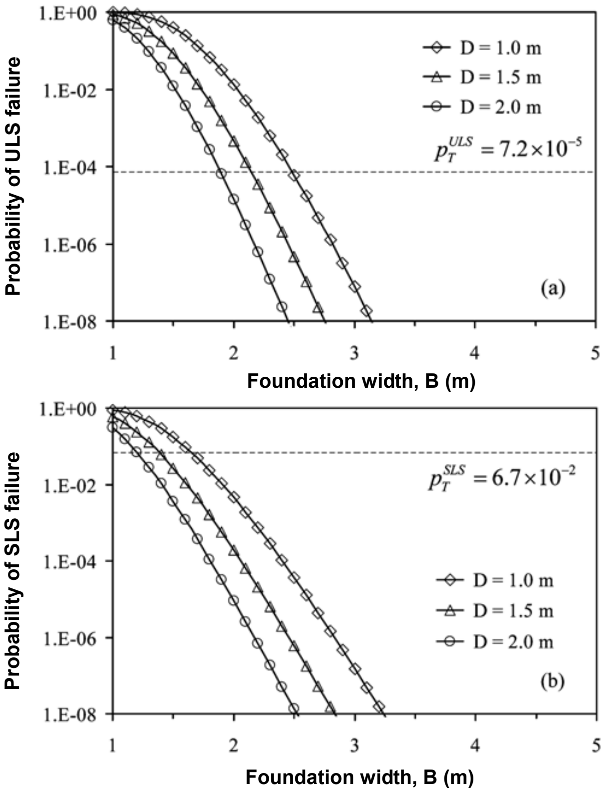


**Fig. 6 Probabilities of failure of selected designs with fixed mean and standard deviation of noise factors: (a) ULS failure; (b) SLS failure**

factors. To demonstrate the effect of the uncertainty of these estimated statistics on the reliability-based design, a series of analyses is performed. For demonstration purposes, the mean of each noise factor (soil parameters or model factor) is set at its sample mean and the standard deviation of each noise factor is assumed to vary in the range of 95% confidence interval.

Although not shown here, the uncertainty in the statistics of *SLS* model factor has little effect on the final design, which is consistent with previous finding that the *ULS* failure controls the design. Thus, only the variation in standard deviation of $\phi'$, denoted as $\sigma_S$, and the variation in standard deviation of $BF_Q$, denoted as $\sigma_{BF}$, are considered. For illustration purposes, both $\sigma_S$ and $\sigma_{BF}$ are assumed three different levels, namely, low, medium, and high variation. These three levels of variation are arbitrarily assigned to be at the lower bound of the 95% confidence interval, the mean value, and the upper bound of the 95% confidence interval.

Table 6 shows the least cost designs that satisfy the target failure probability requirement ( $p_f^{ULS} < p_T^{ULS} = 7.2\times10^{-5}$ ) at various levels of $\sigma_S$ and $\sigma_{BF}$. The results show that the least cost designs are sensitive to the assumed $\sigma_S$ and $\sigma_{BF}$. Under the lowest level of $\sigma_S$ and $\sigma_{BF}$ (among all cases in Table 6), the least cost design costs 769.4 USD, while it costs 1404.0 USD under the highest level of variation. Thus, in a traditional reliability-based design that uses target failure probability as a constraint, the selection of "best" design based solely on least cost is meaningful only if the statistics of noise factors (soil parameters and model factors) can be ascertained.

**Table 6 Least-cost designs under various standard deviation levels in noise factors**

| $\sigma_S$ (°) | $\sigma_{BF}$ | *B* (m) | *D* (m) | Cost (USD) |
|---|---|---|---|---|
| 1.12 | 0.148 | 1.6 | 2.0 | 769.4 |
| 1.12 | 0.203 | 1.8 | 1.8 | 910.8 |
| 1.12 | 0.260 | 1.9 | 2.0 | 1026.0 |
| 1.84 | 0.148 | 1.8 | 2.0 | 936.5 |
| 1.84 | 0.203 | 1.9 | 2.0 | 1026.0 |
| 1.84 | 0.260 | 2.1 | 1.9 | 1200.1 |
| 2.43 | 0.148 | 2.0 | 1.9 | 1104.0 |
| 2.43 | 0.203 | 2.1 | 2.0 | 1216.9 |
| 2.43 | 0.260 | 2.3 | 1.9 | 1404.0 |

If the standard deviation of noise factors is underestimated by a certain margin, then it is likely that an acceptable design (a design that meets *ULS* target failure probability) will no longer be satisfactory. For example, the design ($B$ = 1.9 m and $D$ = 2.0 m) was acceptable (meeting the target failure probability) at the uncertainty level of $\sigma_S$ = 1.84° and $\sigma_{BF}$ = 0.203. This design is re-analyzed with various levels of uncertainty. The results are shown in Table 7, which indicate that in many instances (where the uncertainty levels are higher than the level that was assumed in the previous design), the target *ULS* failure probability ( $p_T^{ULS} = 7.2\times10^{-5}$ ) is no longer satisfied.

## 8. RELIABILITY-BASED ROBUST GEOTECHNICAL DESIGN (RGD)

One way to reduce the effect of the uncertainty of the statistical characterization of soil parameters and model factors in a reliability-based design is considering robustness explicitly in the design. In this section, the reliability-based RGD methodology outlined previously is applied to the same shallow foundation design (see Fig. 1). For this demonstration exercise, the statistics of the noise factors listed in Tables 3, 4, and 5 are included in the analysis.

As per the flowchart of the RGD procedure shown in Fig. 4, the mean and standard deviation of the *ULS* failure probability, denoted as $\mu_p^{ULS}$ and $\sigma_p^{ULS}$, respectively, can be obtained for all possible designs in the design space using PEM. Since *ULS* controls the design in this case, only the *ULS* failure probability is of concern here. As an example, Fig. 7 shows the mean *ULS* failure probability ( $\mu_p^{ULS}$ ) for selected designs with $D$ = 1.0 m, 1.5 m and 2.0 m. Similarly, Fig. 8 shows the standard deviation of the *ULS* failure probability ( $\sigma_p^{ULS}$ ) of selected acceptable designs with $D$ = 1.0 m, 1.5 m and 2.0 m.

Because many designs that meet the safety requirement of $p_f^{ULS} < p_T^{ULS} = 7.2\times10^{-5}$ are associated with different levels of robustness (in terms of $\sigma_p^{ULS}$ ) and cost, a multi-objective optimization is needed.

**Table 7 ULS failure probability of a given design ($B$ = 1.9 m, $D$ = 2.0 m) under different uncertainty levels in noise factors**

| $\sigma_S$ (°) | $\sigma_{BF}$ | $B$ (m) | $D$ (m) | ULS failure probability, $p_f^{ULS}$ |
|---|---|---|---|---|
| 1.12 | 0.148 | 1.9 | 2.0 | 2.01E-08 |
| 1.12 | 0.203 | 1.9 | 2.0 | 1.95E-06 |
| 1.12 | 0.260 | 1.9 | 2.0 | 4.68E-05 |
| 1.84 | 0.148 | 1.9 | 2.0 | 6.83E-06 |
| 1.84 | 0.203 | 1.9 | 2.0 | 6.36E-05 |
| 1.84 | 0.260 | 1.9 | 2.0 | 3.83E-04 |
| 2.43 | 0.148 | 1.9 | 2.0 | 1.30E-04 |
| 2.43 | 0.203 | 1.9 | 2.0 | 4.77E-04 |
| 2.43 | 0.260 | 1.9 | 2.0 | 1.50E-03 |

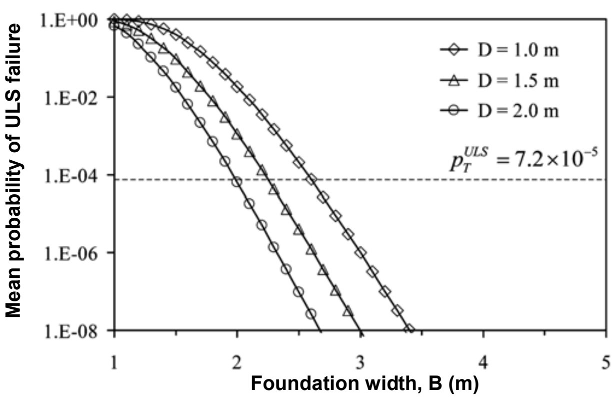


**Fig. 7 Mean *ULS* failure probabilities of selected designs considering variation in statistics of noise factors**

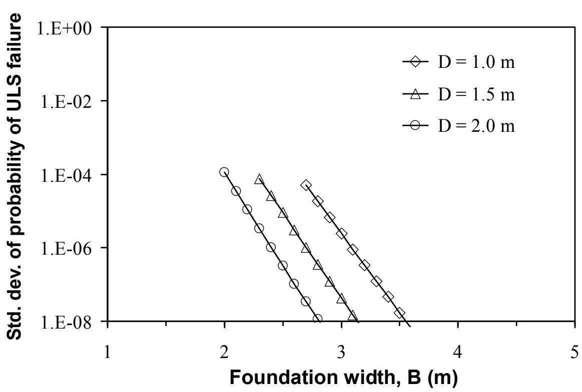


**Fig. 8 Standard deviation of *ULS* failure probabilities of selected acceptable designs considering variation in statistics of noise factors**

## 8.1 NSGA-II Algorithm to Obtain Pareto Front

As noted previously, the NSGA-II algorithm (Deb *et al.* 2002) is employed to search for the Pareto Front in the design space. The NSGA-II algorithm is summarized in the following (with reference to Fig. 9). First, a random "parent population" $P_0$ from the design space is created with a size of $n$. The term "parent population" is widely used in Genetic Algorithm (GA); here,

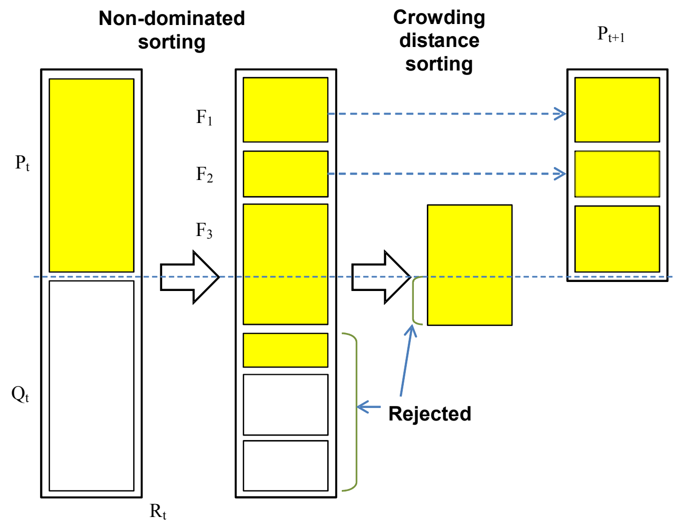


**Fig. 9 An Illustration of NSGA-II algorithm (Deb *et al.* 2002)**

it can be thought of as the first trial set of "optimal" designs. A series of genetic algorithm (GA) operations such as mutation and crossover are performed on "parent population" $P_0$ to generate the "offspring population" $Q_0$ with the same size of $N$. Then, an iterative process is adopted to refine the parent population. In the GA, each step in the iteration is termed as a "generation".

In the $t^{th}$ generation, the parent population $P_t$ and the offspring population $Q_t$ are combined to form an intermediate population $R_t = P_t \cup Q_t$ with a size of $2n$. Non-dominated sorting is next performed on $R_t$, which groups the points in $R_t$ into different levels of non-dominated fronts. For example, the best class is labeled $F_1$, and the second best class is labeled $F_2$, and so on. The best $n$ points are selected into parent population of the next generation, $P_{t+1}$. Using the scenario illustrated in Fig. 9 as an example, if the number of points in $F_1$ and $F_2$ is less than $n$, they will all be selected into $P_{t+1}$. Then, if the number of points in $F_1$ and $F_2$ and $F_3$ exceeds the population size $n$, the points in $F_3$ are sorted using the "crowding distance" sorting technique (Deb *et al.* 2002), which aims to maintain the diversity in the selected points. Thus, the best points in $F_3$ are selected to fill all remaining slots in the next population $P_{t+1}$. After obtaining $P_{t+1}$ in the $t^{th}$ generation, $P_{t+1}$ is then treated as the parent population in the next generation and the process is repeated until $P_{t+1}$ is converged. The final, converged $P_{t+1}$ is the Pareto Front (Juang and Wang 2013).

In the shallow foundation design example, this optimization with NSGA-II may be achieved by using target failure probability as a constraint and robustness and cost as objectives. Symbolically, this optimization can be set up as follows:

**Find** $\mathbf{d} = [B, D]$

**Subject to:** $B \in \{1.0\text{ m}, 1.1\text{ m}, 1.2\text{ m}, \ldots, 5.0\text{ m}\}$ and
$D \in \{1.0\text{ m}, 1.1\text{ m}, 1.2\text{ m}, \ldots, 2.0\text{ m}\}$
$\mu_p^{ULS} < p_T^{ULS} = 7.2\times10^{-5}$

**Objectives:** Minimizing the standard deviation of *ULS* failure probability ($\sigma_p$)
Minimizing the cost for shallow foundation.

As with any Genetic Algorithm (GA) process, the design parameters ($B$ and $D$ in this case) are generated in the discrete space. The population size of 100 with 100 generations is used in the NSGA-II optimization (Deb *et al*. 2002). Although not shown here, the points on the Pareto Front (a set of optimum designs) are initially very scattered, but gradually converge. For this shallow foundation design (Fig. 1), converged results are obtained at 20th generation. At convergence, 62 "unique" designs are selected into the Pareto Front, as shown in Fig. 10. It can easily be observed that there is an obvious trade-off relationship between cost and robustness. The obtained Pareto Front can be used as a design aid for the decision maker to select the "best" design based on the desired target cost or robustness level.

### 8.2 Selection of Best Design Based on Feasibility Robustness

The Pareto Front shown in Fig. 10 uses the standard deviation of the failure probability directly as a measure of robustness. While this Pareto Front provides a trade-off relationship that can aid in making informed design decisions, it may be desirable to use a relative measure of robustness, a more user-friendly index. Thus, the results shown in Fig. 10 are further refined.

"Feasibility robustness," as defined by Parkinson *et al.* (1993), is the design that can maintain feasible (or safe) status relative to the nominal constraint for a definable probability as it undergoes variations. For the design example of shallow foundation, the ultimate limit state (*ULS*) requirement controls the design. In the safety constraint that requires the *ULS* failure probability to be less than the target probability, $p_f^{ULS} \le p_T^{ULS} = 7.2 \times 10^{-5}$, the failure probability ( $p_f^{ULS}$ ) at a given state is a random variable that depends on the uncertainty in statistics of noise factors, and the target probability is a fixed value. Symbolically, feasibility robustness can be formulated as follows:

$$\Pr\left[(p_f^{ULS} - p_T^{ULS}) < 0\right] \ge P_0 \tag{8}$$

where $\Pr\left[(p_f^{ULS} - p_T^{ULS}) < 0\right]$ is the probability that the *ULS* safety constraint is satisfied, and $P_0$ is an acceptable probability pre-defined by the designer. Thus, an index may be created for assessing the feasibility robustness.

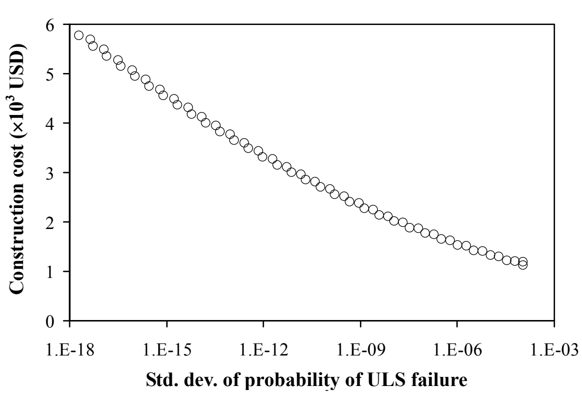


**Fig. 10 Converged Pareto Front for shallow foundation design obtained by NSGA-II based on two-objective (cost and robustness)**

Determination of the probability $\Pr\ [(p_f^{ULS} - p_T^{ULS}) < 0]$ requires the knowledge of the distribution of $p_f^{ULS}$, which is generally difficult to ascertain. Based on the previous studies by Most and Knabe (2010) and Luo *et al*. (2012b), the resulting histogram of the reliability index such as $\beta^{ULS}$ (corresponding to $p_f^{ULS}$ ) caused by variance in sample statistics can be approximated with a normal distribution. Thus, an equivalent counterpart in the form of $\Pr\ [(\beta^{ULS} - \beta_T^{ULS}) > 0]$, where $\beta_T^{ULS} = 3.8$ (corresponding to $p_T^{ULS} = 7.2 \times 10^{-5}$ ), may be used to assess the level of feasibility robustness.

The mean and standard deviation of $\beta^{ULS}$, denoted as $\mu_B$ and $\sigma_B$ respectively, can be obtained using FORM integrated with PEM. Then, Eq. (8) can be replaced by:

$$\Pr\left[(\beta^{ULS} - 3.8) > 0\right] = \Phi(\beta_\beta) \ge P_0 \tag{9}$$

where $\Phi$ is the cumulative standard normal distribution function, and $\beta_\beta$ is defined as:

$$\beta_\beta = \frac{\mu_\beta - 3.8}{\sigma_\beta} \tag{10}$$

The term $\beta_\beta$ may also be used as an index of feasibility robustness. The relationship between $\beta_\beta$ and the cost for the 62 designs on the Pareto Front is shown in Fig. 11. As expected, the results show that a design with higher feasibility robustness (higher $\beta_\beta$) requires a higher cost. Thus, a trade-off between cost and robustness is obvious. It is noted that in the lower cost range, the curve is relatively flat, indicating that a small increase in cost can result in a large increase in feasibility robustness, which is cost-efficient. In the higher cost range, however, the slope is relatively sharp, indicating that it costs a lot more to raise robustness, which is not cost-efficient.

By selecting a target feasibility robustness level ( $\beta_\beta^T$ ), the least-cost design among those on the Pareto Front can readily be identified. For example, when the target feasibility robustness is set at $\beta_\beta^T = 2$, which corresponds to an acceptance probability of $P_0 = 97.72\%$, the least-cost design is $B = 2.3$ m and $D = 2.0$ m, which costs 1423.7 USD. The least cost designs of the shallow foundation corresponding to different target feasibility robustness levels are listed in Table 8. The feasibility robustness offers an easy-to-use quantitative measure for making an informed design decision considering cost and robustness after satisfying the safety requirements.

## 9. ADDITIONAL DISCUSSION: EFFECT OF SPATIAL VARIABILITY

Recent studies (*e.g.*, Schweiger and Peschl 2005; Griffiths *et al*. 2009; Luo *et al*. 2011; Luo *et al*. 2012a) have shown that the traditional reliability analysis without considering spatial variability may yield an overestimation of the failure probability in many geotechnical problems. Thus, it would be of interest to examine the effect of spatial variability of soil parameters on the reliability-based robust design of shallow foundations. To demonstrate the procedure to consider the effect of spatial variability,

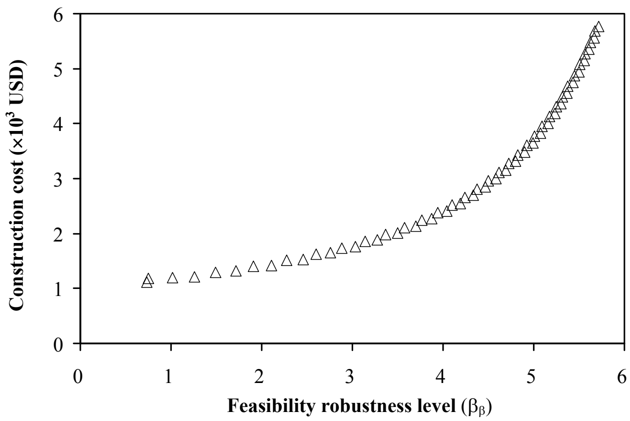


**Fig. 11 Cost versus feasibility robustness for all designs on Pareto Front**

**Table 8 Selected final designs at various feasibility robustness levels**

| $\beta_\beta$ | $P_0$ (%) | $B$ (m) | $D$ (m) | Cost (USD) |
|---|---|---|---|---|
| 1 | 84.13 | 2.1 | 1.9 | 1200.1 |
| 2 | 97.72 | 2.3 | 2.0 | 1423.7 |
| 3 | 99.87 | 2.6 | 2.0 | 1763.7 |
| 4 | 99.997 | 3.1 | 2.0 | 2409.8 |

the ten effective friction angles $\phi'$ (for dry sand, $c' = 0$) listed in Table 2 are assumed to have been obtained from triaxial tests conducted on samples taken at an equal interval of 1 m in this homogeneous sand.

To characterize the soil spatial variability, it is essential to determine a fundamental statistical indicator of spatial variability, namely, scale of fluctuation $\theta$, which is defined as the distance within which the soil properties show relatively strong correlation from point to point (Vanmarcke 1977 & 1983). Determination of scale of fluctuation $\theta$ generally requires a large amount of in-situ or experimental data taken over a wide range at site of concern, and many approaches have been proposed to determine $\theta$ (*e.g.*, DeGroot and Baecher 1993; Baecher and Christian 2003; Fenton and Griffiths 2008). However, in this example, as the sample size of effective friction angles $\phi'$ is quite small, it is difficult to determine the scale of fluctuation of $\phi'$. Nevertheless, according to Vanmarcke (1977), the vertical scale of fluctuation of $\phi'$of a site may be approximately estimated as: $\theta = 0.8\ (\overline{d})$ where $\overline{d}$ is the average distance between intersections of fluctuating property and its trend function. Based on the limited data in Table 2, $\overline{d}$ is estimated to be about 2 m, and thus $\theta \approx 1.6$ m, which is within the typical range of vertical scale of fluctuation, $\theta = 0.5$ m to 2.0 m, reported by Cherubini (2000). In the absence of sufficient data, for demonstration purpose, the vertical scale of fluctuation $\theta$ of $\phi'$ is assumed to be a lognormally distributed random variable with a mean of 1.6 m and a COV of 0.3 (Luo *et al*. 2012a). On the other hand, the horizontal scale of fluctuation is generally much larger than the foundation dimension, typically in the range of 10 m to 30 m; thus, the effect of the horizontal spatial variability may be neglected for the design of shallow foundations (Cherubini 2000).

One way to consider the effect of spatial variability is through a variance reduction technique. Vanmarcke (1983) pointed out that the averaged variability of soil properties over a large domain can be approximated with an equivalent variance. The averaged variance of soil parameter considering the spatial average effect can be obtained as:

$$\sigma_\Gamma^2 = \Gamma^2\ \sigma^2 \tag{11}$$

where $\sigma$ = the standard deviation of soil parameter of concern ($\phi'$ in this study); $\sigma_\Gamma$ = the reduced standard deviation of soil parameter considering the spatial average effect; and $\Gamma$ is the reduction factor defined as (assuming an exponential autocorrelation structure):

$$\Gamma^2 = \frac{1}{2}\left(\frac{\theta}{L}\right)^2 \left\{\frac{2L}{\theta} - 1 + \exp\left[-\frac{2L}{\theta}\right]\right\} \tag{12}$$

where $L$ is the characteristic length, which is generally problem-dependent. For a shallow foundation, the characteristic length may be approximately estimated as the sum of the embedment depth and the foundation width, $L = D + B$ (Cherubini 2000).

To consider the effect of spatial variability in the reliability-based robust design, the scale of fluctuation $\theta$ may be treated as an additional noise factor, and accordingly the statistical characterization of the uncertainty of this noise factor is included in the RGD approach (Fig. 4). The procedure to derive the Pareto Front is the same as presented previously. It is noted, however, that the standard deviation of $\phi'$ used in reliability analysis is automatically reduced to account for the spatial averaging effect through Eq. (11).

Figure 12 shows the feasibility robustness index $\beta_\beta$ for all designs on the derived Pareto Front that considers the effect of spatial variability. As a reference, the data from Fig. 11 (in which the effect of spatial variability is not considered) are also plotted in Fig. 12. It can be observed from Fig. 12 that for the same design (associated with a "unique" cost), the feasibility robustness index ($\beta_\beta$) considering spatial variability is higher than that without considering spatial variability. At a given cost, the percent difference in feasibility robustness caused by the effect of spatial variability is more profound in the lower cost range. As the cost increases, the effect of spatial variability becomes less significant, especially at the higher cost range.

The least cost designs of this shallow foundation at different feasibility robustness levels considering spatial variability effect are listed in Table 9. Compared to the results shown in Table 8, at the same feasibility robustness level the design considering spatial variability costs less than that without considering spatial variability. Thus, for the example shallow foundation studied, the design that achieves the same target feasibility robustness tends to be slightly over-designed (at a slightly higher cost) if spatial variability is not considered. At the same cost level (which implies the same design, as each point in Fig. 12 represent a unique design), the computed feasibility robustness is slightly lower if spatial variability is not considered. The implication is that the design that does not consider spatial variability is biased toward conservative (or safer) side in the shallow foundation design presented this paper.

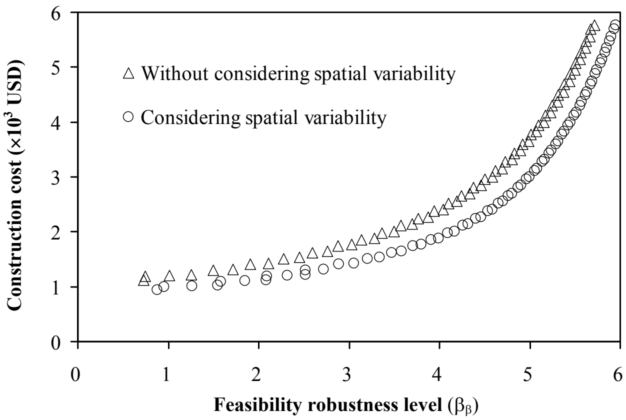


**Fig. 12 Comparison of cost versus feasibility robustness for all designs on Pareto Fronts derived with and without considering spatial variability**

**Table 9 Selected final designs at various feasibility robustness levels considering spatial variability**

| $\beta_\beta$ | $P_0$ (%) | $B$ (m) | $D$ (m) | Cost (USD) |
|---|---|---|---|---|
| 1 | 84.13 | 1.9 | 1.9 | 1011.9 |
| 2 | 97.72 | 2.0 | 2.0 | 1119.4 |
| 3 | 99.87 | 2.3 | 1.9 | 1404.0 |
| 4 | 99.997 | 2.7 | 2.0 | 1885.0 |

## 10. SUMMARY AND CONCLUDING REMARKS

This paper presents the rationale for including robustness explicitly in the design of a geotechnical system. Quantification of uncertainties in soil parameters and geotechnical models is a prerequisite for a reliability-based design. Due to inexactness of geotechnical models and lack of soil parameters data, uncertainties exist in the derived statistics of model factors and soil parameters, which compromises the effectiveness of the reliability-based design. The proposed reliability-based robust geotechnical design (RGD) approach can reduce the effect of these unavoidable uncertainties by achieving a certain level of design robustness, in addition to meeting safety and cost requirements.

When multiple design objectives (including safety, cost, and robustness) are imposed, a single best design often does not exist. In fact, an optimization with multiple design objectives usually leads to a Pareto Front, which is a set of optimal designs that are superior to all other designs in the design space, but within the set, no design is dominated by any other designs. By applying the proposed RGD methodology implemented in a multi-objective optimization framework, a Pareto Front is derived, which describes a trade-off relationship between cost and robustness at a given safety (reliability) level. The derived Pareto Front and the associated feasibility robustness index enable the engineer to make an informed design decision.

It should be noted that RGD is not a design method to compete with the traditional design methods; rather, it is a complementary design strategy to both reliability-based and factor of safety-based design methods. The proposed RGD methodology has been illustrated in this paper with an example of spread foundation design. The significance of this methodology has been elaborated and demonstrated.

This paper represents the first step in developing the RGD methodology. The methodology is being adapted and refined at Clemson University in an ongoing research project. Further investigations by interested third parties are also encouraged to advance this design methodology.

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