Simplified Approach for Reliability-Based Design against Basal-Heave Failure in Braced Excavations Considering Spatial Effect

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Abstract: This paper presents a simplified approach for reliability analysis of basal heave in a braced excavation considering the spatial variability of soil parameters. The first-order reliability method (FORM) with a variance reduction technique is employed to model the spatial variability in lieu of the conventional random field modeling (RFM). The proposed approach yields results that are comparable with those obtained using the conventional RFM approach that relies on Monte Carlo simulation. The proposed approach requires much less computational effort, is easy to use, and has potential as a practical tool for reliability-based design that has to deal with spatial variability of soils.

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Introduction

Conventionally, basal-heave stability in a braced excavation in clay is evaluated with a factor of safety (FS), defined as the ratio of the resistance over the load (Terzaghi 1943; Bjerrum and Eide 1956). In designs on the basis of FS, soil parameters are generally considered as constant inputs for simplicity. However, FS greater than unity does not guarantee basal-heave stability in clay because of the inherent variability of soil parameters, such as undrained shear strength and unit weight. Although uncertainty in the soil parameters is often dealt with by use of conservative parameter values, the probabilistic approach using reliability analysis offers a more direct and consistent way to consider soil variability explicitly. Examples of the reliability-based design for basal-heave stability of a braced excavation can be found in Goh et al. (2008) and Wu et al. (2010), in which a design chart that relates the probability of basal-heave failure (pF) to the FS is provided.

In traditional reliability analysis, uncertain soil parameters are interpreted as continuous random variables defined by their probability distributions or sample statistics (e.g., mean value and standard deviation). The soil parameters are often considered as homogeneous or “spatially constant” fields in such analysis. However, the uncertainty stems not only from the inherent variability, but also from spatial variability. For the latter, the variation of soil parameters may be modeled with the random field theory (Vanmarcke 1977). Spatial variability is generally described by the scale of fluctuation, which is the maximum distance beyond which the spatially random parameters are uncorrelated (Akbas and Kulhawy 2009). As the scale of fluctuation decreases, the soil parameters in the random field tend to vary more rapidly; conversely, as the scale of fluctuation increases, the soil parameters in the random field tend to vary less and become more uniform.

The effect of spatial variations of soil properties can be significant in many geotechnical problems, as demonstrated by recent studies of random field modeling (RFM) (Fenton and Griffiths 2003; Fenton et al. 2005; Griffiths and Fenton 2009; Huang et al. 2010). In these studies, local averaging subdivision techniques were adopted to model the random field. Of course, the random field can also be modeled by using other approaches, such as the Cholesky decomposition method (Fenton 1997; Haldar and Babu 2008; Srivastava et al. 2010; Suchomel and Mašín 2010). The conventional RFM, however, has to be realized with Monte Carlo simulation (MCS), and a large number of simulations are needed to obtain convergent results.

As an alternative to the conventional RFM, simplified methods that implement a proper spatial averaging strategy have been shown to be effective in considering the effect of spatial variability of soil properties (Phoon and Kulhawy 1999a, b; Goh et al. 2008; Klammler et al. 2010; Most and Knabe 2010). To consider spatial averaging in the reliability analysis, variances of soil parameters are reduced by multiplying a reduction factor that is a function of scale of fluctuation and characteristic length (Vanmarcke 1983). Typical scales of fluctuation for commonly used soil properties have been reported by Phoon and Kulhawy (1999b). The characteristic length often depends on the problem under investigation and is generally assumed to be equivalent to the length of the failure surface (Schweiger and Peschl 2005; Most and Knabe 2010) or taken as the distance in the random field over which the variance reduction is calculated (Cherubini 2000; Babu and Dasaka 2008). Recent studies (Peschl and Schweiger 2003; Suchomel and Mašín 2010;
Luo et al. (2012) show that the variance reduction–based simplified approach can capture the overall trend derived from the conventional RFM approach.

Although RFM coupling with MCS is a rigorous approach to account for spatial variability, the use of this approach is very limited in geotechnical reliability-based design for at least two reasons: (1) a rigorous simulation of the random field is very time-consuming, which is not practical, especially for complicated problems, such as braced excavations; (2) MCS is further complicated by the lack of knowledge on spatial variability (e.g., the scale of fluctuation could be uncertain). Contrarily, with the variance reduction-based simplified approach, traditional reliability methods can be adopted in lieu of MCS to reduce the computational effort. However, the application of such a simplified approach requires a proper assessment of the characteristic length, which is problem-specific and may be difficult to determine.

This paper presents a simplified approach that considers the spatial variability of soil parameters for reliability analysis of basal-heave stability in a braced excavation in clay. This approach is developed and presented in five steps. (1) The conventional RFM using the exponential correlation function is conducted in a study of basal-heave stability to provide a reference for further development. (2) By trial and error, the variance reduction factor is determined, with which the simplified approach can yield results that are comparable with those obtained using the conventional RFM. (3) The characteristic length for the stability analysis is back-calculated on the basis of the derived variance reduction factors. (4) The proposed approach, which is a first-order reliability method (FORM) implemented with the variance reduction to account for the spatial variability, is adopted for reliability analysis of basal-heave stability. (5) The effect of uncertainty of the scale fluctuation (because of the lack of knowledge) is further evaluated with the proposed approach. The paper concludes that the proposed simplified approach is easy to use and yields results that are comparable with those obtained with the computationally expensive RFM approach.

Factor of Safety against Basal-Heave Failure

**Slip Circle Method**

The basal-heave failure in a braced excavation in clay occurs when the shear strength of the soil cannot support the weight of the soil within the critical zone around the excavation. Soil outside the excavation zone moves downward and inward because of its own weight, and the soil inside the excavation zone is forced to heave. The bracing system will collapse if the amount of basal-heave movement is excessive. Traditionally, the basal-heave stability is evaluated with FS using the deterministic approach. Semiempirical methods (Terzaghi 1943; Bjerrum and Eide 1956; Eide et al. 1972; Chang 2000) to estimate FS are widely used in the traditional deterministic design.

In this paper, the slip circle method adopted by Japanese, Chinese, and Taiwanese building codes [Japanese Society of Architecture (JSA) 1988; Professional Standards Compilation Group (PSCG) 2000; Taiwan Geotechnical Society (TGS) 2001] to calculate the FS against basal heave is used for its simplicity to consider the increase of undrained shear strength with depth and for its convenience to implement random field theory. With the slip circle method, the FS is defined as (Fig. 1)

\[
FS = \frac{M_R}{M_D}
\]  

(1)

in which \(M_R\) and \(M_D\) = resistance moment and driving moment, respectively. The driving moment \(M_D\) is caused by the weight of soil and possible surcharge

\[
M_D = W \times \frac{r}{2} + q_s \times \frac{r^2}{2}
\]  

(2)

in which \(W = \) total weight of the soil in front of the vertical failure plane and above the excavation surface; \(q_s = \) surcharge; \(r = \) radius of the slip circle; and \(r = H_w - H_r\), in which \(H_w = \) length of diaphragm wall, and \(H_r = \) depth of the final strut. The resistance moment \(M_R\) comes from three arcs \((bc, ed, de)\) along the slip surface, as shown in Fig. 1. Although uniform undrained shear strength may be used in the computation of FS, the undrained shear strength generally increases with depth for most normally consolidated clay. However, the ratio of undrained shear strength over the effective overburden stress \((s_u/\sigma_v^0)\) remains approximately constant (Ladd and Foott 1974). For this reason, the slip circle method can be easily adapted to consider the increase of \(s_u\) with depth. The total resistance moment \(M_R\) is computed by summing the resistances contributed by all the small arcs

\[
M_R = r \times \int_{0}^{\beta} s_u \times r \times d\beta
\]  

(3)

in which \(\beta = \) angle from \(\theta b\) to the current slice, as shown in Fig. 1.

In a deterministic analysis of basal-heave stability in a braced excavation in clay, the required FS generally depends on the method used. The recommended minimum required FS is 1.2 (JSA 1988; PSCG 2000; TGS 2001) when the slip circle method is employed. In the slip circle method, the resistance is computed using the summation of the resistance of numerous small arcs [Eq. (3)]. This formulation makes it easy to implement the RFM, which is the main reason behind the choice of the slip circle method in this study.

**Gamma Sensitivity Index**

Many factors influence the basal-heave stability in a braced excavation in clay, as shown in Eqs. (2) and (3). In this paper, the relative importance of those input variables is first examined using the gamma sensitivity index (Der Kiureghian and Ke 1985), which is a by-product of reliability analysis. This index is expressed as
in which $\gamma_i = \text{gamma sensitivity index for the } i^{th} \text{ input variable}; \alpha = \text{directional cosine at the design point in the original random variable space}; J_{x,y} = \text{Jacobian matrix with element of } \partial y/\partial x, \text{ with } y = T(x) \text{ in which } T(\cdot) \text{ is an orthogonal transformation function}; y_i = \text{uncorrelated standard normal random variable}; \text{ and } M = \text{diagonal matrix of the standard deviation of each parameter } x_i$. The uncertain variables considered in this study include $x_1 = s_u/\sigma_v$ (normalized undrained shear strength); $x_2 = H_e$ (excavation depth); $x_3 = H_w$ (length of diaphragm wall); $x_4 = H_s$ (depth of final strut); $x_5 = D$ (depth of groundwater table); $x_6 = \gamma$ (unit weight of soil); and $x_7 = q_s$ (surcharge). The gamma sensitivity index indicates the relative contribution of each of these input variables to the computed reliability index or failure probability. A greater gamma sensitivity index value indicates a greater influence of the variable of concern on the failure probability.

On the basis of FORM analysis of basal-heave stability without considering spatial variability, the gamma sensitivity index for each of the seven input parameters is obtained with Eq. (4). The statistics of the uncertain parameters used in this FORM analysis are shown in Table 1. For this gamma sensitivity analysis, the mean of $s_u/\sigma_v$ (normalized undrained shear strength) is set at 0.3, and the coefficient of variation (COV) of $s_u/\sigma_v$ is varied between 0.1 and 0.6. As shown in Fig. 2, the parameter $s_u/\sigma_v$ is found to have the greatest influence on the probability of basal-heave failure, and all other factors are relatively insignificant. The gamma sensitivity index of $s_u/\sigma_v$ is also found to increase drastically with the COV of $s_u/\sigma_v$. Thus, this paper is focused on the effect of the spatial variability of the normalized undrained shear strength $s_u/\sigma_v$ on the probability of basal-heave failure in a braced excavation.

**Table 1. Parameters for Basal-Heave Stability Problem Shown in Fig. 1**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Mean</th>
<th>COV of parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normalized undrained shear strength</td>
<td>$s_u/\sigma_v$</td>
<td>0.30</td>
<td>0.1–0.6</td>
</tr>
<tr>
<td>Unit weight of soil</td>
<td>$\gamma$</td>
<td>19 kN/m$^3$</td>
<td>0.05$^a$</td>
</tr>
<tr>
<td>Surcharge</td>
<td>$q_s$</td>
<td>10 kPa/m</td>
<td>0.2$^b$</td>
</tr>
<tr>
<td>Depth of groundwater table</td>
<td>$D$</td>
<td>2 m</td>
<td>0.05$^c$</td>
</tr>
<tr>
<td>Final excavation depth</td>
<td>$H_e$</td>
<td>20 m</td>
<td>0.05$^c$</td>
</tr>
<tr>
<td>Final strut depth</td>
<td>$H_s$</td>
<td>17 m</td>
<td>0.05$^c$</td>
</tr>
<tr>
<td>Penetration depth</td>
<td>$H_p$</td>
<td>24 m</td>
<td>0.05$^c$</td>
</tr>
</tbody>
</table>

$^a$Based on coefficient of variation values given by Harr (1987) and DiMaggio (2008).

$^b$Wu et al. (2010).

$^c$Hsiao et al. (2008).

**Fig. 2. Gamma sensitivity index at various coefficients of variation of $s_u/\sigma_v$ on the basis of reliability analysis**

**Stationary Random Field Modeling of $s_u/\sigma_v$**

To provide a reference for the proposed simplified approach for reliability-based design against basal-heave failure in a braced excavation considering spatial random effect, the conventional RFM of the normalized undrained shear strength $s_u/\sigma_v$ is first conducted. The parameter $s_u/\sigma_v$ is modeled using a stationary lognormal random field. The mean of the undrained shear strength $s_u$ of the clay often increases linearly with depth; however, this trend is removed by adopting the normalized parameter $s_u/\sigma_v$. The stationary random field of $s_u/\sigma_v$ has the characteristics of a “second-order process” (Baecher and Christian 2003): (1) the mean and variance of $s_u/\sigma_v(z)$ are the same regardless of the “absolute” location of $z$; and (2) the correlation coefficient between $s_u/\sigma_v(z_1)$ and $s_u/\sigma_v(z_2)$ is the same regardless of the “absolute” locations of $z_1$ and $z_2$; rather, it depends only on the distance between $z_1$ and $z_2$. All other input parameters are modeled as spatially constant lognormal variables or constants. The assumption of lognormal distribution for inherent variability for soil properties is not uncommon in the RFM (Akbas and Kulhawy 2009; Griffiths et al. 2009). The assumption of lognormal distribution prevents negative values for soil parameters and is supported by previous studies (Phoon and Kulhawy 1999b).

Variations of $s_u/\sigma_v$ in the field are represented by its scale of fluctuation $\theta$, mean value $\mu_n$, and COV$_n$ (the subscript in the last two terms, in, denotes the statistic for lognormal distribution). The standard deviation and mean of the equivalent normal distribution of $s_u/\sigma_v$, denoted as $\ln(s_u/\sigma_v)$, are expressed as

$$\sigma_n = \sqrt{\ln(1 + \text{COV}_n^2)}$$

$$\mu_n = \ln \mu_n = \frac{1}{2} \sigma_n^2$$

in which the subscript $n$ denotes normal distribution. The lognormally distributed random field of $s_u/\sigma_v$ can be obtained by the transformation (Fenton et al. 2005)

$$s_u/\sigma_v(x_i) = \exp[\mu_n + \sigma_n \cdot G_n(x_i)]$$

in which $\mu_n$ and $\sigma_n$ are determined from Eqs. (5) and (6); $x_i$ is spatial position at which $s_u/\sigma_v$ is modeled; and $G_n(x_i) = \text{normally distributed random field with zero mean, unit variance, and correlation function } \rho(\tau)$, in which $\rho(\tau) = \text{an exponentially decaying correlation function}$ (Jaksa et al. 1999; Haldar and Babu 2008)

$$\rho(\tau) = \exp\left(-\frac{2\tau}{\theta}\right)$$

in which $\tau = |x_i - x_j|$ is the absolute distance between any two points in the random field; and $\theta$ = scale of fluctuation. The correlation matrix is built with the correlation function and can be decomposed by Cholesky decomposition (Fenton 1997; Haldar and Babu 2008; Suchomel and Mašín 2010; Srivastava et al. 2010)

$$L \times L^T = \rho$$

With the matrix $L$, the correlated standard normal random field can be obtained by linearly combining the independent variables as follows (Fenton 1997):

$$G_n(x_i) = \sum_{j=1}^{M} L_{ij} Z_j \quad i = 1, 2, \ldots, M$$

in which $M = \text{number of points in the random field}$; and $Z_j = \text{sequence of independent standard normally distributed random variables}$.

To begin with, two random variables uniformly distributed between zero and one, $U_j$ and $U_{j+1}$, are generated first. Then two independent standard normally distributed variables are given by

$$Z_j = \sqrt{-2 \ln(1-U_j)} \cos(2\pi U_{j+1})$$  \hspace{1cm} (11)

$$Z_{j+1} = \sqrt{-2 \ln(1-U_j)} \sin(2\pi U_{j+1})$$  \hspace{1cm} (12)

The stationary random field of the normalized undrained shear strength $s_u/\sigma_v^*$ at each spatial position is obtained by Eq. (7) for a specified mean, standard deviation, and scale of fluctuation. The MCS is then used to generate samples in the lognormal random field. Each MCS process involves the same mean, standard deviation, and scale of fluctuation of $s_u/\sigma_v^*$. However, the spatial distribution varies among these simulations. Given a sufficient number of simulations, the output, such as $\Gamma$ or FS [Eq. (3)] or FS [Eq. (1)], can be obtained and statistically analyzed to produce estimates of the probability density function of $M_R$ or FS and the failure probability $p_f$. The failure probability $p_f$ is computed as the ratio of the number of simulations that yield failure (FS < 1) over the total number of simulations $N$. After a trial-and-error process, it was found that 100,000 simulations are sufficient to reach convergence in all scenarios associated with this basal-heave problem.

**Spatial Averaging Effect**

Spatial averaging is a concept with which the spatial variability of the soil property is averaged to approximate a random variable that represents a soil parameter (Vanmarcke 1977). The variability of the averaged soil property over a large domain is less than over a small domain. The reduced variability of the soil properties over a large domain can be quantified with the variance reduction technique. The reduction is computed using the variance reduction function, which is a function of the scale of fluctuation $\theta$ and characteristic length $L$. The form of the variance reduction function depends on the type of correlation function employed.

To consider spatial averaging in a reliability analysis, the variances of soil parameters may be reduced by multiplying a factor known as the variance reduction factor, which is computed using the variance reduction function (Vanmarcke 1983). Many successful applications of the variance reduction technique have been reported in the literature, for example, the constant model (Cherubini 2000; Schweiger and Peschl 2005), triangular model (Babu and Dasaka 2008), and exponential model (Most and Knabe 2010). The exponential model, which is often employed in the RFM in geotechnical engineering, is adopted herein. The variance reduction function for the exponential model is given as follows (Vanmarcke 1983):

$$\Gamma^2 = \frac{1}{2} \left( \frac{\theta}{L} \right)^2 \left[ 2L \frac{\theta}{\phi} - 1 + \exp \left( - \frac{2L}{\phi} \right) \right]$$  \hspace{1cm} (13)

in which $\theta = \text{scale of fluctuation}$; and $L = \text{characteristic length}$. Given the variance reduction factor $\Gamma^2$, the reduced variance $\sigma^2_{\Gamma}$ can be obtained with the following equation:

$$\sigma^2_{\Gamma} = \Gamma^2 \cdot \sigma^2$$  \hspace{1cm} (14)

**Reliability Analysis of Basal-Heave Stability Considering Spatial Variability**

**Random Field Modeling of Clay for Basal-Heave Stability Analysis**

Past studies (Goh et al. 2008; Wu et al. 2010) on basal-heave stability have shown high failure probability $p_f$ can exist in a design that meets the minimum FS requirement specified in the codes. However, yielding high failure probabilities for those designs that are known to be “safe” raises questions, because the codes are generally conservative, and exceeding the minimum FS requirement would indicate a safe design. One possible reason for having a higher computed failure probability than what the experience or the code would suggest is overestimation of the variation of soil parameters, which may be caused by the negligence of the effect of spatial variability in traditional reliability analysis.

In this paper, the preceding issue is examined within the context of basal-heave stability. Basal-heave stability in a braced excavation is examined using the conventional RFM with the Cholesky decomposition method. The excavation case analyzed by Wu et al. (2010), shown in Fig. 1 and with additional data shown in Table 1, is employed in this study. In reference to Fig. 1 and Table 1, all soil and structural parameters, except the normalized undrained shear strength $s_u/\sigma_v^*$, are treated as spatially constant random variables or constants. The parameter $s_u/\sigma_v^*$ is modeled as a spatially random variable.

The COV of undrained shear strength $s_u$ can be as high as 0.8 but typically is approximately 0.3 (Phoon and Kulhawy 1999a). In this paper, the COV of $s_u/\sigma_v^*$ is first selected as 0.3. However, the effect of the variation of this COV will be subsequently examined. According to Phoon and Kulhawy (1999a), the average horizontal and vertical scales of fluctuation for clay are 50.7 and 2.5 m, respectively. Thus, only the vertical spatial randomness is modeled in this paper, because the horizontal scale of fluctuation is much greater, and its effect is far less significant. For basal-heave stability analysis in a braced excavation, only the random field from the depth of the final strut to the bottom of the diaphragm wall (Fig. 1) needs to be considered because the resistance moment comes only from this region.
The Cholesky decomposition method is not practical if the number of points in the random field exceeds 500 (Fenton 1997). In this study, arc $b c d$ on the slip circle, as shown in Fig. 1, is subdivided by means of equal vertical distance into 100 small arcs (elements), which are considered sufficient in both stability analysis and RFM. Arc $c d e$ is subdivided in the same way as arc $c d$. Further refinement with more than 100 elements (arcs) is not necessary because it yields practically the same results.

Fig. 3 shows an example of the simulated spatial variability of normalized undrained shear strength $s_u / \sigma_0'$ with different scales of fluctuation ($\theta = 0.5, 2.5, 5, 10, 100,\text{ and } 1000\text{ m}$). As expected, the spatial variation in the case of smaller $\theta$ is much more significant than that for larger $\theta$. As $\theta$ decreases toward zero, the random field $s_u / \sigma_0'$ tends to vary drastically from point to point; conversely, as $\theta$ increases toward infinity, the random field $s_u / \sigma_0'$ tends to become uniform (or spatially constant) in each simulation. Traditional reliability analysis often assumes the field to be spatially constant, and the effect of spatial correlation is ignored.

For a given mean, COV, and $\theta$ of $s_u / \sigma_0'$, MCS may be carried out, and in each simulation, the same mean, COV, and $\theta$ are used to generate the random field. After a sufficient number of simulations ($10^5$ in this study), the histogram or the probability density for the output variable (e.g., FS) can be obtained. Fig. 4 shows an example of a histogram of the computed FS using the conventional RFM with 100,000 simulations under the following scenario: mean of $s_u / \sigma_0' = 0.3$; COV of $s_u / \sigma_0' = 0.3$; and $\theta = 2.5\text{ m}$. The shape of the histogram suggests a lognormal distribution. The failure probability $p_f$ is determined by the ratio of the number of simulations with FS $< 1.0$ over the total number of simulations, which is the area under the fitted curve for FS $< 1.0$.

To study the effect of spatial variability, a series of scales of fluctuation ($\theta = 0.5, 2.5, 5, 10, 100,\text{ and } 1000\text{ m}$) is selected in the reliability analysis. For each scale of fluctuation, $10^3$ simulations using MCS are conducted, and the results are shown in Fig. 5. For each data point in Fig. 5, the execution time for $10^3$ simulations is approximately 4 min on a laptop PC equipped with an Intel Pentium Dual CPU T2390 running at 1.86 GHz. The effect of the scales of fluctuation is very obvious; smaller scale of fluctuation results in smaller $p_f$ at the same FS. As shown in Fig. 5, if the target $p_f$ is set at $10^{-3}$, the required FS is approximately 1.7 at $\theta = 2.5\text{ m}$ [this $\theta$ value is mean of the vertical scale of fluctuation for clay as per Phoon and Kulhawy (1999a)] and is approximately 2.6 at $\theta = 1,000\text{ m}$ (this $\theta$ value is close to spatially constant condition). On the other hand, for FS $= 1.2$, the minimum value that is adopted in many codes (JSA 1988; PSCG 2000; TGS 2001) for the design of excavations against basal heave on the basis of the slip circle method, the failure probability $p_f$ is approximately 0.32 under the condition of $\theta = 1,000\text{ m}$ (≈ spatial constant). Because basal-heave failure occurs infrequently, these codes are considered adequate in practice; therefore, the failure probability of 0.32 obtained from the reliability analysis that does not consider spatial variability (emulated by the case with $\theta = 1,000\text{ m}$) is likely to be overestimated.

Finally, the analysis of basal-heave stability presented for RFM of clay is primarily used as a reference for the subsequent study of the effect of spatial variability using the variance reduction-based simplified approach.

**Parametric Study**

A series of parametric analyses are conducted to study the influence of spatial variability on the reliability-based design of braced excavation (basal-heave stability) in clay. For these analyses, only the inherent variability and the spatial variability of $s_u / \sigma_0'$ are considered to assess the effect of the spatial correlation. All other input parameters are treated as constant parameters (only the mean values shown in Table 1 are used in the analysis). For $s_u / \sigma_0'$, the following ranges of parameters are analyzed:
For each pair of COV and $\theta$, $10^5$ MCS runs are executed, and the mean and COV of the resulting $10^5$ resistance moments $M_R$ are obtained. Fig. 6 shows how the mean of the normalized $M_R$, defined as the ratio of the $M_R$ obtained from MCS for a given pair of COV and $\theta$ values over the $M_R$ obtained from a deterministic analysis that uses the mean values of all input parameters, varies with the COV and $\theta$ of $s_u/\sigma'$. The mean of the normalized $M_R$ is shown to be approximately 1.0, indicating that the mean of the $M_R$ through $10^5$ simulations is consistent with the deterministic solution regardless of the inherent variability and the spatial variability of $s_u/\sigma'$. This is expected because the resisting moment $M_R$ in the slip circle method is a linear function of undrained shear strength $s_u$.

Fig. 7 shows how the COV of $M_R$ changes with the COV and $\theta$ of $s_u/\sigma'$. Two observations can be made: (1) at the same COV level, the COV of $M_R$ increases almost linearly with the COV of $s_u/\sigma'$; and (2) at the same COV of $s_u/\sigma'$, the COV of $M_R$ increases with increasing $\theta$ and reaches the maximum value at $\theta = \infty$. The COV of $M_R$ at $\theta = \infty$ approaches the 1:1 line. As shown in Fig. 7, smaller $\theta$ results in smaller variability of $M_R$, which corresponds to smaller scale of fluctuation results in a larger variance reduction in the soil parameters, which would yield a smaller variation of output responses.

Furthermore, the failure probability $p_f$ for each combination of COV and $\theta$ is shown in Fig. 8. The failure probability $p_f$ increases with both COV and $\theta$ of $s_u/\sigma'$.

With both COV and $\theta$ of $s_u/\sigma'$, for a given COV, the maximum failure probability $p_f$ is reached at $\theta = \infty$; the implication is that the design can be too conservative without considering the effect of spatial variability of soil parameters.

**Simplified Approach Using Variance Reduction Technique**

As mentioned previously, the focus of this paper is on the simplified approach using the variance reduction technique. To apply this technique, it is necessary to determine an appropriate characteristic length $L$. In this regard, the variance reduction factor $\Gamma^2$ for the simplified approach is first established by matching the solutions obtained from the variance reduction-based simplified approach with those from RFM. Again, only $s_u/\sigma'$ is modeled as a spatially random variable to study the influence of the inherent variability and the spatial variability of $s_u/\sigma'$. All other parameters are treated as constants. The criterion for matching the two approaches (the simplified approach versus RFM) is to achieve the same level of variability of the response ($M_R$ in this case), because the mean $M_R$ is expected to be approximately the same (as shown in Fig. 6).

Through this calibration, the variance reduction factor $\Gamma^2$ to be used in the simplified approach for a given case is obtained. Fig. 9 shows a flowchart for searching for the reduction factor $\Gamma$ for a given pair of standard deviation ($\sigma$) and scale of fluctuation ($\theta$) of $s_u/\sigma'$.

In Fig. 10, the flow sequence on the left summarizes the procedure of the conventional RFM with the Cholesky decomposition method [Eqs. (7)–(12)]. After 100,000 simulations of the basal-heave stability analysis, the standard deviation of $M_R$ (denoted as $\sigma_{NS}$) is obtained. By assuming a reduction factor $\Gamma$ for this case (with the same $\sigma$ and $\theta$ of $s_u/\sigma'$), the reduced variance and the reduced standard deviation $\sigma_{\Gamma}$ can be obtained from Eq. (14). With the flow sequence on the right, 100,000 simulations of the basal-heave stability analysis are conducted using the simplified approach with known $\sigma_{\Gamma}$ and the standard deviation of $M_R$ (denoted as $\sigma_{\Gamma}$) is obtained. The simplified approach is repeated with different assumed $\Gamma$ values, and the $\Gamma$ value at which $\sigma_{NS} = \sigma_{\Gamma}$ is the optimum reduction factor for a given pair of $\sigma$ and $\theta$ of $s_u/\sigma'$. In this study, the condition of $\sigma_{NS} = \sigma_{\Gamma}$ is signaled by the stopping criterion $|\sigma_{NS} - \sigma_{\Gamma}|/\sigma_{\Gamma} \leq 10^{-3}$.

Fig. 10 shows the back-calculated $\Gamma$ values for various pairs of $\sigma$ (or COV) and $\theta$ of $s_u/\sigma'$ using the MCS-based RFM approach. It is apparent that the inherent variability rarely influences the variance reduction at the same COV level. The reduction factor $\Gamma$ depends only on $\theta$ at the same COV level, which is consistent with the variance reduction models presented in the literature (Vanmarcke 1983).
Alternatively, the reduction factor $\Gamma$ can also be determined using the variance reduction function if the characteristic length is known. To find an appropriate characteristic length for the basal-heave problem, the reduction factors are evaluated using the exponential model [Eq. (13)] with three assumed characteristic lengths $L = 27$, $39$, and $98$ m. The first characteristic length $L = 27$ m is the distance $od$ (from the depth of the final strut to the bottom of the diaphragm wall), as shown in Fig. 1. This length is the vertical scale of the spatially random region. The second characteristic length $L = 39$ m is the length of arc $cd$, and the third characteristic length $L = 98$ m is the length of the sliding surface (arc $abcde$). The reduction factors computed with the assumed characteristic lengths are shown in Fig. 11 and compared with the back-calculated reduction factors obtained previously. As shown in Fig. 11, the assumption of $L = 27$ m yields reduction factors that are most consistent with those back-calculated using the Monte Carlo simulation–based random field modeling approach; however, the former is easier to apply, is less demanding on computing resources, and offers significant advantages in engineering practice.

In summary, the variance reduction–based simplified approach is deemed suitable for basal-heave stability analysis if an appropriate characteristic length (and reduction factor) can be determined. The variance reduction–based simplified approach yields almost identical results to those obtained using the Monte Carlo simulation–based random field modeling approach; however, the former is easier to apply, is less demanding on computing resources, and offers significant advantages in engineering practice.

The approach described previously (Fig. 9) for back-calculating the variance reduction factor and characteristic length is demonstrated to be effective for the problem of basal heave that involves a linear limit state that has an explicit form. For other geotechnical problems that involve more complicated and nonlinear limit states, further study is needed to examine its general applicability.

Reliability-Based Design Considering Spatial Variability

With the validated variance reduction technique for basal-heave stability, the reliability analysis using FORM, in lieu of MCS, can be performed, which is an effective and efficient means to consider spatial variability. The principle and procedure of FORM are well documented (Ang and Tang 1984). A spreadsheet solution implementing FORM (Low and Tang 1997) has been shown to be effective and can be a practical tool in engineering practice. Fig. 12 shows the setup of a spreadsheet solution for reliability analysis of the braced excavation case presented previously (Fig. 1, Table 1).

Because the scale of fluctuation is found to be an important parameter about which knowledge is limited, it should be of interest to also examine the effect of the possible uncertainty of this parameter. Thus, the scale of fluctuation $\theta$ of $s_u/\sigma_u$ is treated as a lognormally distributed random variable in the spreadsheet solution, as shown in Fig. 12. The simplified approach to consider the spatial effect of $s_u/\sigma_u$ is realized using the exponential variance reduction function [Eq. (13)]. All other input parameters are treated as spatially constant random variables or simply constants in the spreadsheet solution.

With the spreadsheet solution setup, as shown in Fig. 12, the influence of spatial variability can easily be assessed by
considering several scales of fluctuation $\theta = 2.5, 5, 10$, and $100$ m. For this series of analysis, the scale of fluctuation is considered as a constant input, which is typical for these kinds of studies. The relationship between failure probability $p_f$ and FS for each scale of fluctuation is numerically derived, as shown in Fig. 13. The reduction factor derived from the spreadsheet solution is also shown in Fig. 13. For comparison purposes, the results from the conventional RFM conducted in this study (shown in Fig. 5) are redrawn and also included in Fig. 13. Again, both approaches (RFM versus the simplified approach using FORM with variance reduction) yield approximately the same results. Furthermore, the significant effect of spatial variability on the computed failure probability can be observed.

Finally, the effect of uncertainty in the scale of fluctuation in the reliability analysis of basal-heave stability is examined. As an example in this demonstration analysis, the COV of the scale of fluctuation is set to 0.1, 0.3, 0.6, and 0.9; the scale of fluctuation is set to the typical mean value of 2.5 m; and the COV of $s_u/\sigma_v$ is set to 0.3. The influence of uncertainty (in terms of COV) in the scale of fluctuation on the computed failure probability is shown in Fig. 14. For FS < 1.2, the variability of the scale of fluctuation has virtually no effect on the computed failure probability $p_f$. For FS > 1.2, the predicted $p_f$ increases with the increasing variability of the scale of fluctuation. Because the required minimum FS in the design against basal heave using the slip circle method is 1.2 (JSA 1988; PSCG 2000; TGS 2001), the effect of the variability of the scale of fluctuation is far less significant than the mean scale of fluctuation itself.

**Procedure for Applying Proposed Approach**

The proposed simplified approach for reliability analysis of basal-heave stability in a braced excavation considering the spatial variability of soils is summarized in the following procedure:

1. Select the analytical model for the basal-heave stability analysis of a braced excavation in clay [e.g., slip circle method (JSA 1988)].

![Fig. 12. Reliability-based procedure for evaluating failure probability of basal-heave](image)

**Fig. 12.** Reliability-based procedure for evaluating failure probability of basal-heave

![Fig. 13. Comparison between the Monte Carlo simulation–based random field modeling and simplified approach](image)

**Fig. 13.** Comparison between the Monte Carlo simulation–based random field modeling and simplified approach

![Fig. 14. Effect of uncertainty in scale of fluctuation on probability of failure against basal-heave in a braced excavation in clay (scale of fluctuation = 2.5 m, coefficient of variation of $s_u/\sigma_v = 0.3$)](image)

**Fig. 14.** Effect of uncertainty in scale of fluctuation on probability of failure against basal-heave in a braced excavation in clay (scale of fluctuation = 2.5 m, coefficient of variation of $s_u/\sigma_v = 0.3$)
2. Determine the variation of soil parameters (e.g., undrained shear strength) described with their COVs and the spatial variability, defined by the correlation function and scale of fluctuation, on the basis of site investigation, soil testing, and engineering judgment guided by published literature.

3. For the basal-heave stability analysis in a braced excavation in clay, the spatial variability of soil parameters, such as undrained shear strength, can be modeled with a one-dimensional (vertical) random field as stated previously. In this RFM, the characteristic length is taken as the distance from the final strut to the bottom of the diaphragm wall. The variation reduction factor \( \gamma^2 \) is then evaluated using Eq. (13) with this characteristic length, and finally the reduced variance \( \sigma^2 \) for this spatially random soil parameter can be determined with Eq. (14).

4. With the reduced variance of the undrained shear strength, reliability analysis can be performed using traditional reliability methods, such as FORM, for the probability of failure against the basal heave. The solution can easily be implemented in a spreadsheet, as shown in Fig. 12. Reliability or probability-based design can be realized by meeting a target probability of failure against the basal heave.

Conclusions

In this paper, a variance reduction–based simplified approach for the reliability-based design against basal-heave failure in a braced excavation is presented and shown to be efficient. This approach requires far less computational effort than the MCS-based RFM approach, is easy to use, and has potential in geotechnical reliability-based design that deals with spatial variability of soils. The following conclusions are drawn on the basis of the presented results:

1. Study of the gamma sensitivity index for all input parameters shows that the normalized undrained shear strength \( s_u/\sigma_u \) is the most influential factor in the basal-heave stability in a given braced excavation in clay, with the unit weight of clay being a distant second. This confirms the common understanding reflected in the existing stability theories on basal-heave stability.

2. The results of RFM of \( s_u/\sigma_u \) using the Cholesky decomposition method are deemed reasonable. The parametric study with the conventional RFM shows that the model with a smaller scale of fluctuation would yield a greater variance reduction in soil parameters (e.g., \( s_u/\sigma_u \)), which in turn would yield a smaller variation in the output responses (e.g., FS against basal heave). The computed probability of basal-heave failure can be too high if the spatial variability is not considered in the reliability analysis. Thus, the basal-heave stability design will be too conservative if the effect of spatial variability is ignored.

3. The algorithm (Fig. 9) developed in this paper for deriving the reduction factor, on the basis of a prescribed equivalency of the first two moments obtained by the RFM approach and the variance reduction–based simplified approach, for a given standard deviation and scale of fluctuation of a spatially random variable \( s_u/\sigma_u \) is found to be effective. For other geotechnical problems that involve more complicated and nonlinear limit states, further study is needed to investigate general applicability of this algorithm.

4. For the analysis of basal-heave stability, the proposed simplified approach with variance reduction technique is shown to be able to produce almost identical results to those obtained using the MCS-based RFM approach, provided that an appropriate characteristic length (and the reduction factor) can be determined. For the basal-heave stability problem, the appropriate characteristic length for the exponential reduction function is determined to be the distance from the final strut to the bottom of the diaphragm wall, which is the vertical scale of the random field in this case.

5. For the braced excavation case examined, when the FS against basal heave is less than 1.2 using the slip circle method, the variation in the scale of fluctuation has virtually no influence on the computed failure probability. For FS > 1.2, the computed failure probability increases with the increasing variability of the scale of fluctuation, but the effect of the variability of the scale of fluctuation is far less significant than that of the magnitude of the scale of fluctuation itself.

6. The proposed simplified approach for basal-heave analysis, which adopts the variance reduction technique, enables the traditional reliability methods, such as FORM, to account for the spatial variability of soil parameters in an efficient way. This approach can easily be implemented in a spreadsheet for probabilistic analysis of basal-heave stability. Reliability- or probability-based design can be realized by meeting a target probability of failure against basal heave.

References


