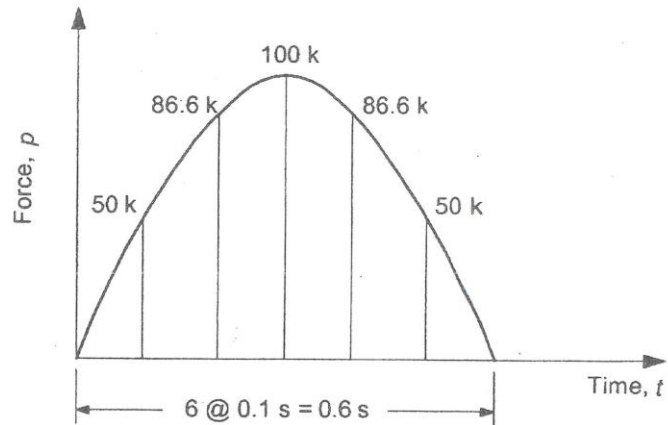
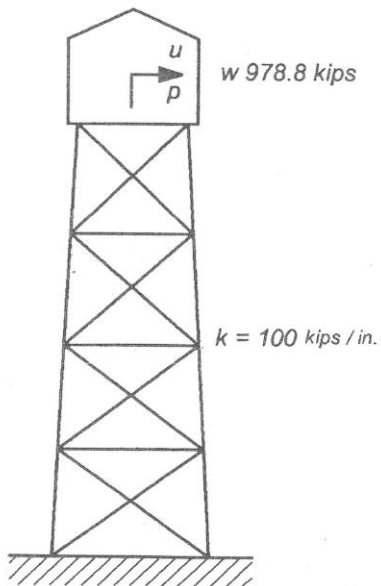


Handout #21

Determine the response of the tower subject to a half sine impulse load. Assume the structure is initially at rest.



$$m = 2.533 \text{ kip} \cdot \text{s}^2 / \text{in.}$$

$$k = 100 \text{ kips} / \text{in.}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{100}{2.533}} = 6.283 \text{ rad} / \text{s}$$

$$\xi = 10\%$$

$$c = 2\xi\omega_n m = 3.183 \text{ kip} \cdot \text{s} / \text{in.}$$

1. Central Difference Method

$$269.21u_{n+1} = p_n + 406.6u_n - 237.39u_{n-1} \tag{a}$$

$$\dot{u}_n = 5(u_{n+1} - u_{n-1}) \tag{b}$$

$$\ddot{u}_n = 100(u_{n+1} - 2u_n + u_{n-1}) \tag{c}$$

Time	p_n	u_{n-1}	u_n	u_{n+1} (Eq. a)	\dot{u}_n (Eq. b)	\ddot{u}_n (Eq. c)	u_n (theoretical)
0.0	0.0	0.0000	0.0000	0.0000	0.0000	0.000	0.0000
0.1	50.0	0.0000	0.0000	0.1857	0.9286	18.573	0.0323
0.2	86.6	0.0000	0.1857	0.6022	3.0110	23.074	0.2254
0.3	100.0	0.1857	0.6022	1.1172	4.6574	9.854	0.6204
0.4	86.6	0.6022	1.1172	1.4780	4.3792	-15.418	1.0961
0.5	50.0	1.1172	1.4780	1.4329	1.5785	-40.594	1.4251
0.6	0.0	1.4780	1.4329	0.8609	-3.0859	-52.693	1.3772
0.7	0.0	1.4329	0.8609	0.0366	-6.9813	-25.216	0.8683
0.8	0.0	0.8609	0.0366	-0.7038	-7.8231	8.381	0.1105
0.9	0.0	0.0366	-0.7038	-1.0953	-5.6593	34.893	-0.5974
1.0	0.0	-0.7038	-1.0953				-1.0073

2. Constant-Acceleration Method

$$u_{n+1} = u_n + 0.1\dot{u}_n + 0.005\ddot{u}_n \tag{a}$$

$$\dot{u}_{n+1} = \dot{u}_n + 0.1\ddot{u}_n \tag{b}$$

$$\ddot{u}_{n+1} = \frac{1}{2.533}(p_{n+1} - 100u_n - 13.183\dot{u}_n - 0.8183\ddot{u}_n) \tag{c}$$

Time	u_n	\dot{u}_n	\ddot{u}_n	p_{n+1}	u_{n+1} (Eq. a)	\dot{u}_{n+1} (Eq. b)	\ddot{u}_{n+1} (Eq. c)	u_n (theoretical)
0.0	0.0000	0.000	0.00	50.0	0.0000	0.000	19.74	0.0000
0.1	0.0000	0.000	19.74	86.6	0.0987	1.974	27.81	0.0323
0.2	0.0987	1.974	27.81	100.0	0.4351	4.755	16.32	0.2254
0.3	0.4351	4.755	16.32	86.6	0.9922	6.387	-13.01	0.6204
0.4	0.9923	6.387	-13.01	50.0	1.5659	5.086	-48.47	1.0961
0.5	1.5660	5.086	-48.47	0.0	1.8322	0.239	-72.63	1.4251
0.6	1.8320	0.239	-72.63	0.0	1.4927	-7.024	-50.11	1.3772
0.7	1.4930	-7.024	-50.11	0.0	0.5401	-12.036	-6.19	0.8683
0.8	0.5400	-12.036	-6.19	0.0	-0.6949	-12.655	43.33	0.1105
0.9	-0.6950	-12.655	43.33	0.0	-1.7430	-8.322	79.29	-0.5974
1.0	-1.7430	-8.322	79.29					-1.0073

3. Average-Acceleration Method

$$1176.9u_{n+1} = p_{n+1} + 1076.9u_n + 104.5\dot{u}_n + 2.533\ddot{u}_n \quad (d)$$

$$\dot{u}_{n+1} = 20(u_{n+1} - u_n) - \dot{u}_n \quad (e)$$

$$\ddot{u}_{n+1} = 400(u_{n+1} - u_n) - 40\dot{u}_n - \ddot{u}_n \quad (f)$$

Time	u_n	\dot{u}_n	\ddot{u}_n	p_{n+1}	u_{n+1} (Eq. d)	\dot{u}_{n+1} (Eq. e)	\ddot{u}_{n+1} (Eq. f)	u_n (theoretical)
0.0	0.0000	0.0000	0.000	50.0	0.0425	0.8497	16.994	0.0000
0.1	0.0425	0.8497	16.994	86.6	0.2245	2.7902	21.816	0.0323
0.2	0.2245	2.7902	21.816	100.0	0.5851	4.4218	10.815	0.2254
0.3	0.5851	4.4218	10.815	86.6	1.0248	4.3736	-11.779	0.6204
0.4	1.0248	4.3736	-11.779	50.0	1.3433	1.9943	-35.806	1.0961
0.5	1.3432	1.9943	-35.806	0.0	1.3291	-2.2767	-49.613	1.4251
0.6	1.3291	-2.2767	-49.613	0.0	0.9073	-6.1607	-28.066	1.3772
0.7	0.9073	-6.1607	-28.066	0.0	0.2227	-7.5297	0.686	0.8683
0.8	0.2227	-7.5297	0.686	0.0	-0.4633	-6.1909	26.089	0.1105
0.9	-0.4633	-6.1909	26.089	0.0	-0.9175	-2.8928	39.871	-0.5974
1.0	-0.9175	-2.8928	39.871					-1.0073

4. Linear-Acceleration Method

$$1715.3u_{n+1} = p_{n+1} + 1615.3u_n + 158.35\dot{u}_n + 5.225\ddot{u}_n \quad (g)$$

$$\dot{u}_{n+1} = 30(u_{n+1} - u_n) - 2\dot{u}_n - 0.05\ddot{u}_n \quad (h)$$

$$\ddot{u}_{n+1} = 600(u_{n+1} - u_n) - 60\dot{u}_n - 2\ddot{u}_n \quad (i)$$

Time	u_n	\dot{u}_n	\ddot{u}_n	p_{n+1}	u_{n+1} (Eq. g)	\dot{u}_{n+1} (Eq. h)	\ddot{u}_{n+1} (Eq. k)	u_n theoretical
0.0	0.0000	0.0000	0.000	50.0	0.0291	0.8745	17.490	0.0000
0.1	0.0291	0.8745	17.490	86.6	0.2119	2.8603	22.227	0.0323
0.2	0.2119	2.8603	22.227	100.0	0.5896	4.4991	10.549	0.2254
0.3	0.5896	4.4991	10.549	86.6	1.0532	4.3819	-12.893	0.6204
0.4	1.0532	4.3819	-12.893	50.0	1.3862	1.8707	-37.331	1.0961
0.5	1.3862	1.8707	-37.331	0.0	1.3644	-2.5299	-50.680	1.4251
0.6	1.3644	-2.5299	-50.680	0.0	0.8969	-6.4303	-27.328	1.3772
0.7	0.8969	-6.4303	-27.328	0.0	0.1678	-7.6476	2.981	0.8683
0.8	0.1678	-7.6476	2.981	0.0	-0.5389	-6.0547	28.875	0.1105
0.9	-0.5389	-6.0547	28.875	0.0	-0.9785	-2.5210	41.789	-0.5974
1.0	-0.9785	-2.5210	41.789					-1.0073

Reference:

Humar, J. L. (2012). *Dynamics of structures*. 2nd ed., ISBN 9058092453, CRC Press.