

Handout #25

$$k_{ij} = \int_0^L EI(x) \varphi_i \varphi_j dx$$

$$K = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

Elastic Stiffness matrix

$$m_{ij} = \int_0^L \bar{m} \varphi_i \varphi_j dx$$

$$M = \frac{\bar{m}L}{420} \begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix}$$

Consistent Mass Matrix

$$k_{ij}^g = \int_0^L N \varphi_i \varphi_j dx$$

$$K^g = \frac{N}{30L} \begin{bmatrix} 36 & 3L & -36 & 3L \\ 3L & 4L^2 & -3L & -L^2 \\ -36 & -3L & 36 & -3L \\ 3L & -L^2 & -3L & 4L^2 \end{bmatrix}$$

Geometric Stiffness Matrix