Chapter 1: The Corporation

The Three Types of Firms

-Sole Proprietorships
  -Owned and ran by one person
  -Owner has unlimited liability

-Partnerships
  -Sole proprietorship with more than one owner
  -Partners can have unlimited liability or limited liability
  -Limited Liability Partnership (LLP): Partners are only liable in cases of own person negligence

-Corporation
  -Owners of a company, shareholders, have limited liability
  -Corporations must be legally formed and have articles of incorporation
  -A corporation’s profits are subject to taxation separate from their owners

Ownership Versus Control of Corporations

The Corporate Management Team
-The board of directors is elected by the shareholders and have ultimate decision-making authority
-Shareholder wealth maximization is the one goal that generally unites shareholders because they all benefit from a higher stock price

Ethics and Incentives Within Corporations

-Principal Agent Problem
  -The principal agent problem is when managers, despite being hired as the agents of the shareholders, put their self-interest ahead of the interests of the shareholders

-The CEO’s Performance
  -A hostile takeover is when an individual or organization can purchase a large amount of a company’s stock and get enough votes to replace the BOD and CEO
  -Often the mere threat of a hostile takeover is enough for action to occur

-Shareholder Versus Stakeholders
  -Stakeholders of a corporation are those with an interest (or stake) of how the corporation operates, including employees, customers, or suppliers
  -The view of stakeholder satisfaction is one that counters the view of shareholder wealth maximization
-However, the satisfaction of one does not necessary mean the dissatisfaction of the other and often it benefits both

-Corporate Bankruptcy
  -If the corporation fails to repay its debts the debt holders are entitled to seize the assets of the corporation in compensation for default

The Stock Market

-Public companies have shares that are traded on stock markets

Primary and Secondary Stock Markets

-Primary markets are when the company issues new shares of a stock and sells it to investors
-Secondary markets occur after and it is between investors
-Bid price is highest price being quoted to buy
-Ask/Offer price is lowest price being quoted to sell
-When they equal, a transaction takes place

Chapter 2: Disclosure of Financial Information

The Disclosure of Financial Information
-Generally Accepted Accounting Principles (GAAP) provide framework and set of standards and rules for financial statements

Types of Financial Statements

The Balance Sheet

\[ Assets = liabilities + shareholder's\ equity \]

Assets
- current assets have a conversion to cash time of less than one year, includes cash, A/R, inventories, and other current assets
- Long-term assets produce tangible benefits for more than one year, includes property, plant, and equipment as well as other long term assets such as goodwill

Liabilities
- Current liabilities will be satisfied within one year and includes A/P, Notes payable, and other current liabilities
- Non-current liabilities will extend over one year and includes long-term debt, capital leases, and future income taxes

Shareholder’s Equity
- The total market value of a firm’s equity equals the market price per share times the number of shares, referred to as the company’s market capitalization
EX: If a company has a market price per share of $10 and has 200 shares, its market capitalization = 10 x 200 = 2000

**Balance Sheet Analysis**

- The liquidation value is the value that would be left after assets sold and liabilities paid

Market-to-Book Ratio

\[
\text{Market – to – Book Ratio} = \frac{\text{Market Value of Equity}}{\text{Book Value of Equity}}
\]

- measures the market capitalization to the book value of shareholder’s equity

Debt-Equity Ratio

\[
\text{Debt – Equity Ratio} = \frac{\text{Total Debt}}{\text{Total Equity}}
\]

- Measures leverage, or the extent to which a firm relies on debt as a source of financing

Enterprise Value

\[
\text{Enterprise Value} = \text{Market Value of Equity} + \text{Debt – Cash}
\]

- The enterprise value of a firm assesses the value of the underlying business assets, unencumbered by debt and separate from any cash and marketable securities
EX: A company has a market value of equity of 1.7, debt of 11.1 and cash of 4.6
Enterprise value = 1.7 + 11.1 – 4.6 = 8.2

**The Income Statement**

Gross Profit
- difference between sales revenues and the costs

Operating Expenses
- Expenses from the ordinary course of running the business that are not directly related to producing the good or services being sold

\[
\text{Earnings Per Share (EPS)} = \frac{\text{Net Income}}{\text{Shares Outstanding}}
\]
Income Statement Analysis

Profitability Ratios

\[ \text{Gross Margin} = \frac{\text{Gross Profit}}{\text{Sales}} \]

-Gross margin reflects ability to sell product for more than the cost of producing it

\[ \text{Operating Margin} = \frac{\text{Operating Income}}{\text{Sales}} \]

-Operating margin reveals how much a company earns before interest and taxes from each dollar of sale

\[ \text{Net Profit Margin} = \frac{\text{Net Income}}{\text{Total Sales}} \]

-net profit margin reflects fraction of each dollar in revenues that is available to equity holders after the firm pays interest and taxes

\[ \text{Asset Turnover} = \frac{\text{Total Sales}}{\text{Total Assets}} \]

-Asset turnover shows amount of sales generated per assets used

\[ \text{Accounts Receivable Days} = \frac{\text{Accounts Receivable}}{\text{Average Daily Sales}} \]

-Firm’s accounts receivable amount in terms of number of days’ worth of sales it represents

\[ \text{Return on Equity} = \frac{\text{Net Income}}{\text{Book Value of Equity}} \]

-ROE provides a measure of the return the firm has earned on its past investments

The Dupont Identity

-Expresses ROE in terms of the firm’s profitability, asset efficiency, and leverage

\[ \text{ROE} = \left( \frac{\text{Net Income}}{\text{Sales}} \right) \times \left( \frac{\text{Sales}}{\text{Total Assets}} \right) \times \left( \frac{\text{Total Assets}}{\text{Book Value of Equity}} \right) \]

EX: A company has a net profit margin of 0.04, asset turnover of 1.8 and equity multiplier of 44/18. ROE = 0.04 x 1.8 x 44/18

Price Earnings (P/E) Ratio

\[ \text{PE Ratio} = \frac{\text{Market Capitalization}}{\text{Net Income}} = \frac{\text{Share Price}}{\text{Earnings per Share}} \]

-The value of the equity to the firm’s earnings, either on a total basis or on a per-share basis
The Statement of Cash Flows
-Determines how much cash the firm has generated, and how that cash has been allocated during a set period

Operating Activity
-Adjusts net income by all non-cash items related to operating activity

Investing Activity
-Purchases of new property, plant, and equipment are referred to as capital expenditures
-Also deduct assets purchased or other investments made by the firm

Financing Activity
-Dividends paid to shareholders are a cash outflow
-Net income – dividends = retained earnings

Other Financing Statement Information

Management Discussion and Analysis
-Preface to the financial statements in which the company’s management discusses the recent year

Statement of Shareholders Equity
-Breaks down shareholder’s equity computed on the balance sheet into the amount that came from issuing new shares versus retained earnings

Statement of Comprehensive Income
-Show the total income and expenses for period by combining net income with information not reported on income statement

Notes to Financial Statements
-Extensive notes with further detail on information provided with statements

Chapter 3: Arbitrage and Financial Decision Making

Valuing Decisions
- Corporate financial decisions can be assessed in terms of their benefits and costs
  - Complicated because they are often difficult to quantify
- Assessing costs and benefits can involve expertise from other management disciplines such as strategy, operations, and marketing
- In order to compare costs and benefits, it is best to assess them in the same terms - “cash today”
- Competitive Market: One in which a good can be bought and sold for the same price
  - A good’s price determines its worth
- **Valuation Principle:** When the value of the benefits exceeds the value of the costs, the decision will increase the market value of the firm
  - When competitive market prices are unavailable, must take into account the preferences/views of the decision-maker
  - Price determines the good’s maximum value (since it can always be purchased at that price), but it might be valued less depending on one’s preferences
    - Ex: You can purchase a shirt for its stated price, but you cannot sell it to the store for that price

**Interest Rates and the Time Value of Money**
- Costs and benefits always occur at different points in time, so interest rates are used to bring them into the same terms
- **Time Value of Money:** The difference in value between money today and money in the future
  - **Interest Rate**
    - Acts as an exchange rate across time
    - **Risk-Free Interest Rate** ($r_f$): Interest rate at which money can be borrowed or lent over that period without risk
    - **Interest Rate Factor:** $(1+r_f)$
  - **Net Present Value**
    - The present value of a cost or benefit is its value calculated in terms of cash today
    - $NPV = PV(\text{Benefits}) - PV(\text{Costs})$
      - $= PV(\text{all project cash flows})$
    - **NPV Decision Rule** implies that firms should accept projects with NPV > 0 and reject projects with NPV < 0
    - If projects are mutually exclusive, select the one with the highest NPV (provided it has NPV > 0)
    - **First Separation Principle** states that regardless of our consumption preferences that dictate whether we prefer cash today versus cash in the future, we should ALWAYS maximize NPV first

**Arbitrage and the Law of One Price**
- **Arbitrage:** The practice of buying and selling equivalent goods in different markets to take advantage of a price difference
  - **Arbitrage Opportunity:** A situation in which it is possible to make a risk-free profit without making any investment
  - **Normal Market:** A competitive market in which there are no arbitrage opportunities
- **Law of One Price** states that if equivalent investment opportunities trade simultaneously in different competitive markets, then there must trade for the same price in both markets.

- **Example of Arbitrage**
  - Suppose an investment guarantees $1,000 after a year at a risk-free rate of 5% and there is a bond currently being traded for a price of $940, is there an arbitrage opportunity?
  - **1ST STEP:** Solve for PV of $1,000 cash flow
    - PV($1000 in one year) = $1,000/(1.05) = $952.38 today
    - Due to Law of One Price, the price of a bond should also be $952.38
  - **2ND STEP:** Identify arbitrage opportunity
    - Buy bond at $940 and borrow $952.38 from the bank
    - As a result, there is a $12.38 net cash inflow
    - In one year, sell the bond for $1,000 and pay $1,000 loan to bank
    - Therefore, overall, you gain a total of $12.38 from this arbitrage opportunity

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**No-Arbitrage and Security Prices**

- **Financial Security:** An investment opportunity that trades in a financial market
- **No-Arbitrage Price:** A price where no arbitrage opportunities exist
  - To calculate this price, Price(Security) = PV(all cash flows paid by the security)
- To determine the interest rate from bond prices,
  - Return = Gain at End of Year / Initial Cost
  - Ex: A risk-free bond that pays $1,000 in one year is trading with a current competitive market price of $929.80
    - Return = 1 + r_f = $1,000/$929.80 = 1.0755
    - Therefore, the interest rate from this bond is 7.55%
  - All risk-free investments should offer investors the same return
- When securities trade at no-arbitrage prices, NPV = 0
- Value is not created by trading securities in a normal market, but rather by the real investment projects undertaken by corporations
- **Second Separation Principle** states that the NPV of an investment decision can be separate from the decision on how to finance the investment
- **Portfolio:** A collection of securities
  - Ex: There are 3 securities, Security A and Security B, and Security C, which has the same cash flows as A + B
  - Therefore, the price of C is equal to the the price of the portfolio A + B
- **Value Additivity:** The price of C must equal the sum of the prices of A + B
If value additivity did not hold, there would be an immediate arbitrage opportunity available. A firm’s cash flows are the sum of the cash flows for all of its projects, so the value of the entire firm equals the sum of the values of all of its projects. To maximize firm value, must maximize NPV.

### The Price of Risk
- **Risk**: When an actual outcome may be different from its expected outcome
- Investors prefer to invest with certainty
  - The cost of losing a dollar in bad times is greater than the benefit of an extra dollar in good times
- **Risk Aversion**: The idea that investors prefer to have a safer income rather than a risky one of the same expected amount
  - Higher risk → Higher return
- **Expected Return of a Risky Investment** = Expected Gain at End of Year/Initial Cost
- **Risk Premium**: Expected Return - Risk-free Interest Rate
  - Represents the additional return that investors expect to earn to compensate them for the security’s risk
- Risk is relative to the overall market
  - The risk of a security must be evaluated in relation to the fluctuations of other investments in the economy
- The more the returns vary with the overall economy/market index, the higher the risk premium of a security
- If a security’s returns vary in the opposite direction of the market index, it offers insurance and has a negative risk premium
- **Discount Rate for a Risky Cash Flow**, \( r_s = r_f + \text{risk premium for investment, } s \)

### Chapter 4: The Time Value of Money

#### The Timeline
- Cash flow involves either receiving or paying money
- **Stream of Cash Flows**: Several cash flows occurring at different points in time
- **Timeline**: Used to represent a stream of cash flows

#### Three Fundamental Rules
1. Only cash flow values at the same point in time can be compared/combined
2. To move a cash flow forward in time, you must COMPOUND it
   a. To compound a cash flow for \( n \) periods, multiply it by \((1+r)^n\)
b. **Future Value**: The value of a cash flow moved forward in time

c. **Simple Interest**: When you earn interest on principal but not accrued interest

d. **Compound Interest**: When you earn interest on both principal and accrued interest

e. Growth resulting from compounding is called “geometric” or “exponential” growth

f. Ex: You invest $100 in an account paying 6% interest per year. How much will you have in 8 years? 25 years?

i. 8 years: $100 * (1 + 0.06)^8 = $159.38

ii. 25 years: $100 * (1 + 0.06)^{25} = $429.19

3. To move a cash flow backward in time, you must DISCOUNT it

a. To discount a cash flow for \( n \) periods, divide it by \((1+r)^n\) or multiple it by \((1+r)^{-n}\)

b. **Discounted Value**: The value of a cash flow moved backward in time

c. **Present Value**: The value today of a future cash flow

d. Ex: You invest in a bond that will pay you $1000 in 3 years, what is the price of the bond today if the risk-free interest rate is 5%? In 7 years?

i. 3 years: $1,000 / (1 + 0.05)^3 = $863.84

ii. 7 years: $1,000 / (1 + 0.05)^7 = $710.68

**Valuing a Stream of Cash Flows**

- \( PV_0 = \text{Sum of } C_t / (1+r)^t \), for each date \( t \) from 0 to \( n \)

- Calculates the present value of a stream of cash flows

- **Future Value of a Cash Flow Stream**: \( FV_n = PV_0 * (1+r)^n \)

- Ex: How much can you borrow today if you can make equal payments of $2000 at the end of each year for 4 years with a current market interest rate of 3%? How much are those payments worth at the end of 4 years?

- \( PV(\text{Cash flows for 4 years}) = 2000/(1+0.03)^1 + 2000/(1+0.03)^2 + 2000/(1+0.03)^3 + 2000/(1+0.03)^4 = $7,434.20 \)

- You can borrow approximately $7,434.20 today

- \( FV(\text{Cash flows at end of year 4}) = $7,434.20 * (1+0.03)^4 = $8,367.26 \)

- The creditor will receive $8,367.26 at the end of 4 years

**Calculating the Net Present Value**

- \( NPV = PV(\text{Benefits}) - PV(\text{Costs}) \)

- Cash inflows are the benefits

- Cash outflows are the costs

- The NPV of an investment opportunity is the PV of its stream of cash flows

**Perpetuities and Annuities**
- **Regular Perpetuities**
  - A stream of equal cash flows received at constant time intervals that goes on forever
  - Assume that the 1st cash flow occurs at the end of the first period
  - \( PV_0 = \frac{C}{r} \)
  - \( C \) is the cash flow and \( r \) is the interest rate
  - The present value of receiving \( C \) in perpetuity is the upfront cost of \( PV_0 = \frac{C}{r} \)
  - Ex: How much will it cost to endow an annual party for forever if you budget $30,000 per year and the interest rate is 8%?
    - This is a perpetuity because you are receiving equal cash flows every year and it will last forever
    - To calculate the amount needed for the endowment,
      - \( PV_0 = \frac{C}{r} = \frac{30,000}{0.08} = $375,000 \) today
    - Therefore, to receive $30,000 each year, $375,000 needs to be donated today

- **Regular Annuities**
  - A stream of \( n \) equal cash flows paid over constant time intervals
  - Assume 1st payment is received one period from today
  - \( PV_0 = C \times \frac{1}{r} \times \left( 1 - \frac{1}{1+r}^n \right) \)
  - Ex: You win a $30 million lottery prize and will receive the money in 30 payments of $1 million per year (payment starts in a year). How much are the payments worth today, if the interest rate is 8%?
    - This is an annuity because you are receiving $1 million per year for only 30 years
    - To calculate the present value of the 30 payments,
      - \( PV_0 = C \times \frac{1}{r} \times \left( 1 - \frac{1}{1+r}^n \right) \)
      - \( = \frac{1,000,000}{0.08} \times \left( 1 - \frac{1}{1.08}^{30} \right) \)
      - \( = 11,257,783.34 \)
    - Therefore, 30 payments of $1 million is worth $11,257,783.34 today, if the interest rate is 8%
  - \( FV_n = PV_0 \times \left( 1+r \right)^n = C \times \frac{1}{r} \times \left( \left( 1+r \right)^n - 1 \right) \)
  - Ex: How much in total will be saved in your RRSP when you are 65 years old if you save $10,000 each year, start at age 35, and the account earns 10% per year?
    - This is an annuity because $10,000 is being deposited each year for 30 years
    - To calculate the RRSP total,
      - \( FV_n = C \times \frac{1}{r} \times \left( \left( 1+r \right)^n - 1 \right) \)
Therefore, the amount you will have in the RRSP at age 65 is $1,644,940.23

Growing Perpetuities
- A stream of cash flows that occur at regular intervals forever that grow at a constant rate
  \[ PV_0 = \frac{C}{r-g} \]
- Assume that \( g < r \) for a growing perpetuity
- In a growing perpetuity, the amount you withdraw and reinvest increases by \( g \)
- Ex: How much do you need to donate if the $30,000 cost of a party rises by 4% per year (assume cost of 1st party is $30,000), with an interest rate of 8%?
  - This is a growing perpetuity because the costs of the party are growing at a constant rate and the cash flows are occurring forever
  \[ PV_0 = \frac{C}{r-g} \]
  \[ = \frac{30000}{0.08-0.04} \]
  \[ = 750,000 \text{ today} \]
  - Therefore, to finance the growing cost, you need to donate $750,000 today

Growing Annuities
- A stream of \( n \) growing cash flows paid at regular intervals that grow at a constant rate
  \[ PV_0 = \frac{C}{r-g} * \left[ 1 - \frac{(1+g)/(1+r)^n}{1} \right] \]
  \[ FV_n = \frac{C}{r-g} * \left[ (1+r)^n - (1+g)^n \right] \]
- Assume that \( g \neq r \) for a growing annuity
- Ex: Suppose you save $10,000 per year for retirement but expect to increase savings by 5% per year. How much will you save at age 65 if you start when you are 35 and the savings account earns 10%?
  - This is a growing annuity because your savings are growing at a constant rate for 30 years.
  \[ PV_0 = \frac{C}{r-g} * \left[ 1 - \frac{(1+g)/(1+r)^n}{1} \right] \]
  \[ = \frac{10000}{0.1-0.05} * \left[ 1 - (1.05)/(1.1)^{30} \right] \]
  \[ = 150,463.15 \text{ today} \]
  - Therefore, the present value of her savings is $150,463.15
  \[ FV = 150,463.15 * 1.1^{30} \]
  \[ = 2,635,491.98 \text{ in 30 years OR...} \]
  \[ FV_n = \frac{C}{r-g} * \left[ (1+r)^n - (1+g)^n \right] \]
  \[ = \frac{10000}{0.1-0.05} * \left[ (1+0.1)^{30} - (1+0.05)^{30} \right] \]
  \[ = 2,635,491.98 \text{ in 30 years} \]
Therefore, you will have saved $2,635,491.98 at age 65

**Solving for Cash Flows**

- **Loan Payments**
  - \( C = \frac{PV}{r \left[ 1 - (1+r)^n \right]} \) OR \( C = \frac{FV}{r \left( (1+r)^n - 1 \right)} \)
  - If payments are annuities, solve for cash flow by inverting annuity formula
  - The amount borrowed is the present value of the payments
  - Ex: What is the annual loan payment if you borrowed a 30-year $80,000 loan with an interest rate of 8% per year?
    - We have the present value of the loan which is $80,000 so we use this formula,
    - \( C = \frac{PV}{r \left[ 1 - (1+r)^n \right]} \)
      \[ = \frac{80,000}{0.08 \left[ 1 - (1+0.08)^{30} \right]} \]
      \[ = \$7,106.19 \text{ per year} \]
    - Therefore, you will need to pay $7,106.19 each year.

**The Internal Rate of Return**

- The interest rate that makes the NPV of the investment equal to zero
- If you invest \( P \) today in return for \( F \) after \( n \) periods, then:
  - \( \text{IRR} = \left( \frac{F}{P} \right)^{1/n} - 1 \)
  - Equation only works for 2 cash flows
  - Ex: Suppose an investment opportunity that requires $1,000 today and will have a $5,000 payoff in 6 years
    - \( \text{IRR} = \left( \frac{F}{P} \right)^{1/n} - 1 \)
      \[ = \left( \frac{2000}{1000} \right)^{1/6} - 1 \]
      \[ = 0.122462 \]
    - Therefore, the internal rate of return is approximately 12.2462%

- **IRR of a Growing Perpetuity** = \( (C_1/P) + g \)
  - Ex: Suppose an investment opportunity that requires $1 million today will generate a cash flow of $100,000 at the end of year one, and this amount will grow by 4% per year thereafter. What is the IRR of this opportunity?
    - \( \text{IRR} = (C_1/P) + g \)
      \[ = \left( \frac{100,000}{1,000,000} \right) + 0.04 \]
      \[ = 0.14 \]
    - Therefore, the internal rate of return is 14%
  - No general formula to solve for interest rate \( r \), the only way to solve for \( r \) is to use guess and check

**Solving for the Number of Periods**

- In this case, the \( r \), \( PV \), and \( FV \) are all known
- \[ \ln[(1+r)^n] = n \cdot \ln(1+r) \]
- In order to use this formula, you must also use the PV or FV function of annuities
- Ex: Suppose you save $5,000 at the end of each year in an account that earns 7.25%.
  How long will it take you to save $60,000?
  \[
  \begin{align*}
  FV_n &= 5,000 \cdot \frac{1}{0.0725} \cdot (1.0725^n - 1) = 60,000 \\
  &= 60,000/(5,000/0.0725) + 1 = 1.0725^n \\
  &= 1.87 = 1.0725^n
  \end{align*}
  \]
  We can now solve for \( n \) by using:
  \[
  n = \frac{\ln(1.87)}{\ln(1.0725)} = 8.94 \text{ years}
  \]
  Therefore, it will take about 9 years to save $60,000

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**Chapter 5 – Interest rates**

**The effective annual rate**

- Interest rates are often stated as an **effective annual rate (EAR)**, which indicates the total amount of interest that will be earned at the end of one year.
- For example, with an **EAR** of 5%, a $100,000 investment grows to $100,000 \times (1 + r) = $100,000 \times (1.05) = $105,000 in one year. After two years it will grow to $100,000 \times (1+r)^2 = $100,000 \times (1.05)^2 = 110,250

**Adjusting the effective annual rate to an effective rate over different time periods**

- The preceding example shows that earning an effective annual rate of 5% for two years is equivalent to earning 10.25% in total interest over the entire period:
  $100,000 \times (1.05)^2 = $100,00 \times 1.1025 = $110,250
- In general, by raising the interest rate factor \((1+r)\) to the appropriate power, we can compute an equivalent effective interest rate for a longer time period.

- In general, we can convert an effective rate of \( r \) for one period to an equivalent effective rate for \( n \) periods using the following formula:
  \[
  1 + \text{Equivalent}\ n\text{-Period Effective Rate} = (1+r)^n \quad \text{or Equivalent}\ n\text{-Period Effective Rate} = (1+r)^n - 1
  \]
  - In this formula, \( n \) can be larger than 1 (to compute an effective rate over more than one period) or smaller than 1 (to compute an effective rate over a fraction of a period).

- Example 5.1 (textbook) - If your bank account pays interest monthly with an **EAR** of 6%.
  What amount of interest will you earn each month?
  A 6% EAR is equivalent to earning \((1.06)^{1/12} - 1 = 0.4868\%\) per month.
  Since \( FV(\text{Annuity}) = C \times \frac{1}{r}[1-(1+r)^n] \), you can solve for the payment, \( C \), using the equivalent monthly interest rate, \( r = 0.4868\% \), and \( n = 120 \) months:
C = FV(annuity)/1/r[(1+r)^n – 1) = 100,000/1/0.004868[(1.004868)^120 – 1] = $615.47 per month
Thus, if we save $615.47 per month and we can earn interest monthly at an EAR of 6%, we will have $100,000 in 10 years.

**Annual Percentage Rates**
- Banks quote interest rates in terms of an **annual percentage rate (APR)**, which indicates the amount of **simple interest** earned in one year, that is, the amount of interest earned without the effect of compounding even though compounding may occur.
- APR does not reflect the true amount you will earn over one year, **the APR itself cannot be used as a discount rate and it is not an effective annual rate**.
- APR with k compounding periods is a way of indirectly quoting the effective interest rate, r, earned each compounding period
- **Implied Effective Interest Rate per Compounding Period = r = APR/k periods per year**
- it shows the actual interest earned over the compounding period and, by convention, it is what is implied from an APR quote
- The effective annual rate corresponding to an APR with k compounding periods per year is determined as follows:
  - Convert the APR to its implied effective interest rate, r, per compounding period
    \[
    r = \frac{APR}{k} \text{ compounding periods per year}
    \]
  - Convert the implied effective interest rate per compounding period, r, into the EAR
    \[
    1 + \text{EAR} = \left(1 + \frac{r}{k}\right)^k
    \]
  - Use k as the exponent as we are converting from an effective rate per compounding period to an effective rate per k periods because the APR specifies k compounding periods in a year and we are converting to an EAR.
  - **1+\text{EAR} = \left(1 + \frac{\text{APR}}{k}\right)^k**
    - **continuous compounding** = compound the interest every instant
  - Consider a Canadian mortgage quote of 8% APR with semiannual compounding.
  - Step 1: Convert the 8% APR to its implied effective interest rate, r, per semiannual period
    \[
    r = \frac{0.08}{2} = 0.04 = 4\% \text{ per 6 months}
    \]
  - Step 2: 1 + Effective monthly rate = \((1 + 0.04)^{1/6} = 1.006558197\), Effective monthly rate = 0.6558197%
  - Step 3: Convert the effective monthly rate into an APR with monthly compounding.
    \[
    r = \frac{\text{APR}}{k} \text{ periods per year, therefore, APR = r x k}
    \]
    \[
    \text{APR} = 0.0065588197 \times 12 = 0.07869836 = 7.869836\%
    \]
  - can be applied to any interest rate quote including quotes that are not on an annual basis
  - a quoted rate of 100% per decade compounded semiannually can be converted into a rate quoted as a rate per six months compounded monthly as follows:
  - Step 1: divide by the compounding frequency to get the implied effective rate per compounding period
    \[
    r = \frac{100\%}{20 \text{ six-month periods in a decade}} = 5\% \text{ effective six-month rate}
    \]
- Step 2: convert the effective six-month rate into a rate per month as the final rate quote has monthly compounding
  \[(1 + .05)^{1/6} = 1 + r \text{ per month}\]
  \[= 1.008164856\]
  \[= 0.8164846\%\]
- Step 3: convert the effective monthly rate into a rate quoted per six compounded monthly by multiplying 6
  \[= 0.8164846\% \times 6 = 4.898908\%\]

**Application: discount rates and loans**

- **COMPUTING LOAN PAYMENTS:** To calculate a loan payment, we first compute the discount rate from the quoted interest rate of the loan, and then equate the outstanding loan balance with the present value of the loan payments and solve for the loan payment.
- Loans like consumer loans and car loans, have monthly payments and are quoted in terms of an APR with monthly compounding. These types of loans are amortizing loans, which means that each month you pay interest on the loan plus some part of the loan balance.
- **Example 5.3 from textbook:** Suppose you need to borrow $400,000 and the Bank of Montreal offers you a five-year term for a mortgage with a rate of 7.252% to be amortized over 25 years of monthly payments. To determine your monthly payments, you need to convert the 7.252% APR with semiannual compounding into an equivalent effective rate per month. This requires two steps.
  - Step 1, you can use Eq. 5.2 to get the implied effective interest rate per semiannual period: 7.252%\(\times\)2 = 3.626% per semiannual period
  - Step 2, you can use Eq. 5.1 to convert the effective semiannual rate to the equivalent effective monthly rate: \((1.03656)^{1/6} – 1 = 0.005954\) or 0.5954% per month
  - \[C = P/1/r(1-1/(1+r)^n) = 400,000/1/0.005954(1-1/(1+0.005954)^{300}) = $2864.17\]
- **Example 5.4 from textbook:** Returning to our hypothetical mortgage at the Bank of Montreal from Example 5.3, we are interested in what your outstanding balance will be at the end of the mortgage’s initial term.
  - **Balance after 5 years = $2864.17 \times 1/0.005954 (1-1/1.005954^{240}) =$365,321.11**

**5.3 The Determinants of Interest Rates**

- Interest rates that are quoted by banks and other financial institutions, and that we have used for discounting cash flows, are nominal interest rates, which indicate the rate at which your money will grow if invested for a certain period.
- The rate of growth of your purchasing power, after adjusting for inflation, is determined by the real interest rate, which we denote by \(r - i\). If \(r\) is the nominal interest rate and \(i\) is the rate of inflation, we can calculate the rate of growth of purchasing power as follows:
  - **Growth in Purchasing Power = 1 + r, 1 + r/1 + i = Growth of Money/Growth of Prices**
  - **The Real Interest Rate \(r\) = r – i/1 + i ≈ r – i**
Example 5.5 – Calculating the Real Interest Rate - In 2000, short-term Canadian government bond rates were about 5.8% and the rate of inflation was about 3%. In 2003, interest rates were about 2.7% and inflation was about 3.1%. What was the real interest rate in 2000 and 2003?

Solution: the real interest rate in 2000 was \((0.058 - 0.03)/(1.03) = 0.0272\) (which is approximately equal to the difference between the nominal rate and inflation: 5.8% - 3% = 2.8%. In 2003, the real interest rate was \((0.027 - 0.031)/(1.031) = -0.0039\) or -0.39% Note that the real interest rate was negative in 2003, indicating that interest rates were insufficient to keep up with inflation: Investors in Canadian government bonds were able to buy less at the end of the year than they could have purchased at the start of the year.

Investment and Interest Rate Policy

- Consider a risk-free investment opportunity that requires an upfront investment of $10 million and generates a cash flow of $3 million per year for four years. If the risk-free interest rate is 5%, this investment has an NPV of

  \[
  NPV = \$10 + \frac{\$3}{1.05} + \frac{\$3}{(1.05)^2} + \frac{\$3}{(1.05)^3} + \frac{\$3}{(1.05)^4} = \$0.638 \text{ million}
  \]

- If the interest rate is 9%, the NPV falls to

  \[
  NPV = -\$10 + \frac{\$3}{1.09} + \frac{\$3}{(1.09)^2} + \frac{\$3}{(1.09)^3} + \frac{\$3}{(1.09)^4} = -\$0.281 \text{ million}
  \]

- Investment no longer profitable as the positive cash flows are being discounted at a higher rate which reduces their present value.

The Yield Curve and Discount Rates

- The relationship between the investment term and the interest rate is called the term structure of interest rates. We can plot this relationship on a graph called the yield curve.

  - In general, a risk-free cash flow of \(C_n\) received in \(n\) years has present value

    \[
    PV_0 = \frac{C_n}{(1+r_n)^n}
    \]

  - \(r_n\) is the risk-free effective annual interest rate for a cash flow that occurs in \(n\) years and
  - is known as the spot rate of interest for an \(n\)-year term.

  - Present Value of a Cash Flow Stream Using a Term Structure of Discount Rates

    \[
    PV_0 = C_1/(1+r_1) + C_2/(1+r_2)^2 + \ldots + C_n/(1+r_n)^n
    \]

The Yield Curve and the Economy

- The Bank of Canada determines very short-term interest rates through its influence on the overnight rate, which is the rate at which banks can borrow cash reserves on an overnight basis.

Risk and Taxes

- Risk and Interest Rates

  - Example 5.8 (textbook): Suppose the Canadian government owes your firm $1000, to be paid in five years. Based on the interest rates in Table 5.2, what is the present value of this cash flow today? Suppose instead Sherritt owes your firm $1000. Estimate the present value in this case.
Using risk-free interest rate of 1.31%
- \( P_{V_0} = \frac{1000}{(1.0131)^5} = 937 \)
- The obligation from Sherritt is not risk free. There is no guarantee that Sherritt will not have financial difficulties and fail to pay the $1000. Because the risk of this obligation is likely to be comparable to the five-year loan quoted in Table 5.2, the 6.38% interest rate of the loan is a more appropriate discount rate to use to compute the present value in this case:
- \( P_{V_0} = \frac{1000}{(1.0638)^5} = 734.01 \)

After-Tax Interest Rates
- Taxes reduce the amount of interest the investor can keep, and we refer to this reduced amount as the after-tax interest rate.
- if the interest rate is \( r \) and the tax rate is \( t \), then for each $1 invested you will earn interest equal to \( r \) and owe tax of \( t \times r \) on the interest. The equivalent after-tax interest rate is therefore
- \( \text{After-Tax Interest Rate} \rightarrow r - (t \times r) = r(1 - t) \)
- Example 5.9 (textbook) = Suppose you have a credit card with a 19.9% A PR with daily compounding, a bank savings account paying 5% EAR, and a car loan with a 4.8% APR with monthly compounding. Your income tax rate is 40%. The interest on the savings account is taxable, and the interest on the credit card and on the car loan is not tax-deductible. What is the effective after-tax interest rate of each instrument, expressed as an E\( AR \)? What should your priorities be in terms of your financial situation?
- 5% EAR rate, \( E\text{AR} \) of credit card \( (1 + 0.199/365)^{365} - 1 = 22.01\% \), \( E\text{AR} \) of the car loan is \( (1 + 0.048/12)^{12} - 1 = 4.91\% \)
- After-tax interest rate earned on savings account is 5% x (1 - 0.40) = 3%
- should pay off the credit card, as its after-tax interest cost (over 22%) is higher than the after-tax interest you can earn in your savings account (only 3%). In addition, even though your savings appear to have a higher rate of interest than the car loan, the after-tax return on your savings, 3%, is less than the after-tax cost of the car loan, 4.91%. Thus, you should use your savings to pay down the car loan, too. So your priorities are first to pay off the credit card (in full) and, second, pay down the car loan. If you have money left over, you can put it in your savings account.

The Opportunity cost of capital
- the discount rate that we use to evaluate cash flows on the investor’s opportunity cost of capital (or more simply, the cost of capital), which is the best available expected return offered in the market on an investment of comparable risk and term to the cash flow being discounted