Information conservation is fundamental: recovering the lost information in Hawking radiation

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Abstract

In both classical and quantum world, information cannot appear or disappear. This fundamental principle, however, is questioned for a black hole, by the acclaimed "information loss paradox". Based on the conservation laws of energy, charge, and angular momentum, we recently show the total information encoded in the correlations among Hawking radiations equals exactly to the same amount previously considered lost, assuming the non-thermal spectrum of Parikh and Wilczek. Thus the information loss paradox can be falsified through experiments by counting the covariances of Hawking radiations from black holes, such as the manmade ones speculated to appear in LHC experiments. The affirmation of information conservation in Hawking radiation will shine new light on the unification of gravity with quantum mechanics.

Essay written for the Gravity Research Foundation 2013 Awards for Essays on Gravitation.

Submitted on March 30, 2013.

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Information is physical [1], it cannot simply disappear in any physical process. This basic principle of information science constitutes one of the most important elements for the very foundation to our daily life and to our understanding of the universe. Unfortunately, it is in stark confrontation with the information loss paradox, which claims information can be lost in a black hole, allowing many distinct physical states to evolve into the same final state.

So what is information? According to the standard textbook definition, information refers to an inherent property concerning the amount of uncertainty for a physical system. Shannon provided a mathematical formula to quantify and measure information given the state of a system [2]. Classically, a physical state is specified by a distribution function in the multi-dimensional phase space for all its degrees of freedom. This distribution evolves according to Liouville's theorem, which conserves the phase space volume and gives rise to the conservation of entropy or information under Hamiltonian dynamics. Classical information is stored, processed, and communicated according to classical physics. In parallel, quantum information follows the laws of quantum mechanics. For either a pure state specified by a wave function or a mixed state specified by a density matrix, its quantum information content is measured by von Neumann entropy instead [3, 4]. In quantum physics, the fact that information is physical takes on a more demanding role. For instance, we know that quantum information can neither be cloned [5] nor deleted [6].

Shannon's entropy, is analogous to thermodynamic entropy, which measures the degree of uncertainty in a physical system. Perhaps the example of a classical bit of information in a modern computer device studied by Landauer [7] best illustrates this point. Consistent with the second law of thermodynamics, the entropy for the environment increases after the erasure of the bit of information because useful work is consumed in the process. Entropy (or information) for the whole system of the bit coding device plus its environment, nevertheless, remains conserved.

The second example concerns the famous Maxwell's demon [8, 9], who paradoxically prepares a more ordered state (with less entropy) by performing microscopic operations to each individual atoms in a gas ensemble. Its resolution comes from the realization of the demon's ever increasing amount of information for all atoms in order to properly act on each individual passing ones. Given the demon's finite capacity to store information, it must update earlier information with newly acquired information, consequently increasing entropy for the environment [3]. The entropy or information remains conserved for the total system of the demon, the atomic gas ensemble, and the environment.

The third example concerns quantum teleportation [10], the paradigm of quantum information science whereby an unknown quantum state is disassembled at the sender and regenerated at the receiver. It was once thought that faster than the speed of light communication may become possible with teleportation [3]. However, information is physical. According to the special theory of relativity, no physical system can travel faster than light, including the information encoded in an unknown physical state. In the teleportation process, reconstruction of the unknown initial state at the receiver end requires a classical communication channel linking to the sender, which is ultimately limited by the speed of light.

The above examples illustrate that information is conserved for a closed system, be it classical or quantum, because information itself is physical. The discovery of Hawking radiation from a black hole [11, 12], however, brings up a serious challenge to the principle of information conservation. While controversial, the conjecture that information could disappear or is not conserved, in a black hole, is nevertheless supported by its share of believers [13]. This so-called paradox of black hole information loss contradicts with the fundamental laws of physics that support conservation of entropy or information.

Historically, a black hole was first viewed as only capable of absorbing but not emitting particles according to classical physics. This was not considered to contradict information conservation principle. As particles disappear into a black hole, the information encoded remains contained inside the black hole, although inaccessible to an outside observer. Thus this information is simply considered hidden inside a black hole, not a real loss to the outside. However, the contradiction arises after the discovery of Hawking radiation [11, 12], which is a natural outcome according to the theory of relativity, quantum mechanics, and thermodynamics.

Hawking asserted that the emitted radiation from a black hole is thermal and its detailed form is independent of the structure of matter that collapsed to form the black hole [16]. The radiation state is thus considered a completely mixed one which is incapable of carrying information about how a black hole is formed. The fact that radiation outside the black hole is in a mixed or a thermal state is in itself not in contradiction with quantum mechanics. When a system is composed of several parts, the reduced state for any part or a subsystem may be mixed and typically the information for each part decreases. The mechanism for this information loss is quantum entanglement. When two subsystems are entangled, their respective entropies increase, which is equivalent to the decease of information contained in each subsystem. The apparent loss to the total information of the system calculated from summing up the information from its two subsystems is due to information hidden inside correlations between the subsystems. In the case of a radiating black hole, it is the correlation between the outgoing radiation states and the internal inaccessible states of the black hole. After the black hole is exhausted due to Hawking radiation, however, a complete thermal state of Hawking radiation contains no information at all. This portraits the most rudimentary form of the paradox for black hole information loss according to our understanding.

In a recent article, the possibility that information about infallen matter could hide in the correlations between the Hawking radiation and the internal states of a black hole is ruled out [17]. This made the information loss paradox more severe, *i.e.*, the paradox arises immediately after the black hole starts to emit thermal Hawking radiations.

We have followed up, recently visited and revisited this same problem [14, 15]. In our opinion, we have provided a satisfactory resolution based on the fundamental principles of physics and statistical theory. At the heart of our proposal is the discovery by us of the existence of correlations among Hawking radiations. Our calculations show that the amount of information encoded in this correlation exact equals to the same amount of information claimed lost by the information loss paradox [14, 15]. In this essay, we present our current understanding of the information conservation principle in the context of Hawking radiation as tunneling [18]. Additionally, we compute the ratio for entropy production and show in detail how the lost information is balanced by the correlation hidden in Hawking radiation.

The original treatment of Hawking radiation [11, 12] assumes a fixed background geometry, such an approximation causes Hawking's result to be inconsistent with energy conservation. In contrast, the approach of Hawking radiation due to quantum tunneling, first treated by Parikh and Wilczek [18], strictly enforces energy conservation for the s-wave outgoing tunneled particles. Particles are supplied by consideration of the geometrical limit due to the infinite blue shift of the outgoing wave-packets near the horizon. The tunneling barrier is created by the outgoing particle itself, which is ensured by energy conservation. Making use of the Painlevé coordinate system that is regular at the horizon, a non-thermal spectrum for Hawking radiation, or the energy dependent tunneling probability

$$\Gamma(M;E) \sim \exp\left[-8\pi E\left(M - E/2\right)\right], \tag{1}$$

is found for a Schwarzschild black hole of a mass M [18]. We will adopt the convenient units of $k = \hbar = c = G = 1$. M is omitted when no ambiguity arises. This result is consistent with the change of the Bekenstein-Hawking entropy $S_{\rm BH}$ for a black hole as shown in Ref. [18], or $\Gamma(M;E) = \exp(\Delta S_{\rm BH})$. It is clearly distinct from a thermal distribution of $\exp(-8\pi EM)$, thus Hawking radiations must be correlated, and their correlations can carry away information encoded within [14, 15].

How to probe the existence of this correlation? Given two statistical events, like two emissions of Hawking radiation, with their joint probability denoted by $p(A,B)$, the probabilities for the two respective emissions are given by $p(A) = \int p(A,B)dB$ and $p(B) = \int p(A,B)dA$. We can proceed with a simple check to see whether $p(A,B) = p(A)\cdot p(B)$ holds true or not. If the equality sign holds true, then there exists no correlation between them. The two events are statistically independent. This is the case when the emission spectrum is thermal. Otherwise for a non-thermal spectrum as given by Eq. (1), the equality sign does not hold. The two emissions are dependent or correlated. Alternatively, we can check if the conditional probability $p(B|A) = p(A,B)/p(A)$ of event B to occur given that event A has occurred is equal to the probability $p(B)$ [19].

We have previously shown [14, 15], the joint probability $\Gamma(E_1,E_2)$ for two emissions of Hawking radiation, one at an energy E_1 and another at an energy E_2, is given by

$$\Gamma(E_1,E_2) = \exp\left[-8\pi(E_1+E_2)\left(M - (E_1+E_2)/2\right)\right], \tag{2}$$

i.e., the probability of two emissions at energies E_1 and E_2 is precisely the same as the emission probability for a single Hawking radiation at an energy $E_1 + E_2$. This can be stated explicitly as

$$\Gamma(E_1,E_2) = \Gamma(E_1+E_2), \tag{3}$$

which holds true because *energy conservation* is enforced within the treatment of Hawking radiation as tunneling associated with the non-thermal spectrum of Eq. (1). The existence of correlation is thus affirmed because we find

$$\Gamma(E_1,E_2) \neq \Gamma(E_1)\cdot\Gamma(E_2). \tag{4}$$

Alternatively, the existence of correlation is revealed by the conditional probability

$$\begin{aligned}\Gamma(E_2|E_1) &= \Gamma(E_1, E_2)/\Gamma(E_1) \\ &= \exp\left[-8\pi E_2 \left(M - E_1 - E_2/2\right)\right], \end{aligned} \tag{5}$$

for an emission with energy E_2 given the occurrence of an emission with energy E_1, and we find $\Gamma(E_2|E_1) \neq \Gamma(E_2)$.

How to measure correlation in terms of the amount of information it can encode? A closely related topic exists in quantum information theory, where mutual information between two parties constitutes a legitimate correlation measure for a bipartite system. The mutual information between two Hawking radiations with energies E_1 and E_2 is defined accordingly as [3]

$$\begin{aligned}S(E_2 : E_1) &\equiv S(E_2) + S(E_1) - S(E_1, E_2) \\ &= S(E_2) - S(E_2|E_1), \end{aligned} \tag{6}$$

where $S(E_1, E_2)$ is the entropy for the system and $S(E_2|E_1)$ is the conditional entropy. A few simple substitutions yield $S(E_2 : E_1) = -\ln\Gamma(E_2) + \ln\Gamma(E_2|E_1) = 8\pi E_1 E_2$, confirming what we discovered earlier: *there exist hidden messengers in Hawking radiation* [14].

For a queue of emissions E_i $(i = 1, 2, \cdots, n)$, the total correlation can be calculated along any one of the independent partitions [14, 15]. We choose a partition according to the queue of the subscripts, expressing the total correlation as the sum of the correlations between emissions E_1 and E_2, $E_1 \oplus E_2$ and E_3, $E_1 \oplus E_2 \oplus E_3$ and E_4, $\cdots$, and $E_1 \oplus E_2 \cdots \oplus E_{n-1}$ and E_n, which is different from the sum of correlations between pairs of E_i and $E_{j\neq i}$. $E_A \oplus E_B$ denote the combined system of E_A and E_B. With the total correlation among all emissions obtained, the amount of information it can encode is again found to be exactly equal to the amount previously considered lost by the information loss paradox [14, 15]. Thus, while its properties are fascinating, a black hole is nothing fundamentally special when it comes to information conservation principle. It remains a physical system governed by the laws of physics we are accustomed to.

If Hawking radiation is thermal, its entropy is previously found to be approximately equal to 4/3 times the reduced amount of the entropy for a Schwarzschild black hole [20]. Expressed in terms of entropy production ratio, this becomes $R = dS'/dS_{\rm BH} \simeq 4/3$, where

dS' denotes the change of entropy for the thermal radiation. The increase of entropy implies the loss of information, thus the information loss paradox.

For a non-thermal spectrum, an extra term in the entropy $S(E_f|E_i) = -\ln\Gamma(E_f|E_i)$ arises, which is associated with the correlation between the two emissions [14], where $\Gamma(E_f|E_i) = \exp\left[-8\pi E_f\left(M - E_i - E_f/2\right)\right]$ is the conditional probability of an emission with energy E_f given an emission of energy E_i, or conditional on the total energy of all previous emissions being E_i according to the relationship (3) from energy conservation.

We now prove that the microscopic process of a Hawking radiation with an energy dE is unitary. The entropy carried by an emission dE is

$$dS = -\ln\Gamma(dE) = 8\pi dE\left(M - dE/2\right), \tag{7}$$

which can be compared with the increase of the Bekenstein-Hawking entropy of a black hole

$$dS_{\rm BH} = 4\pi[(M - dE)^2 - M^2] = -8\pi dE\left(M - dE/2\right). \tag{8}$$

This gives an entropy production ratio of $R = |dS/dS_{\rm BH}| = 1$ for the non-thermal spectrum of Eq. (1). Thus we show the Hawking radiation process is thermodynamically reversible, *entropy or information is conserved*. The decrease of the entropy for a black hole is exactly balanced by the increase of the entropy in the emitted radiations.

According to Bekenstein [21], there exists a one-to-one correspondence between the internal state of a black hole and its precollapsed configuration when its entropy is interpreted in terms of the Boltzmann's formula. The proof of the conservation of total entropy for a black hole and its Hawking radiation, together with the Bekenstein conjecture, thus implies a one-to-one correspondence between the precollapsed configuration and the state of Hawking radiation [22]. This shows that the complete process of Hawking radiation is unitary.

What happens when a black hole is exhausted due to emission of Hawking radiation? For the non-thermal spectrum of Eq. (1), the entropy of Hawking radiation can be computed through a counting of the microstates denoted by $(E_1, E_2, \cdots, E_n)$, constrained by *energy conservation* $\sum_i E_i = M$. Within such a description, a fixed queue of E_i specifies a microstate. The probability for the microstate $(E_1, E_2, \cdots, E_n)$ to occur is given by

$$\begin{aligned} P_{(E_1,E_2,\cdots,E_n)} &= \Gamma(M;E_1)\cdot\Gamma(M-E_1;E_2)\cdot \\ &\quad \cdots\Gamma\left(M - \sum_{j=1}^{n-1} E_j; E_n\right). \end{aligned} \tag{9}$$

Given the non-thermal spectrum, each of the factors on the right hand side can be easily computed. For example, we find for the last emission $\Gamma(M - \sum_{j=1}^{n-1} E_j; E_n) = \exp(-4\pi E_n^2)$. Collecting all factors together we obtain $P_{(E_1,E_2,\cdots,E_n)} = \exp(-4\pi M^2) = \exp(-S_{\rm BH})$. According to the fundamental postulate of statistical mechanics all microstates of an isolated system are equally likely, thus the number of microstates $\Omega = 1/P_{(E_1,E_2,\cdots,E_n)} = \exp(S_{\rm BH})$, and the total entropy of Hawking radiation becomes $S = \ln\Omega = S_{\rm BH}$, according to the Boltzmann's definition. The total entropy carried away by all emissions is thus shown to be precisely equal to the entropy in the original black hole. Therefore, due to *entropy conservation*, Hawking radiation must be *unitary*.

We have recently extended our resolution for the black hole information loss paradox [14] to an extensive list of other types of black holes, including charged black holes, Kerr black holes, and Kerr-Neumann black holes [15]. In their respective cases, our resolution is shown to remain valid provided the spectra for Hawking radiation as tunneling are non-thermal as required by the conservation laws of physics: *charge conservation, angular momentum conservation, and energy conservation*. Even when effects of quantum gravity [15] and non-commutative black holes [23] are involved, our discovery of correlation among Hawking radiations remains effective in providing a consistent resolution to the black hole information loss paradox.

After the discovery by us that the non-thermal spectrum of Parikh and Wilczek allows for the Hawking radiation emissions to carry away all information of a black hole, a natural question to ask is whether Hawking radiation is indeed non-thermal or not? Although the derivation of the non-thermal spectrum is based on solid theory, it remains to be confirmed experimentally or in observations. A recent analysis by us shows the non-thermal spectrum can be distinguished from the thermal spectrum by counting the energy covariances of Hawking radiations [24]. With the relatively low energy scale for quantum gravity and the large dimensions, the production of micro black holes and the corresponding observation of Hawking radiations has already been studied [25–30]. If Hawking radiations from a micro black hole were observed in an LHC experiment, our results show that it can definitely determine whether the emission spectrum is indeed non-thermal [24] or not. Thus it provides an avenue towards experimentally testing the long-standing “information loss paradox”.

The series of work by us reveal that information is conserved in an isolated system for a black hole including its radiations. The principle of information conservation remains true when correlations among Hawking radiations are properly taken into account. Information conservation principle thus states Hawking radiation is unitary, which shows that the dynamics of a black hole obey the laws of quantum mechanics. Since a black hole is a result of Einstein's field equation, the unitarity of a black hole definitely indicates the possibility of a unified gravity and quantum mechanics.

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