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Dear Prof. Rideout,

I would like to submit my paper titled:

"Spacetime is 4-dimensional"

to the gravity research foundation for 1985 Awards for Essays on Gravitation. ~~After~~ As soon as you receive my manuscript, please let me know! Thank you!

With best wishes!

Sincerely yours

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Spacetime is 4-dimensional

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Abstract

The quantum state of the universe is described by Hartle and Hawking's ground state which is defined by a path integral over all compact metrics. The most probable classical evolution of the universe can be considered come from some gravitational instanton by a quantum tunnelling. These arguments have been generalized to the case of Kaluza-Klein models. It is found that in d=11 simple supergravity,with a minisuperspace ansatz, all instantons must have a 4-dimensional sector. It suggests that this is the main reason why spacetime is 4-dimensional.

Gravity Essay 1985

Spacetime is 4-dimensional

Spacetime in which we grow up and live is apparently 4-dimensional. This fact is something so familiar to everybody's experience that we seldom pause to consider its origin. Most explanations made to date essentially are based, implicitly or explicitly, on the anthropic principle[1]: the reason that the universe looks as it does is simply a consequence of our existence. Specifically, if space is more than 3-dimensional, no stable bound orbits of planets around the Sun, which are necessary for planetary life to develop, are possible in both Newton's and Einstein's gravitational theories. In quantum mechanics the same argument applies to the existence of stable atoms, which is the basis for chemistry and biology. Also, if the development of intellegence crucially depends on the information-process or communication, it seems that space must have an odd number of dimensions to avoid the reverberation phenomenon. The disadvantage of the anthropic principle, however, is that our inhabitable universe is implicitly picked out from many alternative universes unsuitable for human life.

There is a common expectation in gravity theory that although spacetime appears smooth, nearly flat and 4-dimensional on large scales, at sufficiently small distances, the Planck length, it is highly curved with all possible topologies and of arbitrary dimension. The quantum state exhibits fluctuation among these possibilities. It is learned from quantum gravity that this so-called spacetime foam structure has about one unit of topology per Planck volume and can be viewed as the fluctuation with chaotic creation and annihilation of mini black holes.[2] From the discussion of this spacetime foam, it turns out that the 4-dimensionality of spacetime is by no means to be taken for granted. It critically depends on how the initial conditions are chosen for the universe. We are brought face-to-face with a dilemma : the initial condition for the evolution of the universe must be imposed by hand, but the universe must be self-contained, since there is nothing outside of it !

Before the new developments of quantum cosmology[3], a 4-dimensional macroscopic spacetime was chosen by hand, simply because it must be consistent with our experience. In quantum cosmology by using a Kaluza-Klein model, we can show that if the universe is in the ground state, it most probably evolves with 4 macroscopic dimensions[4].

The long standing "First Cause" problem in cosmology is dispelled by recent developments in quantum cosmology. In the Big Bang cosmological model, in the very early stage of Planck era, quantum gravitational effects played the most important role. In Euclidean quantum theory of gravity interacting with matter, the quantum state of the universe takes the ground state of Hartle and Hawking:[3]

$$\Psi(h_{ij}, \tilde{\phi}) = \int_C \delta[g_{\mu\nu}]\,\delta[\phi]\,\exp -\bar{I}[g_{\mu\nu}, \phi], \qquad (1)$$

where ϕ describes the matter fields, h_{ij} is the compact spacelike 3-geometry and $\bar{I}$ is the Euclidean action. The path integral is summed over all compact positive definite 4-metrics $g_{\mu\nu}$ with matter fields ϕ which have the configuration (h_{ij}, $\tilde{\phi}$) as a boundary. In general, one can consider models with (n-1) - dimensional compact spacelike geometrys and sum over n-dimensional manifolds. Furthermore, for some non-compact geometrys, if their complexified manifolds have some compact real sectors, we can first apply the proposal to these sectors, and then obtain the wave function for the original non-compact configurations by analytic continuation. For example, a 3-hyperboloid and a 3-sphere can be considered as two real sectors of a complexified manifold. Thus the wave function for the former is an analytic continuation of the latter [4] .

The evolution of the universe must obey the Einstein equations which contain constraint equations and time development equations. The constraint equations imply the fact that the evolution is independent of the time label and the 3-coordinate description of the spacelike 3-geometry in the 3+1 decomposition of spacetime. The quantum counterparts of these constraints are the Wheeler-DeWitt equation and the momentum constraints. Quantum gravity is even more economical than general relativity in that the time coordinate does not appear explicitly in the theory. The time lapse is somehow to be defined by all four manifolds sandwiched between two specified spacelike 3-surfaces.

We are now equipped to discuss the problem of dimension. The first simple Kaluza-Klein model discussed is of a positive cosmological constant Λ and a metric form $R \times S_3 \times S_n$ ($n \geq 2$) [4].

The contribution of classical Euclidean solutions of the field equations dominates the path integral (1), its action must be very small when both sizes of S_3 and S_n are very small. When these sizes are not large enough to reach the zero-potential surface of the Wheeler-DeWitt equation

$$\frac{6}{a^2} + \frac{n(n-1)}{b^2} - 2\Lambda = 0, \qquad (2)$$

the wave function takes an exponential behaviour, where a, b are the radii of S_3 and S_n respectively. However, as soon as (a, b) reach the spacelike (with respect to the super-metric) segment of this surface

$$\frac{3n - \sqrt{3n^2+6n}}{6} < \left(\frac{b}{a}\right)^2 < \frac{3n + \sqrt{3n^2+6n}}{6}, \qquad (3)$$

the wave function starts to oscillate. Then the wave function behaves as a wave packet $\Psi = C \cos S$, where C is a slowly changing amplitude. The rapidly oscillating phase S can be identified with the action of classical evolution orbits in the WKB approximation with the correspondence[3,5]

$$p_x = \frac{\partial S}{\partial x}, \quad (x = a, b) \qquad (4)$$

Thus the wave function represents an ensemble of classical trajectories which satisfy equation (4). The ground state proposal imposes a necessary and sufficient restriction so that there is one and only one trajectory through each point in the wave packet region. All classical orbits represented by the wave packet begin with a zero initial velocity from segment (3). The configuration region where the wave function oscillates is called Lorentzian region, otherwise is called Euclidean region. In the Euclidean region the time concept breaks down and the classical evolution becomes meaningless.

It is reasonable to assume that the universe we live in is of the greatest probability. The relative probability is evaluated by a path integral

$$\Psi^* \Psi = \int_{C+C^*} \delta[g_{\mu\nu}]\, \delta[\phi]\, \exp - I[g_{\mu\nu}, \phi] , \qquad (5)$$

the path integral is summed over all compact positive definite (n + 4) - metrics which have the configuration $S_3(a) \times S_n(b)$ as a section. One can view the transition to the wave packet region as a quantum tunnelling from Euclidean compact (n + 4) - geometries. A gravitational instanton is defined to be a complexified solution of classical field equations with some real compact sector. Therefore it dominates the contribution to the path integral (5). It follows that the most probable evolution of the universe must be quantum mechanically penetrated through the zero-potential surface from an instanton. For our trial model, the only instantons are $S_4 \times S_n$ and $S_3 \times S_{n+1}$. Turning to the Lorentzian region, it follows that the universe most probably evolves with an exponentially expanding external space and a static internal space of a given size.

Using this toy model, we have shown that, if the universe takes the ground state, the most probable evolution must have a static internal space. However, since the dimension of the metric form is put into by hand, the dimension problem remains unsolved. One must appeal to more realistic models.

Kaluza-Klein theory with d = 11 simple supergravity is a more realistic model [4]. Supersymmetry compels us to introduce a $\frac{3}{2}$-spin gravitino field and a 3-index antisymmetric tensor A_{MNP} into the theory. For simplicity, we assume the gravitino field to be zero. Then the field equations are

$$R_{MN} - \frac{1}{2} g_{MN} R = \frac{1}{48}\left(8 F_{MPQR} F_N{}^{PQR} - g_{MN} F_{SPQR} F^{SPQR}\right), \tag{6}$$

$$F^{MNPQ}{}_{;M} = \left[\frac{-\sqrt{2}}{2\cdot(4!)^2}\right]\cdot \eta^{M_1\cdots M_8 NPQ} F_{M_1\cdots M_4} F_{M_5\cdots M_8} \tag{7}$$

$$(0 \leq M, N, P, Q, S, R \cdots \leq 10)$$

where $F_{MNPQ} = 4!\, \partial_{[M} A_{NPQ]}$ and $\eta^{A\cdots N} = |g|^{-\frac{1}{2}} \epsilon^{A\cdots N}$.

In the following we shall work with a minisuperspace model with only a finite number degrees of freedom: the d=11 spacetime takes homogeneous $M_m \times M_n$ (m+n = 11) form, and all F_{MNPQ} components with mixed indices m and n are assumed to vanish. One is tempted to find the wave function of ground state, specifying M_m to be $R \times S_{m-1}$ and M_n to be S_n, where R denotes time, S_{m-1} and S_n are interpreted as the external and internal spaces respectively. The Wheeler-DeWitt equation is hyperbolic with the geometric average of the scale lengths Y as the only timelike coordinate. In the configuration space, when γ is sufficiently small, the action for Euclidean metrics is very small. The wave function takes some polynomial and exponential forms there, its exact form depends on the operator ordering in the Wheeler-DeWitt equation. One expects that as soon as it reaches a spacelike segment of the zero-potential surface, the wave function would start to oscillate and the universe would begin to evolve. Unfortunately, the zero-potential surface remains timelike everywhere and no Lorentzian region exists . This corresponds to the fact that there does not exist any classical Lorentzian solution under this ansatz.

However, if we assume $M_m = R \times H_{m-1}$ (H_{m-1} is a (m-1) - dimensional hyperboloid), the situation changes drastically. One can find classical solutions of metric signature $(\underbrace{- - \cdots -}_{m} \underbrace{+ + \cdots +}_{n})$ with $S_{m-1} \times S_n$ as a boundary, (S_{m-1} is considered as a compact real sector of complexified H_{m-1}). These solutions dominate the path integral (1). Returning to the H_{m-1} configuration, the wave function oscillates and represents the classical orbits of ground state. Possible orbits exist for $m=2,3 \cdots 9$, it means one can have macroscopic spacetime of $2,3 \cdots 9$ dimensions. Considering dimensionality, one hopes maybe the most probable evolution of the universe must have $m=4$. This is indeed the case.

In order to obtain the most probable evolution, one has to find the instanton solutions. In the subspace S_m ($m=1,2,3$) F vanishes and for our ansatz the right hand side of equation (7) for S_{11-m} vanishes. It follows that F must be a harmonic in S_{11-m}. However, we find from de Rham cohomology that $H^4(S_4)=1$ and $H^4(S_m)=0$ ($m\neq 4$). So there is no nontrivial instanton. If $m=5$, both F components in S_5 and S_6 must be a harmonic and so vanish. By the dimension duality, there does not exist nontrivial instanton for $m=10, 9, 8, 6$ either. The only nontrivial instanton is of $m=4$ or $m=7$ and the F components in S_7 does not need to be a harmonic. Fortunately, some instantons have been found, their Lorentzian versions are Freund-Rubin, Englert, Duff-Pope and Englert-Rooman-Spindel spaces[6]. These spaces are products of a 4-dimensional Anti-de Sitter space and a round or squashed 7-sphere. Perhaps, these cases exhaust all instanton solutions with this ansatz. Note that, de Sitter space and Anti-de Sitter space are two real sectors of the 4-sphere. Before the discovery of quantum cosmology, one did not know how to exclude the $m=5, 6$ cases.

It is believed that this 4-dimensional Anti-de Sitter space is our macroscopic spacetime. The static internal space represents the internal symmetry of particle physics. The gauge coupling constant inversely related to the size of the internal space S_7 must be time-independent. The 7-dimensional sector of Freund-Rubin space can be replaced by products $S_2\times S_2\times S_3$, $S_2\times S_5$ or $S_3\times S_4$. In any case, one cannot avoid a 4-dimensional sector. A conjecture is as follows: for the most general ansatz any instanon must have a 4-dimensional sector. The nonexistence of instanton without a 4-dimensional sector may be the main reason why spacetime is apparently 4-dimensional.

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