THE CAUSAL UNIVERSE

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ABSTRACT

Creation of matter is possible in the cosmological context, without cost of energy. This creation is regulated by the laws of quantum mechanics and general relativity. These elements are used to conceive a singularity-free causal open homogeneous isotropic cosmology. The history of the universe unfolds in two stages : the "fireball" production stage which occurs as the response to a spontaneous local disturbance which is followed by free expansion. The latter extrapolates back to the former so as to avoid the initial "big-bang" singularity.

Evidence drawn from astronomical observation points strongly to an expanding open structure of the universe [1]. Straightforward extrapolation of this cosmological state of affairs to the distant past according to the laws of general relativity leads to a singularity. This circumstance has lead to the so-called big-bang hypothesis [1], more a confession of desperation and bewilderment than the outcome of logical argumentation rooted in the known (or even unknown !) laws of physics. It is our endeavour to repair this situation [2], [3].

We show below how it is possible within the confines of general relativity and quantum mechanics, to conceive a non-singular universe as the response of matter and the gravitational field to a local disturbance in space-time. Thus we supplement the old adage "Nature abhors a vacuum" with the principle "Science abhors a singularity".

We have constructed a model, a toy world as it were, which exemplifies this conception. Matter emerges from a vacuum quantum fluctuation which is seized upon by the matter-gravitational coupling. The rest is an application of quantum mechanics.

Perhaps the most puzzling point in the epistemology of material existence is the origin of matter ex-nihilis. Where does the energy come from ? [And, at least for the present, we discard recourse to theology]. The answer is both simple and elegant. Namely when matter is created it carries a gravitational field. The extraordinary fact is that the energy associated with the gravitational field in this "cosmological context" is exactly equal and opposite to the energy carried by the matter from which it has been engendered. The latter is then created in accordance with the fundamental conservation laws of physics.

The key to the understanding of this important point lies in the phrase "in the cosmological context". The only relevant cosmological piece of the Einsteinian metric tensor $g_{\mu\nu}$ is its determinant g [4]. The remaining five relevant components are concerned with the response of the gravitational field to local agglomerates of matter whilst g contains the response to the global distribution characteristic of cosmology. Thus almost all present cosmologies deal only with the evolution of g, familiarly parametrized by the Robertson-Walker cosmic scale factor.

The proof of the energy cancellation theorem between the matter and g is elementary and informative. We represent matter by a scalar field, Ψ, of mass m. The action in usual notation is $(\hbar = c = 1)$

$$S = \frac{1}{2}\int d^4x \sqrt{-g}\left\{ g^{\mu\nu}\partial_\mu \Psi \, \partial_\nu \Psi - m^2\Psi^2 + \frac{R}{6}\Psi^2 - K^{-1}R \right\} \qquad (1)$$

According to the above, the cosmological metric is of the form

$$g_{\mu\nu} = \left[(-g)^{1/4}/(-\mathring{g})^{1/4}\right]\mathring{g}_{\mu\nu} \equiv \mathring{g}_{\mu\nu}\left(K/6 \ \phi^2\right)$$

where $\mathring{g}_{\mu\nu}$ is the Minkowski metric. ϕ turns out to be a convenient parametrization of g. By making the Weyl transformation, $\Psi = \psi/\sqrt{K/6}\,\phi$, S takes on a Lorentz invariant form in an underlying Minkowski space :

$$S = \frac{1}{2}\int d^4x \sqrt{-\mathring{g}}\left\{\left[\mathring{g}^{\mu\nu}\partial_\mu\psi \, \partial_\nu\psi - \frac{K}{6}m^2\phi^2\psi^2 + \frac{\mathring{R}}{6}\psi^2\right] - \left[\mathring{g}^{\mu\nu}\partial_\mu\phi \, \partial_\nu\phi + \frac{\mathring{R}}{6}\phi^2\right]\right\} \qquad (2)$$

Accordingly, the condition of energy-momentum conservation becomes

$$\partial_\mu T^\mu_\nu = \partial_\mu \left[T^\mu_\nu(\phi) + T^\mu_\nu(\psi) \right] = 0 \qquad (3a)$$

$$T_{\mu\nu}(\phi) = -\left\{ \partial_\mu\phi\,\partial_\nu\phi - \tfrac{1}{2}\mathring{g}_{\mu\nu}\partial_\sigma\phi\,\partial^\sigma\phi - \tfrac{1}{6}\left(\partial_\mu\partial_\nu - \mathring{g}_{\mu\nu}\Box\right)\phi^2 \right\} \qquad (3b)$$

$$T_{\mu\nu}(\psi) = +\left\{ \partial_\mu\psi\,\partial_\nu\psi - \tfrac{1}{2}\mathring{g}_{\mu\nu}\partial_\sigma\psi\,\partial^\sigma\psi - \tfrac{1}{6}\left(\partial_\mu\partial_\nu - \mathring{g}_{\mu\nu}\Box\right)\psi^2 + \tfrac{1}{2}\mathring{g}_{\mu\nu}(\kappa/6)m^2\phi^2\psi^2 \right\} \qquad (3c)$$

Equations (3) formulate precisely the required theorem. The minus sign in Eq. (3b) is essential. Space-time variations of g give rise to a negative contribution to the energy density. In consequence, matter carrying positive energy can be created yet the total Minkowskian energy kept fixed and equal to that of empty space. This theorem is the exact expression of the qualitative consideration that the gravitational attractive energy effectively counterbalances the rest mass. Thus the answer to the enigma of the creation of matter ex-nihilis is contained in Einstein's action ab initio, i.e. the term $-\kappa^{-1}R$ in Eq. (1) [5].

The essential feature in the whole analysis is the reduction of the cosmological problem to that of a dynamics which unfolds in the underlying Minkowski space (Eq. 2). This setting of cosmology is truly in the spirit of the interpretation of gravity as a gauge field which ensures the local conservation of energy-momentum [6], and more particularly in the context of spontaneous breakdown of gauge symmetry [7]. It follows that the response to a point-like isotropic disturbance at some origin, t = 0, r = 0, is a function of $t^2 - r^2 (\equiv \tau^2)$. This property opens the way to the formulation of a causal singularity-free cosmology.

Thus we postulate that for $\tau^2 < 0$ and $t < 0$, space is empty i.e. the universe is in the forward light-cone of the origin. At this origin, a quantum fluctuation [they always exist in empty space] of extension m^{-1} seeds the universe. We therefore set ϕ = constant for $\tau < 0$ and let it vary for $\tau > 0$.

The interval $0 \leq \tau \lesssim m^{-1} (\equiv \tau_0)$ is the seeding period. Our present considerations do not treat of this important "quantum" interval where the uncertainty principle forbids the formulation of the concept of the presence or absence of a physical particle. This necessary quantum "haziness" renders the definition of a point-like origin purely operational. We shall then show that ψ- particles, are produced for $\tau > \tau_0$. Thus we realize the classical conservation law through $T_{\mu\nu}(\phi) = T_{\mu\nu}(\psi) = 0$ for $\tau^2 < 0$, $t < 0$ but integrate it to $T_{\mu\nu}(\phi) + T_{\mu\nu}(\psi) = 0$ for $\tau > \tau_0$, each piece being separately non-vanishing.

What mechanism is available to produce ψ 's ? The answer lies in the mass term in $S \left(= K/6\, m^2 \phi^2 \psi^2\right)$. The dependence of ϕ on τ then gives rise to non-conservation of the number of ψ 's [8]. It is a straightforward matter to produce a formula for the number of ψ 's produced at time τ. One finds $N_k(\tau) = N_k(\tau_0) + [2N_k(\tau_0) + 1]\,|\gamma_k(\tau,\tau_0)|^2$. Here $N_k(\tau)$ is the number of ψ 's present at time τ in the mode k.

k is a momentum-like variable. One notes the remarkable quantum coherence property reflected in the "memory" of the initial situation through the factor $[2N_k(\tau_0)+1]$. The function $\gamma_k(\tau,\tau_0)$ is obtained from the solution of an ordinary differential equation with initial condition such that $\gamma_k(\tau_0,\tau_0)=0$. Furthermore our initial conditions require $N_k(\tau_0)=0$ whereupon integration over k yields for the density of ψ 's at time τ $(\equiv n(\tau))$ the formula $n(\tau)=Am^3(1+\delta(\tau))$; $\delta(\tau)$ is a small correction which vanishes asymptotically. A is a constant of $O(1)$. Thus, in the fireball stage, the universe expands and it leaves behind a sea of ψ -particles of almost uniform density in which the particles are a Compton wavelength apart.

The evolution of the fireball is found by integrating Einstein's equations with $T_{\mu\nu}$ obtained from the matter density $n(\tau)$. One finds an open universe which closely approximates a de Sitter space. It has the property that for some $\tau\,(=\tau_\infty)$, ϕ goes to infinity. The function $\delta(\tau)$ is always small, but vanishes strictly at $\tau=\tau_\infty$ as is required by self-consistency. Namely at $\tau=\tau_\infty$, one tends to a steady-state cosmology where the matter density is strictly constant. The theory is of the type encountered in phase transitions wherein ϕ is a self-consistent field which is engendered by and engenders $n(\tau)$. In particular there is a second trivial solution : $\phi\equiv$ constant, $n(\tau)\equiv 0$, which evidently is not realized.

Until now we have described how a universe can be produced. Will this production continue ad infinitum as it would appear from the preceding discussion ? The answer is no. Indeed, detailed analysis of the production mechanism reveals that it depends critically on the quantum coherence which is intrinsic to the toy model. Any perturbation from this model, in which Ψ is allowed to interact with fields other than gravity will lead to incoherence. We now observe that in the vicinity of $\tau = \tau_\infty$, the Ψ- particles accelerate rapidly with respect to each other. This implies radiation of other fields to which they are coupled and hence heating and incoherence. It is an easy matter to show that for temperatures of $O(m)$, production stops leaving behind massy matter as well as the ancestor of the present black-body radiation. This is the end of the fireball stage.

Production having stopped, general relativity leads naturally to a solution of free expansion. It is precisely this solution which previously had been extrapolated back to the big-bang singularity. Remarkably, this very analytic structure allows for a smooth joining between the fireball and expansion stages, thereby avoiding the singularity. The usual theorems [9)] which predict the inevitability of an initial singularity are foiled by the quantum production mechanism. This has its phenomenological expression in a negative pressure [2, 10)]. The causal universe so constructed is free of singularities as well as event and particles horizons [11)].

The detailed theory of the transition between the production and expansion regimes has not been developed. Rather continuity requirements permit an interpolation between the two in terms of two parameters. These are chosen to be the total present number of baryons and the entropy of radiation per baryon, s . Together, these determine the time, τ_1 , which marks the end of production. One finds $\tau_1 = 10^{-3}$ sec for m = 1 Gev corresponding to a proper time, τ_1^* , equal to 10^{-2} sec.

A noteworthy point is that the theory has a simple and distinct analytic character for a particular value of ς equal to 10^8 ! Why this special value agrees with observation [1)] is presently beyond our ken.

In the figure is presented the cosmological function $\ln\left(\kappa/6\,\phi^2\right) \equiv \lambda$, as a function of τ .

There are naturally several problems that must be confronted before our toy world is promotable to candidacy for the real one. Is there only one universe ? Are other fluctuations quenched and only one seeded ? Secondly what is the detailed quantum theory of the region $0 \leq \tau \lesssim m^{-1}$? And finally what is the nature of the primeval soup described here by ψ . In particular is quantum field theory at a sufficient stage of development to give us the proper conceptual outlook on such problems as hadron synthesis and baryon unbalance ? Some of these problems can be examined in the light of present concepts, but we are surely still missing some essential ideas. Nevertheless we do hope that we have happened upon some of them.

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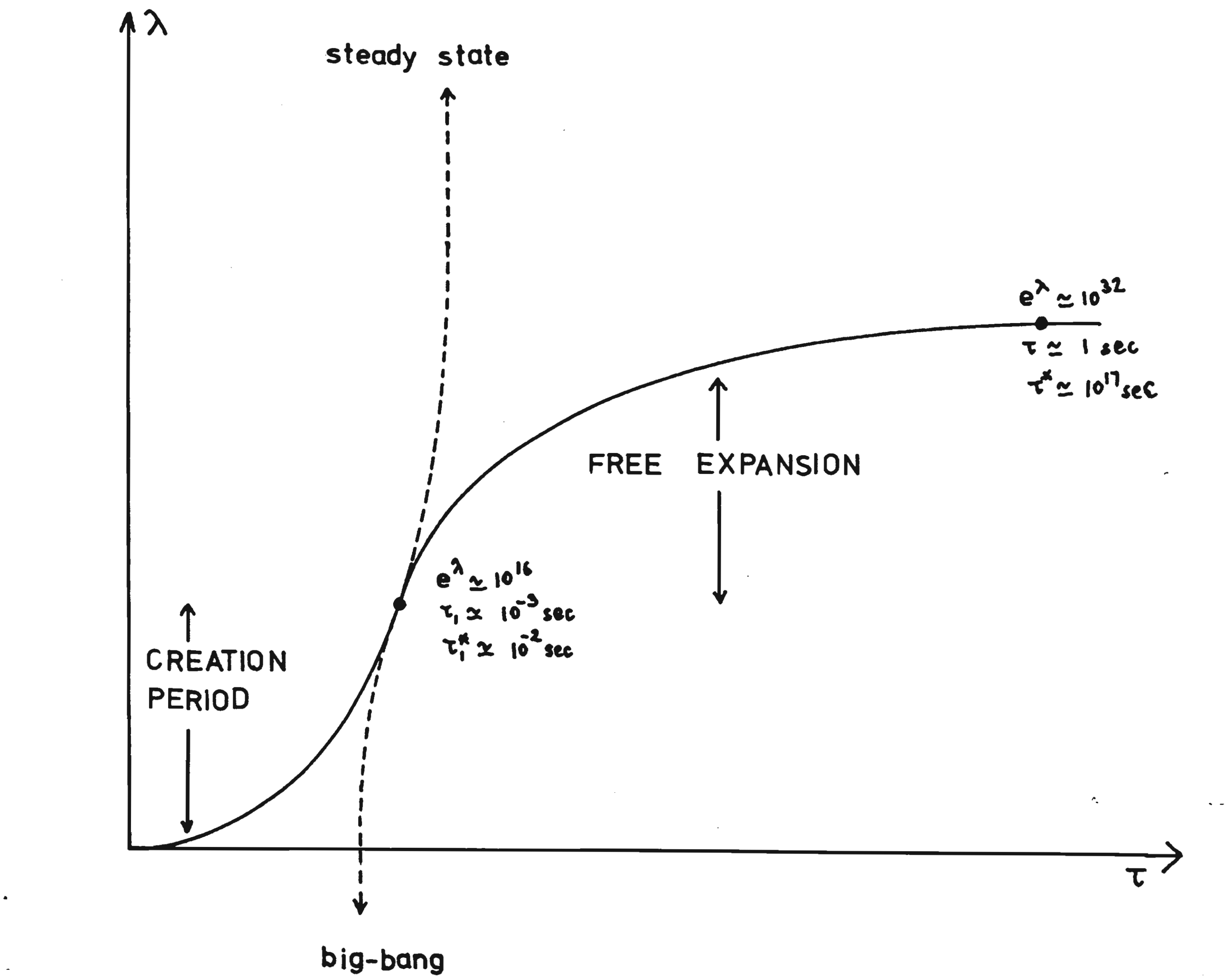
FIGURE CAPTION

λ as a function of τ . Time scales are calculated for m = 1 Gev.



λ
steady state
$e^{\lambda} \simeq 10^{32}$
$\tau \simeq 1$ sec
$\tau^{*} \simeq 10^{17}$ sec
FREE EXPANSION
$e^{\lambda} \simeq 10^{16}$
$\tau_1 \simeq 10^{-3}$ sec
$\tau_1^{*} \simeq 10^{-2}$ sec
CREATION PERIOD
τ
big-bang

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