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ON A POSSIBLE UNIFICATION OF GRAVITATIONAL AND WEAK INTERACTIONS

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SUMMARY

Within the framework of Cartan's generalization of Einstein' theory of gravitation one can achieve a unification of gravitational and weak interactions by appropriate choice of the parameter which couples spin and torsion. The proposed spin-torsion coupling has negligible cosmic effects except at stages of evolution when 10^{81} nucleons are confined to a sphere with a radius of about one astronomical unit. For a single nuclear particle the gravitational effects of mass and spin balance at a radius of about one percent of its Compton wavelength, thus stabilizing it against gravitational collapse.

Over half a century ago Elie Cartan[1] pointed out that Einstein's concept of gravitation as a geometrical feature in a Riemann space free of torsions is unnecessarily restricted. The lines of reasoning that led Einstein to his field equations can be followed through even if one aims at conceiving gravitation as a geometrical feature in a more general affine space (the "Cartan space") which admits the presence of torsions. In the resulting theory of gravitation, which was rediscovered and given its modern form by Kibble[2], the field equations

$$[1] \qquad G_{ik} = \varkappa \tau_{ik} \quad ,$$

which specify the part played by the matter stress tensor τ_{ik} as a source of gravitation, are complemented by a second set of equations

$$[2] \qquad C^{\cdot j}_{ik} = \varkappa \sigma^{\cdot j}_{ik}$$

which recognize the matter spin tensor $\sigma^{\cdot j}_{ik}$ as a source of torsion. The tensor G_{ik} in equation [1] differs from the corresponding tensor $G^{(E)}_{ik}$ in Einstein's theory by terms linear and quadratic in the Cartan tensor $C^{\cdot j}_{ik}$,

$$G_{ik} = G^{(E)}_{ik} + O(C^{\cdot j}_{ik}) + O(C^{\cdot j}_{ik})^2 \quad ,$$

and on account of the equations [2] the theory of Cartan and Kibble amounts to replacing Einstein's field equations by

$$[3] \qquad G^{(E)}_{ik} = \varkappa (T_{ik} + \varkappa X_{ik})$$

where T_{ik} is the symmetrized matter stress tensor[3], and X_{ik} is a tensor quadratic in the matter spin tensor $\sigma^{\cdot j}_{ik}$.

This theory will reduce to Einstein's theory in absence of material spin provided the coupling parameter in equation [1] is the usual

$$[4] \qquad \varkappa = 8\pi G c^{-4} = 2.08 \times 10^{-43} \text{ kg}^{-1} \text{ m}^{-1} \text{ sec}^{2} \ .$$

If the coupling parameter governing the relation [2] is accepted to have the same value [4], then the spin correction to Einstein's field equations described by the last term in equation [3] is going to be exceedingly small for spin densities encountered in matter of any but the most extreme densities. As Trautman[4] has shown, an assembly of 10^{80} spinning nucleons must be confined to a sphere of about 1 cm radius to yield a gravitational spin effect of the same order of magnitude as the gravitational mass effect. Kibble's assessment[2] is most succinct: "...it seems impossible that [the terms in $\varkappa^2$] would lead to any observable difference between the predictions of the two theories. Hence we must conclude that for all practical purposes the theory presented here is equivalent to the usual one."

Now, as has been pointed out by several authors[2,5,6], the term X_{ik} in equation [3] for the case of a minimally coupled spinor field has precisely the form of the four-fermion contact interaction familiar from the theory of weak interactions. In particular, for axial coupling one has

$$\varkappa X_{ik} \doteq g_o(\bar{\phi}\gamma_5\gamma_i\phi)(\bar{\phi}\gamma_5\gamma_k\phi)$$

where

$$g_o = \frac{3\pi G\hbar^2}{2c^2} = 3.79 \times 10^{-95} \text{ kg m}^5 \text{ sec}^{-2} .$$

On the other hand, the experimentally observed axial coupling constant of the weak interactions is

$$g_A = 1.01 \times 10^{-62} \text{ kg m}^5 \text{ sec}^{-2} .$$

The enormous mismatch between g_o and g_A,

$$\frac{g_A}{g_o} = 2.67 \times 10^{32} ,$$

seems to preclude the possibility of identifying the coupling of spin and torsion as the origin of the actually observed weak interactions.

However, there is no compelling reason why the coupling parameter governing equation [2] should be the same as the coupling parameter governing equation [1]. This fact emerges when one reconsiders, without prejudice, the standard derivation[7] of the field equations [1] and [2] from an action principle with the Lagrangian density

$$\mathcal{L} = k^{-1}\mathcal{R} + \mathcal{L}_m$$

where $\mathcal{R}$ is the curvature scalar density of the Cartan space, $\mathcal{L}_m$ the scalar action density of the matter field, and k a factor of dimension $kg^{-1}\ m^{-1}\ sec^2$ whose numerical value is completely undetermined at this stage. The field equations [1] are obtained upon variation with respect to the basis vectors $\underline{e}_i$ of the Cartan space.

$$\underline{e}_i \cdot \frac{\delta \mathcal{L}}{\delta \underline{e}^k} = 0 \ .$$

They acquire physical content if one makes the identification

$$e\, \tau_{ik} \equiv \alpha\, \underline{e}_i \cdot \frac{\delta \mathcal{L}_m}{\delta \underline{e}^k} \quad ; \quad e \equiv \det|e^\gamma{}_k|$$

where α is a dimensionless scale factor which acknowledges the proportionality, not the equality, of inertial and gravitational stress. Accordingly, the coupling parameter $\varkappa$ in equation [1] should be written

$$\varkappa \equiv k\alpha \ ,$$

and it is this product only which must be given the value [4] to yield agreement with observations.

Similarly, field equations of the type [2] are obtained upon variation with respect to the contortion tensor $K^{ik}{}_j$ of the Cartan space.

$$\frac{\delta \mathcal{L}}{\delta K^{ik}{}_j} = 0 \ .$$

They acquire physical content if one makes the identification

$$e\sigma^j{}_{ik} \equiv \beta\ \frac{\delta \mathcal{L}_m}{\delta K^{ik}{}_j}$$

where β is another dimensionless scale factor which acknowledges the proportionality, not the equality, of inertial and gravitational spin. Accordingly, one should write the field equation

[2'] $$C^{j}_{ik} = \varkappa' \sigma^{j}_{ik}$$

where the coupling parameter is the product

$$\varkappa' \equiv k\beta \ .$$

If that is accepted, equation [3] is modified into the form

[3'] $$G^{(E)}_{ik} = \varkappa(T_{ik} + \varkappa' X_{ik}) \ ,$$

and a unification of gravitational and weak interactions can be achieved by choosing on account of equation [5], $\varkappa' = 8\pi G' c^{-4}$ so that

[6] $$\frac{\varkappa'}{\varkappa} = 2.67 \times 10^{32} \quad \text{i.e.} \quad \varkappa' = 5.55 \times 10^{-11}\ kg^{-1}\ m^{-1}\ sec^{2}$$

eliminates any mismatch between $g_o' = 3\pi G' \hbar^2/2c^2$ and g_A.

Although this proposed coupling between spin and torsion is strong enough to yield observable effects in the microcosmic domain, it is still not strong enough to yield observable effects on a macrocosmic scale except under conditions of extreme material density. For example, in the case of spinning dust equation [3'] amounts to replacing in the Newtonian approximation the usual law of gravitation $\nabla^2\phi = 4\pi G\rho$ by a law of the form

[7] $$\nabla^2\phi = 4\pi G(\rho - \varkappa'\sigma^2)$$

where σ is the spin density of dimension $kg\ m^{-1}\ sec^{-1}$. When used in conjunction with the Euler equation

$$(\partial \underline{v}/\partial t) + (\underline{v}\cdot\underline{\nabla})\underline{v} = -\underline{\nabla}\phi$$

and the conservation laws of mass and spin

$$(4\pi/3)\rho R^3 = M \quad ; \quad (4\pi/3)\sigma R^3 = S$$

in the context of classical cosmology, characterized by the homogeneous velocity field

$$\underline{v} = (\dot{R}/R)\underline{x} \quad ; \qquad \varrho = \varrho(t) \quad .$$

one obtains for the scale factor R(t) the differential equation

$$\ddot{R} = -\frac{GM}{R^2} + \frac{3G\varkappa' S^2}{4\pi R^5} \quad .$$

An exact solution of this equation is

$$R(t) = \left(\frac{3\varkappa' S^2}{16\pi M} + \frac{9GM}{2} t^2\right)^{1/3} \quad .$$

Except for the replacement of $\varkappa$ by $\varkappa'$ this result is formally identical with the result of Trautman[4] obtained by a different method. It means that a sphere containing N particles of nuclear mass μ and spin $\hbar/2$ has a minimum radius

$$R_{min} = \left(\frac{3\varkappa'\hbar^2}{64\pi\mu} N\right)^{1/3} \approx 10^{-18} N^{1/3} \text{ m} \quad .$$

Thus, a metagalaxy of 10^{81} particles must be confined to a sphere of about 10^9 m radius, i.e. about one astronomical unit, if the spin-torsion interaction is to become dominant. At densities prevailing in the universe, at the present time, the effects of spin-torsion interaction are obviously quite negligible.

However, the quasi-antigravitating tendency of this proposed spin-torsion interaction, as indicated by equation [7], may possibly be the origin of the stability of so-called elementary particles against gravitational collapse. For a nuclear particle the radius at which the gravitational effects of mass density ϱ and spin density σ are in balance is

$$r_o = \left(\frac{3\varkappa'\mu^2 c^3}{16\pi\hbar}\right)^{1/3} \times \frac{\hbar}{\mu c} \approx 10^{-2} \times \frac{\hbar}{\mu c}$$

i.e. at about one percent of its Compton wavelength.

The proposal outlined above amounts to introduction of the dimensionless

$$K = \varkappa'\mu^2 c^3 \hbar^{-1}$$

as a new universal constant of nature.

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BIOGRAPHICAL SKETCH

Frederick Augustus Kaempffer, born November 29 1920 in Görlitz, Germany. Married June 14 1944 to Gisela Elisabeth Wirth. Two sons. ages 29 and 23. Graduate Work at the University of Göttingen. Dipl.Phys.1943. Dr.rer.nat.1948. Lady Davis Fellowship 1948 pays for emigration to Canada. followed by acquisition of Canadian Citizenship in 1955.

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