Using Gravity to Determine the Nature of Superluminous Astronomical Objects

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Summary

Power spectrum analysis of the optical variations of Sco X-1 reveals periodicities which may be interpreted as atmospheric vibrations of an underlying star. This hypothesis leads to an estimate for the effective gravity in the atmosphere which implies that the star associated with this X-ray source is either a white dwarf or a neutron star. We suggest that similar analysis of the optical variations of other astronomical objects can provide similar information about their basic structure; in particular, the problem of the nature of quasars might be resolved by this means.
1. Introduction

It does not take a particularly close observer of recent progress in high-energy astrophysics to realise that current understanding of many energetic cosmic phenomena is still rudimentary. The nature of such super-luminous objects as quasars and X-ray stars as well as their relation to other astronomical objects remains uncertain. While observational work has continued to increase the available store of data, the theoretical situation is not much improved over its position several years ago.

Recently we have been carrying out a detailed study of the 'flickering' of the optical star associated with the brightest extra-solar X-ray source, Sco X-1. In the course of this work we have discovered a useful technique whereby it is possible to diagnose the effective gravity of not only this source but also other thermally radiating astrophysical plasmas. Such a technique is likely to prove invaluable, since knowledge of the strength of the gravitational field associated with an astronomical body is a decisive factor in determining its fundamental nature.

2. Periodicities in Sco X-1

The spectral features, optical 'flickering' activity, and time variations of the star which has been identified with the prototypical discrete X-ray source Sco X-1 are outwardly characteristic of old novae. Since novae are generally believed to be members of highly-evolved close binary systems, a number of attempts have been made in the past few years to find a periodicity in the optical light curves. But all such investigations have led to fairly inconclusive results, partly because they employed data that were not particularly suitable to the task.
Following the publication of Sandage, Westphal, and Kristian's extensive photometric observations of Sco X-1 during five nights in 1967\(^1\), we began our own search for the presence of periodic structure in the optical 'flickering'. Almost immediately, we were able to identify stretches of data in which power spectrum analysis revealed significant periodicities. In retrospect, this piece of good fortune was largely due to the inability of our computer program (as then written) to handle very large numbers of data points, which prevented us from falling into the trap that has led several other investigators to conclude that no significant stable periodic structure exists in the optical flux of Sco X-1 over a whole night's observations.

The results of our detailed investigation of the optical fluctuations of Sco X-1 using the technique of power spectrum analysis\(^2\) are currently being prepared for publication. The most striking discovery, which has already been published in the British journal Nature\(^3\), shows what happened to the rapid 'flickering' after a particularly large and well-defined optical flare on the night of April 3, 1967. Prior to the occurrence of this flare event, no evidence exists for any statistically significant periodicity in the power spectrum of the optical variations. However, just after the flare a fairly broad band of frequencies centered on 0.006 cps, corresponding to a period of approximately three minutes, has clearly been excited (see Figure 1). Thereafter, a periodic component at 0.0055 cps grows to a peak amplitude of about one per cent of the mean optical intensity of the star before decaying with a time-scale of roughly one hour. Similar results have been found for the data of other nights where power spectrum analysis has provided evidence for relatively large periodic components.
One of the most important aspects of our analysis of the optical fluctuations of Sco X-1 is the evidence we have found for the presence of 'harmonics' of the basic frequency at 0.0055 cps. These small amplitude periodicities generally appear as marginally significant peaks in the power spectra of data samples taken from all four nights on which observations were made by Sandage et al.\(^{(1)}\) with either the 100-inch or the 200-inch reflector at Palomar. The amplitudes of these 'secondary' periodic components are generally found to be about a few tenths of one per cent of the mean optical flux. In our experience, the power spectrum of any given set of photometric data may show comparably significant peaks at several frequencies or a single prominent peak accompanied by several lesser ones. The power spectrum in Figure 1 shows a 'harmonic' peak at 0.0085 cps that is about four standard deviations above the mean 'noise' level.

3. Implications for Sco X-1 and other X-ray stars

There is now fairly general agreement that the low-energy X-ray flux from Sco X-1 is produced by thermal bremsstrahlung of an essentially-isothermal optically-thin plasma at kilovolt temperatures\(^{(4,5)}\). It has also been shown that the observed optical continuum can be interpreted as bremsstrahlung radiation from the same plasma that produces the soft X-rays, but which is becoming optically thick at long visible and near-infrared wavelengths\(^{(5,6)}\).

We shall adopt the simplest possible model for the hot plasma atmosphere of Sco X-1:

(a) a compressible, isothermal, fully-ionized hydrogen gas in a 'vertical' uniform gravitational field;

(b) 'effective' boundary conditions of a fixed base and a free boundary at the top of the atmosphere (open-end organ-pipe analogy).
The dispersion relation for 'vertical', small amplitude, adiabatic waves in such a plasma is

\[ \omega^2 = \omega_a^2 + c_s^2 K^2 \]  

(1)

where \( \omega_a = \frac{c_s}{2H} \) is the acoustic frequency, \( c_s = \left( \frac{\gamma kT}{\mu g} \right)^{\frac{1}{2}} \)

is the speed of sound, and \( H = \frac{kT}{\mu g} \) is the scale height.

\( \gamma \) is the ratio of specific heats, \( k \) is Boltzmann's constant, \( T \) is the plasma temperature, \( \mu \) is the mean mass of particles in the plasma, and \( g \) is the gravitational acceleration. The effect of the boundary conditions is that only certain discrete values of the wave number \( K \) are allowed. These are

\[ K_n = \left( \frac{2n+1}{2L} \right) \pi, \quad n=0,1,2,3, \ldots \]  

(2)

where \( L \) is the 'effective' thickness of the atmospheric plasma. Thus, the ratio of the \( n^{th} \) harmonic to the fundamental \( (n = 0) \) frequency is given by

\[ \frac{f_n}{f_0} = \left[ \frac{(2n+1)^2 \xi^2 + 1}{\xi^2 + 1} \right]^{\frac{1}{2}} \]  

(3)

where

\[ \xi \equiv \frac{\pi H}{L}. \]
The lowest-frequency significant peak that recurs in the power spectra of the optical flux of Sco X-1 is at approximately 0.0055 cps. It is also the strongest periodicity that we have been able to find (cf. Figure 1). If we (provisionally) designate this frequency as the 'fundamental' it is possible to interpret successfully the weaker periodicities as acoustic modes of atmospheric oscillation. This leads to a value for the parameter $\xi$ which is approximately 0.5.

With values for the fundamental frequency $f_o$ and the parameter $\xi$, it is possible to estimate the effective values of the gravitational acceleration $g$ and the atmospheric thickness $L$. These quantities may be expressed as

$$g = 4\pi f_o \left[ \frac{\left( \frac{k T}{\mu} \right)}{\sqrt{\xi (\xi^2 + 1)}} \right]^{1/2}$$

and

$$L = \frac{1}{4 \pi f_o} \left[ \frac{1}{\sqrt{\xi (\xi^2 + 1)}} \left( \frac{k T}{\mu} \right) \right]^{1/2}.$$  

Taking $\xi = 0.5$, $f_o = 0.0055$ cps, $\gamma = 5/3$, $T = 7 \times 10^7 \degree K$, and $\mu = m_H/2$ (where $m_H$ is the mass of the hydrogen atom), we obtain the following numerical estimates:

$$g \approx 5 \times 10^6 \text{ cm sec}^{-2}$$

and
\[ L \approx 10^{10} \text{ cm.} \]

Since the value determined for \( L \) depends critically on both the details of the theoretical model and identification of marginally significant periodicities with specific modes of atmospheric vibration, this value is much less certain than our determination of \( g \). It is not really necessary to adopt a specific theoretical model in order to estimate the effective gravitational acceleration in the radiating atmosphere of Sco X-1, provided only that the strongest periodicity at 0.0055 cps really is produced by the oscillation of the hot plasma in its fundamental (or a low harmonic) mode. The effective value of \( g \) is then expected to be of order

\[ 2\pi \left( 0.0055 \right) c_s \approx 5 \times 10^b \text{ cm sec}^{-2}, \]

just as calculated above. The important point here is that this result is more than two orders of magnitude greater than the solar surface gravity. Such a high value for the effective \( g \) in the X-radiating atmosphere of Sco X-1 implies that the underlying star must be a condensed object — either a white dwarf or a neutron star (see Figure 2). This conclusion seems to confirm the widely-held belief among theoreticians that high-energy X-ray sources must be associated with compact stellar objects.\(^{(7,8,9)}\)
4. Possible Application to Quasars

It is not difficult to think of several other interesting astronomical situations where the same sort of analysis that we have employed for Sco X-1 could prove fruitful, provided we have a relatively bright source of hot gas radiating either thermal bremsstrahlung or line radiation which can be physically excited into oscillation by flares or other explosive phenomena. Quasars seem to be particularly suitable candidates since many of them have flat, weakly-polarized optical spectra that could be produced by the thermal radiation of an ionized plasma at $10^4$-$10^5 \, ^\circ K$, and optical 'flaring' seems to occur in a number of these objects—notably 3C48, 3C273, 3C279, 3C345, and 3C446.

It is instructive to consider the case of 3C273, the quasar archetype, because its properties are relatively well-known. Its optical continuum has a fairly flat spectrum and is essentially unpolarized, suggestive of emission from an optically-thin thermal gas at a temperature of approximately $30,000 \, ^\circ K$\(^{(10)}\). Whatever mechanism is responsible for the intense, highly-polarized\(^{(11)}\) infrared flux does not seem to produce much of the optical continuum radiation. Assuming that the masses of quasar nuclei are of the same order as the masses of galactic nuclei, $\sim 10^{10} M_\odot$, and that the characteristic size of the radiating quasar plasma is given by the light-travel distance for optical flares, $\sim$ one light day $\approx 10^{15.4} \, \text{cm}$, we might expect the natural oscillations of the emitting plasma to have periods $\sim 10^2 \, \text{sec}$.
Fortunately, 3C 273 is a 12 to 13th mag optical object which can be observed with the same counting statistics as Sandage et al.\(^{(1)}\) used in the case of Sco X-1. It would be very useful to have such detailed observations of 3C 273, particularly after a large optical flare, so that power spectrum analysis could be employed usefully to search for weak periodicities in its 'flickering' light flux. Just as for Sco X-1, we would hope to determine the effective gravitational acceleration in the radiating plasma of 3C 273 and thereby obtain a clue to the basic nature of the source. Conceivably, one might be able to determine whether the redshifts of quasars are 'cosmological' or 'gravitational', since the latter hypothesis requires masses \(\sim 10^{14} M_\odot\) and correspondingly larger values of the effective gravity.
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References

Figure Captions

**Figure 1.** Power spectrum of the optical flux of Sco X-1 obtained from 500 post-flare data points of 3 April 1967.\(^1\) The number of lags used was 150. The peaks that we have identified as the fundamental and the first harmonic modes of atmospheric oscillation are indicated.

**Figure 2.** Surface gravity \(g_{\text{surface}}\) of various types of stars as a function of stellar radius \(R\) (logarithmic plot). The effective value of \(g\) estimated for the radiating atmosphere of Sco X-1 is shown as the lower limit to the surface gravity of the underlying star of Sco X-1. White dwarfs or neutron stars are the only possibilities.
Figure 1.

SCO X-1

- Fundamental at 0.0055 cps
- First Harmonic at 0.0085 cps

SPECTRAL DENSITY (arbitrary units, linear scale) vs FREQUENCY (cps)
Figure 2.

Log \( g_{\text{surface}} \) vs. Log \( R \) (cm).

- Neutron Stars
- White dwarf Stars
- Surface Gravity of Underlying Star of Sco X-1
- Main-sequence Stars
- Sun
- Jupiter
- Earth
- Mercury

Log \( R \) (cm)
Biographical Sketches

Dr. Paul A. Feldman was born on 22 April 1940 in Everett, Massachusetts. He attended the public schools of Chelsea, Massachusetts, where his parents still reside. Dr. Feldman is a graduate of the Massachusetts Institute of Technology and holds a Ph.D. in Physics from Stanford University in California. He spent last year as a N.A.T.O. Post-doctoral Fellow at Cambridge University's Institute of Theoretical Astronomy where he is now a Visiting Research Fellow. His current research interests center on topics in the fields of non-thermal, high-energy, and plasma astrophysics. He is married with one child, a six-year-old daughter.

Mr. John R. Gribbin is twenty-four years old and resides with his wife in Bottisham, Cambridgeshire. He has been awarded both a Bachelor's degree and a Master's degree from the University of Sussex in England. Currently, Mr. Gribbin is a research student at the Institute of Theoretical Astronomy where he holds Cambridge University's Isaac Newton Studentship in Gravitational Astronomy. Mr. Gribbin expects to receive his Ph.D. this year with a thesis on the pulsations of high-density stars.