

Axion Experiments to Algebraic Geometry

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Testing Quantum Gravity via the Weak Gravity Conjecture

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Abstract

Common features of known quantum gravity theories may hint at the general nature of quantum gravity. The absence of continuous global symmetries is one such feature. This inspired the Weak Gravity Conjecture, which bounds masses of charged particles. We propose the Lattice Weak Gravity Conjecture, which further requires the existence of an infinite tower of particles of all possible charges under both abelian and nonabelian gauge groups and directly implies a cutoff for quantum field theory. It holds in a wide variety of string theory examples and has testable consequences for the real world and for pure mathematics. We sketch some implications of these ideas for models of inflation, for the QCD axion (and LIGO), for conformal field theory, and for algebraic geometry.¹

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1 Introduction

Quantum gravity is famously difficult. Its effects are tiny at accessible energies, whereas our theoretical understanding of it relies largely on supersymmetry, notably absent from the real world. Making predictions about our universe, rather than a pristine toy model, is challenging. For this reason, any feature shared by all known consistent quantum gravity theories could be an important clue to their general nature.

We recently identified a candidate: the Lattice Weak Gravity Conjecture (LWGC) [1]. A powerful statement about the nature of quantum gravity, it passes stringent theoretical checks and makes nontrivial predictions for real-world observable physics, properties of as-yet-unformulated theories of quantum gravity, and pure mathematics. In this essay we motivate and provide evidence for the LWGC (the many well-understood examples at least call for a deeper explanation) and explain some of its implications. A virtue of this conjecture is that it points the way toward many tractable theoretical questions whose answers will provide more evidence for the conjecture or shed light on the nature of quantum gravity theories in which it fails.

The Weak Gravity Conjecture (WGC) [2] makes quantitative the qualitative belief that theories of quantum gravity should have no continuous global symmetries [3–6]: as Hawking radiation is unaffected by global symmetry charges, this would lead to an infinite number of stable black holes remnants near the Planck scale, with associated thermodynamic problems. A gauge theory at extremely weak coupling $e \rightarrow 0$ is nearly indistinguishable from a global symmetry, so we expect quantum gravity theories to resist the $e \rightarrow 0$ limit.

In the real world charged black holes will discharge by emitting electrons. This observation was elevated to a postulate by [2]: in the absence of special BPS conditions forbidding decay, all charged black holes should spontaneously discharge. Charged black holes in general relativity have a maximum charge for a given mass. For the decay of maximally-charged black holes to be kinematically possible, a charged particle must exist with:

$$m \leq \sqrt{2}eqM_{\text{Planck}}. \quad (1)$$

A further argument in [2] based on magnetic monopoles suggested the theory has a cutoff length scale $(eM_{\text{Planck}})^{-1}$ and hence breaks down as $e \rightarrow 0$.

Our refined WGC gives a different perspective on the $e \rightarrow 0$ limit:

Lattice Weak Gravity Conjecture. *For any charge \vec{Q} allowed by charge quantization, a particle of charge \vec{Q} exists with charge-to-mass ratio at least as large as that of a semiclassical extremal black hole with $\vec{Q}_{\text{BH}} \propto \vec{Q}$.*

Here \vec{Q} is a multicomponent vector in theories with multiple massless photons, and refers to the Cartan charges when there are non-abelian gauge symmetries. This statement is much stronger than the original WGC as it requires the existence of infinitely many charged particles (though most are effectively black holes). This is not a step we take lightly. We were led to this hypothesis by investigating Kaluza-Klein theories, as shown in Figure 1.

The LWGC demands one particle lighter than eM_{Pl} , another lighter than $2eM_{\text{Pl}}$, another $3eM_{\text{Pl}}$, and so on: an infinite tower of charged states. Quantum field theory always breaks down

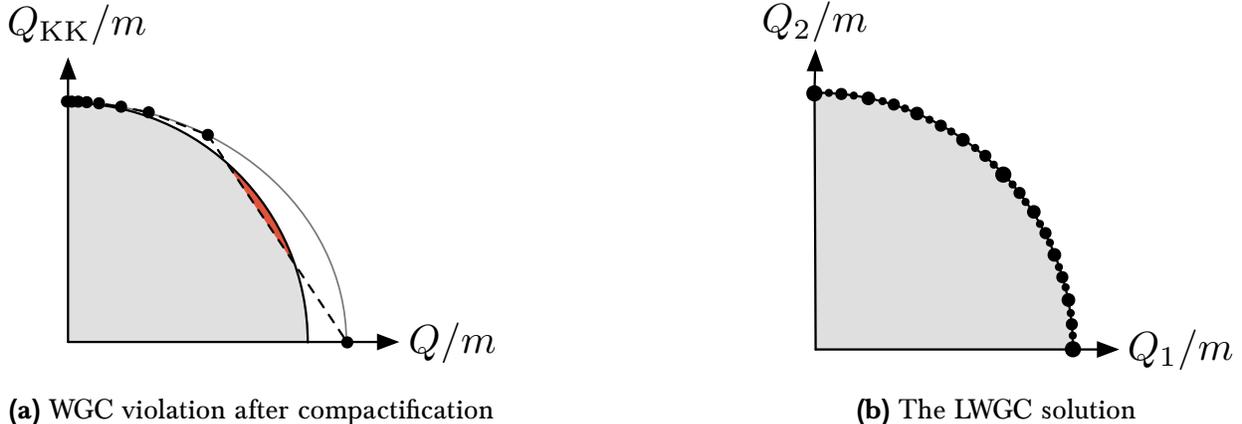


Figure 1: (a) Kaluza-Klein reduction of theories satisfying the WGC can lead to violations in the lower dimensional theory. Sub-extremal black holes with charge-to-mass ratios in the red region are stable. (b) If the higher dimensional photon is itself a Kaluza-Klein photon, then there is an extremal charged particle of every charge, and the WGC is satisfied.

in the presence of such a tower, giving a completely different explanation for the cutoff length scale $(eM_{\text{Pl}})^{-1}$ from that of [2]. With the LWGC, infinitely many particles become massless when $e \rightarrow 0$, implying the theory breaks down completely in the limit of continuous global symmetries, as expected.

We speculate that the LWGC is required for a completely consistent picture of black hole thermodynamics [7] (or holography [8]), but at present the primary evidence for the conjecture comes from the wide variety of well-understood quantum gravity theories where it holds. We have checked it in perturbative heterotic string theory, including toroidal compactifications with Wilson lines; supersymmetric compactifications on \mathbb{Z}_N orbifolds of T^4 or T^6 without Wilson lines; certain examples of Higgs branches; and some T^8 orbifolds lacking one-loop tadpoles. The only examples we know where it appears to fail are nonsupersymmetric theories with instabilities, for which our perturbative calculations are unreliable. The LWGC also predicts that finite size black holes can be slightly superextremal, as verified in some string examples by [9]. Thus we have a vast (and growing) set of examples of quantum gravity theories obeying the LWGC.

2 Applications

The WGC connects to a remarkable array of topics in high energy physics, of which we highlight a few: axion inflation, the QCD axion, AdS/CFT, and Calabi-Yau geometry.

2.1 Axion Inflation

The best-developed application of the WGC is to large-field axion inflation, which predicts detectable primordial gravitational waves. Viewing axions as 0-form analogues of gauge fields,

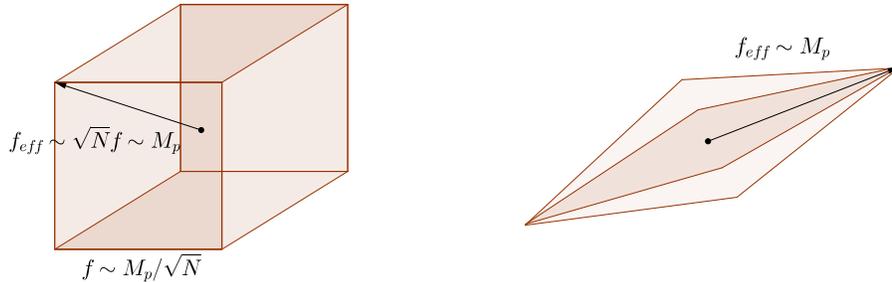


Figure 2: N-flation (left) and decay constant alignment (right) seek to produce a large effective decay constant from multiple sub-Planckian ones. However, the WGC implies $f_{\text{eff}} \lesssim M_{\text{Planck}}$ for the simplest versions of these models.

the WGC generalizes to the existence of an instanton of action S satisfying

$$S \lesssim M_{\text{Planck}}/f. \quad (2)$$

Axions in controlled regimes of string theory obey this bound [10]. String theory requires $S \gtrsim 1$ to maintain perturbative control while successful inflation requires $f \gtrsim 5M_{\text{Planck}}$; the WGC implies these are incompatible.

Models evading this bound posit many axions each traversing a smaller field range during inflation (“N-flation” [11–13]) or at least two axions, with the inflaton taking a long diagonal path within a small field space (“alignment” [14]), as in Figure 2. Recent arguments cast doubt on these models [15–18] using a generalized WGC for theories of multiple gauge fields [19]. For N-flation as well as many alignment models, the total inflaton displacement is bounded to be sub-Planckian.

Large field natural inflation models consistent with the WGC are being actively explored [20–23], but may not exist as consistent quantum gravity theories [24–28]. The WGC’s consequences for axion monodromy [29, 30] and the relaxion [31] are under intense investigation [18, 32, 33].

2.2 QCD Axion

A major particle physics puzzle is the tiny QCD theta angle: $\bar{\theta} \lesssim 10^{-10}$. The most plausible solution is a light pseudoscalar field, the QCD axion, which dynamically relaxes $\bar{\theta}$ [34]. The QCD instanton action is

$$S_{\text{QCD}} = 4 \ln M_*/\Lambda_{\text{QCD}} \approx 160, \quad (3)$$

where M_* is an ultraviolet scale such as M_{GUT} . Using the general bound (2) we predict

$$f_{\text{QCD}} \lesssim 10^{16} \text{ GeV}, \quad (4)$$

with mild logarithmic uncertainties. Experimental observation of a QCD axion with decay constant well above the GUT scale would contradict the WGC.

A common claim is $f_{\text{QCD}} < 10^{12}$ GeV to avoid dark matter overproduction, but alternative cosmologies [35–37] or initial conditions [38, 39] allow larger f_{QCD} . The absence of QCD axions with decay constants above the GUT scale is a nontrivial, falsifiable prediction of the WGC. It can be tested in terrestrial searches for axion dark matter [40–42]. These axions also cause superradiant instabilities in black holes, detectable through astrophysical observations including gravitational wave signals [43–45]. The new window on gravitational physics that LIGO has opened can also test the WGC!

2.3 AdS/CFT

The AdS/CFT correspondence encompasses many quantum gravity theories. The 4d CFT translation of the WGC in AdS₅ is the existence of a charge Q , dimension Δ operator with

$$\frac{\Delta}{\sqrt{12c}} \leq \frac{Q}{\sqrt{b}}, \quad (5)$$

with c the central charge and b the coefficient of the current two-point function. However, the flat-space WGC may be modified due to the curvature of AdS [46] (cf. [47] on AdS₃/CFT₂). We have checked that the naive analog of the LWGC based on (5) holds for $\mathcal{N} = 4$ super-Yang-Mills theory. Further exploration of the LWGC in conformal field theory should be rewarding.

2.4 Geometry

Compactifying M-theory on a Calabi-Yau three-fold gives a 5d supergravity theory with BPS states corresponding to M2-branes wrapping holomorphic curves in the three-fold. These BPS states generate the “effective” cone in the charge lattice, inside which the BPS bound requires $|\vec{Q}| \leq M$ in Planck units. Simultaneously satisfying this bound as well as the LWGC requires a BPS state of $|\vec{Q}| = M$ at every lattice point within the cone. Geometrically, this means every effective 2-cycle class of a compact Calabi-Yau should contain a holomorphic curve representative. This conjecture is similar in spirit to Vafa’s inference of the existence of an exponentially large number of holomorphic curves in 2-cycle classes far out in the homology lattice [48] in order to match microscopic BPS degeneracies with black hole entropy [49]. The conjecture is quite surprising from a mathematical perspective, though it has been verified in the simple case of the quintic three-fold [50, 51] and Sen [52] has proven it for T^6 using U-duality [53].

2.5 The Nature of Quantum Gravity

The LWGC holds in a wide variety of perturbative string vacua and beyond. It encapsulates a central feature of string theory: the existence of a tower of increasingly heavy states of ascending charge. We imagine that a further refinement of the WGC might capture even more of the essence of string theory. For instance, it is tempting to postulate that states of arbitrary *spin* exist at any point in the charge lattice, with masses gradually increasing for higher spins. The challenge is to formulate such conjectures in a precise manner, consistent with all available

theoretical data. Our dream is that by examining many individual quantum gravities provided by string theory, we can abstract away details of specific vacua and gain a clear, testable vision of the essential nature of quantum gravity.

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