Spatial coherence of a strongly interacting Bose gas in the trimerized kagome lattice

Thomas H. Barter,1 Tsz-Him Leung 1, Masayuki Okano 1, Maxwell Block,1 Norman Y. Yao 1,2, and Dan M. Stamper-Kurn 1,2
1Department of Physics, University of California, Berkeley, California 94720, USA
2Materials Sciences Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA

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We produce a trimerized kagome lattice for ultracold atoms using an optical superlattice formed by overlaying triangular lattices generated with two colors of light at a 2:1 wavelength ratio. Adjusting the depth of each lattice tunes the strong intratrimer ($J$) and weak intertrimer ($J'$) tunneling energies, and also the on-site interaction energy $U$. Two different trimerization patterns are distinguished using matter-wave diffraction. We characterize the coherence of a strongly interacting Bose gas in this lattice, observing persistent nearest-neighbor spatial coherence in the large $U/J'$ limit, and that such coherence displays asymmetry between the strongly and the weakly coupled bonds.

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Ultracold atoms in optical lattices provide tunable implementations of condensed-matter models. The lattices realized thus far include plaquette lattices, in which identical few-site plaquettes, with strong intraplacquett coupling, are regularly arrayed and weakly coupled to one another. Both experimentally and theoretically, such plaquette lattices allow for a controlled approach to the complexity of many-body quantum systems. At the limit of weak interplacquett coupling, the few-body states within isolated plaquettes can be precisely determined and finely controlled, enabling, for example, cold-atom demonstrations of superexchange, resonant valence states, quantum magnetism, and anyonic statistics [1–4]. These single-placquett states serve as starting points for describing strongly correlated states that arise when interplacquett tunneling is increased.

In this Rapid Communication, we realize a lattice of triangular plaquettes known as the trimerized (or “breathing”) kagome lattice (TKL). The TKL [Fig. 1(d)] is obtained by choosing one orientation of triangular plaquettes of the kagome lattice to have strong intersite tunneling ($J$) while plaquettes of the other orientation have weak intersite tunneling ($J'$). This lattice has received theoretical interest [5–7] as a route to understanding quantum antiferromagnetism in the kagome lattice. For antiferromagnetically coupled spins, the strongly coupled trimers support microscopic spin frustration. Weak intertrimer coupling then leads to robust spin-liquid ground states on macroscopic scales [5–7]. Solid-state materials with a TKL structure have been synthesized and studied in recent experiments [8,9].

Our focus here is on the behavior of strongly interacting bosons in a plaquette lattice. Specifically, we consider interacting Bose gases of $^{87}$Rb atoms within a TKL with a widely tunable trimerization ratio $J/J'$. In the tight-binding limit, our system is modeled by the Bose-Hubbard Hamiltonian

$$H = -\sum_{\langle p,q \rangle} J_{pq} (a_p^\dagger a_q + H.c.) + \frac{U}{2} \sum_p n_p (n_p - 1),$$

(1)

where $\langle p,q \rangle$ denotes summation over all pairs of neighboring lattice sites $p$ and $q$, $J_{pq} = J(J')$ for strongly (weakly) coupled bonds, and $n_p$ is the number operator for site $p$. Here, $J$ and $J' > 0$. For strong trimerization ($J/J' \gg 1$) and strong interactions $U/J' \gg 1$, the TKL is predicted to support Mott insulating states in which atoms remain coherently delocalized within trimer plaquettes while intertrimer coherence is suppressed by interactions [10–12].

We present two main results. First, we develop an atom optical technique in which the momentum distribution of a superfluid trapped and transiently excited within the TKL reveals the inversion asymmetry of the TKL and distinguishes lattices of opposite trimerization. Second, analyzing the distribution of atoms released from the lattice, we find that lattice trimerization causes the nearest-neighbor coherence to remain strong even in the deep Mott insulating limit ($U/J' \gg 1$). By an interferometric technique, we demonstrate that this spatial coherence resides almost exclusively within the trimer plaquettes.

We form the TKL by overlaying two commensurate triangular lattices, one with twice the spacing of the other [13,14]. Each triangular lattice is formed by the intersection of three focused laser beams at equal angles and lying in a single (horizontal) plane [15]. The short-wavelength (SW) lattice (lattice spacing $a_s = 355$ nm) is formed by in-plane polarized 532-nm-wavelength light, while the long-wavelength (LW) lattice is formed by out-of-plane polarized 1064-nm-wavelength light. The relative position of these two lattices is stabilized interferometrically to better than 2 nm. Note that in our previous work on the regular kagome lattice, both sets of lattice beams were polarized in plane.

The unit cell of the superlattice contains four sites of the SW lattice (labeled in Fig. 1), and one site of the LW lattice. The TKL is obtained when the LW lattice site is centered between three equidistant nearest-neighbor sites of the SW lattice, which now form the trimer of the trimerized lattice. For example, two trimerizations, with opposite spatial inversion
asymmetry, are obtained by centering the LW lattice sites at locations shown in Fig. 1(a).

The LW lattice has three effects on the overall superlattice. First, the energies of the three sites in the trimer are lowered relative to the fourth one roughly by $\Delta V \approx \frac{1}{2} V_{LW}$, where $V_{LW}$ is the depth of the LW lattice potential. When $\Delta V$ is sufficiently high, the fourth site is unoccupied and a kagome structure is realized. Second, the tunneling barrier between sites within a trimer is decreased, while the barrier between trimers is increased, leading to the trimerization of tunneling energies ($J > J'$). Third, the combined lattice “breathes,” with the spacing between trimer sites (a) decreasing, and the nearest-neighbor spacing between trimers (a') increasing.

We visualize the spatial asymmetry of the TKL through the coherent diffraction of a superfluid from the lattice. For this, we prepare nearly pure Bose-Einstein condensates of $5 \times 10^4$ $^{87}$Rb atoms in a hybrid optical and magnetic harmonic trap, with trap frequencies ($\omega_x$, $\omega_y$, $\omega_z$) = $2\pi \times (40, 70, 80)$ Hz, with z being the vertical axis. The atoms are loaded into the superlattice by simultaneously increasing depths of the SW and LW lattices to $V_{SW}/\hbar = 45$ kHz and $V_{LW}/\hbar = 15$ kHz by an exponential ramp [16]. The relative position of the two lattices is adjusted to produce $C_6$ symmetric diffraction patterns of gases released from the lattice. The gas remains only loosely confined along the transverse (vertical) direction.

The momentum space distribution of this superfluid in equilibrium, shown in Fig. 2(a), does not manifest the broken inversion symmetry and reduced rotational symmetry of the lattice, since the superfluid order parameter has a uniform phase. To fully characterize the lattice, we imprint complex phases onto the superfluid wave function through transient dynamics. After allowing the superfluid 20 ms to equilibrate in the TKL, we suddenly extinguish the SW lattice potential and allow the atoms to evolve for a variable time $0 < \tau < 150\ \mu$s in the remaining LW lattice. We then switch off all potentials, allow the atoms to expand freely for 20 ms, and image their spatial distribution by absorption imaging.

The two lattice trimerizations lead to strong but opposite inversion-asymmetric diffraction patterns. We focus on the first-order diffraction peaks, occurring at reciprocal lattice vectors $G_1 = k_2 - k_3$ and its cyclic permutations, where $k_i$ are wave vectors of the LW lattice beams. We observe

FIG. 1. Construction of the optical TKL. (a) A SW triangular lattice of spacing $a_0 = 355$ nm is formed by in-plane polarized 532-nm-wavelength light. Sites of a $2 \times 2$ unit cell are labeled A–D. Two locations of the LW lattice site, corresponding to right and left trimerizations, are shown as solid and open white dots, respectively. (b) A LW triangular lattice of spacing $2a_0$ is formed by out-of-plane polarized 1064-nm-wavelength light. Black dots indicate sites of the SW lattice. (c) Combined superlattice potential for the right TKL. (d) Tight-binding model of the TKL, with strong intrattrimer tunneling $J$ and weak intertrimer tunneling $J'$.
the diffracted populations $P_{\nu}$ differ from those at $-G$, [Fig. 2(b)]. Quantifying the asymmetry by the parameter $A = (\sum (P_{\nu} - P_{-\nu})/\sum (P_{\nu} + P_{-\nu})$, [17], we observe equal and opposite oscillations of $A$ as a function of \( \tau \) [Fig. 2(c)], distinguishing the opposite inversion asymmetry of the two trimerization patterns. The data match noninteracting band theory well for $0 < \tau < 80 \mu s$. We attribute the discrepancy between theory and data at later times ($\tau > 80 \mu s$) to dephasing caused by interactions.

To realize the strongly interacting Bose-Hubbard Hamiltonian, we introduce an additional one-dimensional optical lattice, formed by a retroreflected 1064-nm-wavelength light beam propagating along $z$. This lattice, with a depth $V_{\perp}/h = 50$ kHz, divides the gas into about 40 layers, each with trap frequencies ($\omega_x$, $\omega_y$, $\omega_z$) $= 2\pi \times (61, 61, 22 \times 10^3)$ Hz. As the tunneling time between layers of 400 ms is slower than the timescale of the experiment, the system can be considered as an ensemble of isolated, two-dimensional systems [18].

The superlattice is ramped up as above, but to variable final lattice depths. The ramp is adiabatic with respect to $P$. The diffracted populations provide confinement and maintain a constant Thomas-Fermi momentum distribution, indicating the loss of long-range phase coherence. However, even in the strongly interacting regime, the momentum distribution still shows modulations that indicate the persistence of short-ranged spatial coherence.

We quantify the NN coherence by fitting the observed distribution with the function

$$\frac{n(k)}{N} = |\tilde{\nu}(k)|^2 \left( 1 + \sum_{p<q} (\alpha_{pq} \cos(k \cdot a_{pq}) + \beta_{pq} \sin(k \cdot a_{pq})) \right),$$

Compared with Eq. (3), this expression is simplified by including only one momentum-space periodicity along each lattice direction. This simplification is justified both for weak trimerization, where the lengths $a$ and $a'$ are nearly equal, and also for strong trimerization, where (as we show) the intratrimer coherence dominates over the intertrimer coherence. As such, we identify $\alpha_{AB} \simeq \text{Re}(\zeta_{AB} + \zeta_{BA})$ and $\beta_{AB} \simeq \text{Im}(\zeta_{AB} + \zeta_{BA})$, and similar for other bond directions. Both the Gaussian width of $\tilde{\nu}(k)$ and also the bond lengths $a_{pq}$ are used as fitting parameters and extracted from the images.

![FIG. 3. Time-of-flight images of strongly interacting atoms released from either the (a) TKL or (c) triangular lattice. As $U/J'$ is increased at constant $U/J$, sharp diffraction peaks are lost, leaving a broad sixfold symmetric modulation. This modulation persists at large $U/J'$ in the TKL, but disappears in the triangular lattice. The color scale is the same as that in Fig. 2. (b) Extracted NN coherence $\alpha$ for the trimerized kagome (blue, green circles) and triangular lattice (triangles). Data points are the average of 3–9 measurements. Error bars are standard errors of mean.](Image)
lattice. For this, we prepare the gas as before, with the exception that the LW lattice light is left off.

The fitted NN coherences demonstrate the influence of lattice trimerization. At low \( U/J' \), in the superfluid regime, the NN coherences of the triangular and TKL are similar. At large \( U/J' \), there is a stark difference. In the triangular lattice, the NN coherence tends to zero in the Mott insulating limit, scaling as \( \alpha \propto (U/J')^{-0.75(9)} \), which is roughly consistent with a perturbative treatment of a Mott insulator with uniform tunneling energies [15–21]. In contrast, for the TKL, NN coherence remains large due to persistent tunneling within trimers. We also observe that \( \alpha \) is smaller for larger \( U/J \), showing the effect of interactions to suppress coherence in a few-site system.

The simultaneous lack of long-range coherence and persistence of nearest-neighbor coherence implies that spatial coherence in the TKL is spatially asymmetric, with large differences between \( \langle b_{i}^\dagger b_{j} \rangle \) and \( \langle b_{i}^\dagger b_{j} \rangle \). We confirm this fact by an interferometric measurement, in which we imprint a site-specific phase on the spatial coherence [24]. For this, we turn off one beam of the LW lattice for a brief time \( \tau \), raising the energy of one site in each trimer (A) above the energy of the other two sites (B and C) by \( \Delta V \approx 0.2 V_{W} \). This energy offset causes the coherence functions to become complex, evolving as \( \zeta_{AB}(\tau) = e^{i\phi} \zeta_{AB}(0) \) and \( \zeta'_{AB}(\tau) = e^{-i\phi} \zeta'_{AB}(0) \), and similar for \( \zeta_{AC} \) and \( \zeta'_{AC} \), with \( \phi = \Delta V \tau / \hbar \).

This phase imprint has a pronounced effect on the momentum distribution (Fig. 4). In the limit that NN coherence remains only on the intratrimer bonds, we expect \( \alpha_{AB} \propto \cos(\phi) \) and \( \beta_{AB} \propto \sin(\phi) \) to oscillate out of phase and with equal amplitude with \( \tau \). Fitting the observed momentum distribution using Eq. (4) while allowing for nonzero \( \beta_{pq} \) and time-varying spacings \( \alpha_{pq} \), we observe such equal amplitude oscillations.

For the data of Fig. 3, we set \( \beta_{pq} = 0 \) since the coherence in this case is expected to be real valued. A single value for the NN coherence function \( \alpha \) is taken as the average of \( \alpha_{pq} \) along the three bond directions [23].

Our measurements on the TKL are benchmarked by similar measurements performed on atoms in the primitive triangular lattice. For this, we prepare the gas as before, with the exception that the LW lattice light is left off.

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for the case of strong interactions and strong trimerization, demonstrating that, for these settings, NN coherence resides nearly exclusively on the intratrimer bonds. In the case of weaker trimerization and closer to the superfluid regime, we still observe oscillations in $\beta_{AC}$, but with diminished amplitude with respect to those in $\alpha_{AC}$. This observation demonstrates that, while NN coherences in this case are still stronger on the intratrimer bonds, there exists a discernible coherence also on the intertrimer bonds.

The temporal oscillations of $\alpha(\tau)$ [$\beta(\tau)$] are not purely cosinusoidal (sinusoidal), and appear to decay in time. To model this behavior, we study the dynamics of a three-site Bose-Hubbard model. The ground state is calculated and projected onto the eigenstates of a new Hamiltonian $H_{\text{eff}}$ that accounts for an energy offset $\Delta V_A$ on site A. We do not account for changes to $U$ and $V$ in the new potential. We also neglect the transport of atoms into nearby plaquettes or onto the now-accessible D sites of the lattice, assuming that such transport affects the experimental system only over longer timescales than probed by our measurements.

As shown in Fig. 5, we find good agreement between the measured coherences $\alpha_{AB}$ and $\beta_{AB}$ and those calculated from this model. The decay of the oscillations in $\alpha_{AB}$ and $\beta_{AB}$ is explained as a beating between several frequencies in the few-body spectrum of $H_{\text{eff}}$. Note that we have scaled the theoretically determined coherence by a factor of 2/3 to obtain a good agreement, presumably to account for effects of nonzero temperature.

Our present work is performed at high filling, reaching $\nu \approx 8/3$ atoms per site (8 atoms per trimer) at the center of the gas. Reaching filling factors below $\nu = 1$ would allow for studies of the predicted fractional (per site) Mott insulating state [11]. Future experiments may identify few-body eigenstates within single trimers by precise spectroscopy. These eigenstates include circulating states, to which atoms can be driven to realize models for orbital magnetism [25]. Alternately, two-state fermions within the TKL at half filling can simulate the spin-1/2 Heisenberg antiferromagnet, which is expected to have a spin-liquid ground state [25–27].

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[16] The SW and LW lattices are initially turned on at $V_{SW}/h = 2$ kHz and $V_{LW}/h = 1$ kHz respectively to initiate position stabilization. After 20 ms, the lattice depths are increased exponentially in time, with a time constant of $\tau = 35$ ms for the SW lattice, while that for the LW lattice varies between 32 and 58 ms depending on the final depth such that the two ramps always finish at the same time.
[22] Values of $U$, $J$, and $J'$ for each setting of $V_{SW}$, $V_{LW}$, and $V_{\perp}$ were calculated following Ref. [28].
[23] Close to the superfluid regime, finer modulations in momentum space, indicating coherence beyond the nearest neighbor, are also seen. Through numerical simulation, we verify that these additional modulations do not affect the fits by which the NN coherence is extracted.