A Novel Based on the Life of Luca Pacioli

The
Divine
Proportions
Of
Luca Pacioli

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Dedication

Although I’ve spent much time as a friar contemplating the after-life, it seems my own death has crept up on me. I am no longer capable of writing myself. I can no longer put pen to paper. I can no longer produce the quick flick of a pen and the sound it makes as it scrapes the page. I miss feeling fully in charge of the words about to spring forth from my quill. I miss the moments where I feel completely lost, milling about in a dark valley, only to suddenly hear a siren atop a hill beckoning me to the light.

But it is not to be. I am weak and bedridden. Now, I can only speak the words I wish to write. Luckily, my good fortune has given me a young friar on his novitiate to transcribe my words, the very same you are reading now. Though luck doesn’t have too much to play into the current situation since I am the head of the monastery where he and I reside. I gave the charge for him to sit at my bedside myself, but I do consider it lucky to be in the position to make such a charge.

As with most of my books, I am dedicating this one to you, my students. I wish I had taken the time during Alberti or Piero’s last days, as I am now making this young man do, and sat down to receive their final wisdom. Hopefully, when we are done, there will still be enough time to take out all the stuffy language
I’m sure to employ and make it much more enjoyable to read. I have spent the majority of my life writing books for you. Books on mathematics, bookkeeping, the divine proportion, and how to apply these fields to the real world. You will probably find most of them to be better written than this one will be. But in all of them I never explained what motivated me to write so many tomes. May this book show you my inspiration so that when you surpass me, and hopefully soon, I may still be able to inspire others, guiding them to surpass you in turn.

—Luca Pacioli
Chapter One

THE BATTLE OF THE STONES

All of my earliest memories are of trying to sneak out of the house. In all but one, my mother catches me red-handed as I endeavor to slip out the back door or shimmy through a window. I always had one purpose in mind: I wanted to see the Battle of the Stones. In this desire I was not alone in my household. My father went to the game regularly, as did my brother, Piero. This is not the same Piero I mentioned in the dedication, but a lovely man nonetheless. My brother Piero was older than me and already had two boys around my age. And he had just started allowing them to come to the game with him. I was determined that I must be allowed to follow suit. Unfortunately, my mother had another plan in mind. She forbade me from going.

This made me angry, so angry that I could shout. And I often did. I find it cathartic now, looking back on how much the devil was in me in those days, and how much I’ve purged him from my life since then.
I would plead my case to her, however flimsy it must have been, but she remained steadfast. She never replied to any of my protestations. She would simply say, “Miracles can be undone.” Her stoicism made me so angry. Many years later I learned she had a series of miscarriages between Piero’s birth and mine, but that had no bearing on my behavior at the time.

When I was five or six, I came up with a brilliant scheme. I would do all of my daily chores early in the morning, after the rooster but before the sun. The floors would be spotless. The dishes shiny, practically new. She would surely have to let me go under those conditions, no? But alas, the answer was still no. I tried this strategy many times, each time hoping I would get into her good graces and gain her permission, to no avail.

One morning I finished my chores much sooner than usual, even before the rooster. As my mother had not roused yet I had no one to plead with. In that hour, I saw my escape. On game days I would normally be there when she awoke, ready to pester her for her permission. But this morning, I made sure that she found me still in bed, or rather back in bed, having convinced her that I must have fallen back to sleep after doing my chores so early. I lay still with my eyes shut. Although I was quiet, my mind was not. My thoughts raced: What do people do when they’re asleep? Should I keep breathing or hold my breath? My father makes loud grating noises like a pig that has found a truffle, should I?

I can only imagine my mother must have been relieved to find me sleeping, not having to endure my onslaught that day. She didn’t stay long to ponder my con, though. As soon as she left my bedroom I bolted upright and stealthily crept out of the house unmolested.

I had done it.
My feet flew fast on the cobblestones through town. I didn’t chance looking back in case my mother was closing in. I ran through the Piazza San Francesco, weaving through trees all the way past the Cathedral as the sun started peeking over the horizon. I knew the game was played just beyond the city gate.

The stone archway amplified the roar of the crowd as I passed through. It was like they were cheering me on, welcoming me to the game.

Nearly breathless, I poked my head between the spectators on the sidelines. I couldn’t wait to see it. My eager eyes peered onto the field, but all I could see was grass. Was I too late? Was it over? In my panic, I realized that although I had a deep desire to go to the game, I had no idea what it actually entailed. Could this be the game? Standing in a field and yelling?

But no, everyone was cheering, their gazes fixed on the far end of the field. What were they looking at? I could make out some fuzzy shapes darting around, but not much more. Could they see that far? It was hard to imagine they could. I had no concept that my sight was limited. And my ears did me no good either. Any noise the shapes were making was muffled by the screaming men around me.

Suddenly, a loud crack rose above the din, followed by a mix of exhilarated and disgusted outbursts from the crowd. Then the shapes became clearer, turning into men. They raced toward each other. Some carried shields. Some breastplates and others deerskin stockings. One resplendent gentleman wore a helmet in the shape of a sparrow hawk’s head. Although the men all wore all kinds of different garb, they each carried the same accessory: a rock.

Stones littered the air. Men were falling fast. The man with the sparrow hawk’s helmet ran out in front of the pack, raised his
stone to the heavens, and was unceremoniously smashed in the face. Another man’s stone had halted his own. I think the man fell to the ground, but all I remember seeing is his blood flying through the air. Some of it hit an old man near me in the face. To my surprise, the old man wiped the red stain from his brow, smiled and then ran out onto the field. He grabbed a fallen man’s shield and smacked another man with it. He was not disturbed by the blood that struck him, rather it seemed to excite something disturbed within him.

Some of the man’s blood landed on my shirt too, but it did not propel me toward bloodlust. Rather, it spurred in me a deep dread. If my mother wasn’t going to be mad at me for sneaking out to the game, she surely would be now.

When I looked up, there was a big brawl in front of me, a free-for-all of violence and gore. The two sides hurled stones at one another with increasing ferocity. Amongst all this chaos, I noticed the lithe young man was lying on the ground, motionless. Indeed, he had fallen. Although I had never seen a dead body before, I knew this was my first.

Shouts of “Halt!” and “Stop!” arose from the crowd. A couple of men went to the young man, checked to see if he was breathing, and then dragged his body off the field.

His corpse had only just been removed when the two sides commenced bludgeoning each other. The game had begun again as quickly as it had ceased, with scant recognition of the life lost.

I turned to a woman beside me, “That’s not the end of it?”

“That’s just the first part, dear.”

I had seen enough.

I ran.

Tears filled my eyes. I was used to not seeing far, but the tears made it hard to see anything around me. I stumbled through the
stone archway that seemed to welcome me to the game. Now it
ricocheted my sobs back at me, taunting me.

I would find out later that the point of the Battle of the
Stones was to hold the area in the middle of the field long enough
to gain undisputed possession of it. Six years before my birth, the
Battle of Anghiari between Florence and Milan gave Florence
undisputed control over Sansepulcro, my village. I can’t imagine
it was much of a battle. Who would want to fight for control of
my remote little rock in the middle of nowhere? Perhaps for that
reason, Florence and Milan hadn’t fought very vigorously for it.
Only one man died in the Battle of Anghiari and that man fell
off his horse as his regiment got into formation.

The Battle of the Stones was different. My neighbors rel-
ished murdering each other. There was no control at stake
beyond the temporary occupation of a patch of grass. The Battle
of the Stones was brutal and pointless. How could we do this to
one another?

I cried thinking of the man in the sparrow hawk’s helmet.
I wanted to run back to my mother’s arms, but I couldn’t go
home, at least not yet. My mother would still be angry at me this
early after my treachery and she might stone me if I came back
before her anger abated.

I didn’t want people on the street to see me crying, though.
I needed to find a place to hide.

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Transcriber’s note: Fra. Luca Pacioli was adamant that I finish each chapter with a num-
ber, in order, from the Fibonacci sequence (0, 1, 1, 2, 3, 5, 8, 13, 21, 34, etc). I have
done so, in that mode.
Chapter Two

THE BEANS

My cheeks were still wet as I wandered into the Cathedral. Brother Angelo, one of the younger priests, greeted me cheerfully.

“Luca!”

I’m sure he saw the blood on my shirt and the shiny patches of tears on my cheeks. His tone shifted, but he tactfully sidestepped confronting me about it directly.

“Where’s your mother?” he asked.

“I’m sure she’ll be along soon.”

My plan was to wait for my mother at the church. She was a pious woman who came to church every day after her morning chores. I usually came with her and I was hoping to convince her I came early to sneak in a few prayers before the main service. My plan was to charm her with feigned piety and I was pretty sure it was going to work. I didn’t have an excuse for the blood on my shirt yet, but there was still time.
Brother Angelo smiled weakly. “Well. It’s good to have as many souls as we can during Stones season.”

I looked around the nave. Only a handful of people occupied the pews. Although I wanted to distance myself from the proceeding across town, the relative lack of attendance infuriated me.

“Why does everyone watch them kill each other like that?”

This seemed to strike a nerve in Brother Angelo. He corrected me, “Am I there? Are all these good people there?”

“No.”

“Well. That’s not everyone, is it?”

I should have let his words be the last of it, but that was not in my youthful nature.

“Everyone else in town was there!” I screamed.

The handful of parishioners snapped their heads around and stared at me.

Brother Angelo should have put my insolence on display. He should have slapped me across the face and kicked me out of His church. But he bent down and carefully took my shoulder in his hand.

“I know the Battle of the Stones is popular, but our town is full of good people.”

But a majority of them had just seen a man murdered and then gleefully cheered the proceeding on, ravenous for the next victim! Sansepulcro was a bad place full of bad people and I needed to prove this to Brother Angelo. I just needed to show him that the town was hungrier for blood than Scripture. To prove it, undeniably in my mind, I only needed two numbers: One, the population of Sansepulcro and two, the number in attendance at that barbaric enterprise.
Brother Angelo wouldn’t know the latter number so I told myself I would have to swallow hard, go back and count them myself. I couldn’t do much with numbers in those days, but I could count and that savage spectacle was probably still underway.

I was sure Brother Angelo would have the first number.
“How many people live in Sansepulcro?”
“I don’t know.”
“What do you mean you don’t know? Who does?”

Again, Brother Angelo demonstrated his supreme grace. “No one does, but we do keep a record of the baptisms we perform.”
“How many is that?”
“Let me show you.”

He led me over to the baptismal font where he produced a large, ornate wooden box. I was curious. Why do they keep this number in a box? Brother Angelo tilted the box toward me. Whatever I was expecting, nothing prepared me for what was inside.

“Beans?”
“For every baby baptized we add one to it. Black beans for boys and white beans for girls.”
“And how many are there?”
“You can count them if you like.”

The enormity of the undertaking overwhelmed me. The box was almost half as large as I was. Regardless, I knew that I had to convince Brother Angelo of Sansepulcro’s irredeemability. I started counting furiously. Beans flew through my fingers. I decided to place the beans in piles of one hundred. I was making quick work of the task, but then the image of the dead man in the sparrow hawk’s helmet flashed before my eyes. For some
reason, this tempered the intensity with which I was counting and made me wonder something.

“If someone dies. Do you take their bean away?”

“No. This is a record of life, not death,” Brother Angelo answered.

A record of life! But this was no record of the souls alive in Sansepulcro. It was no record of the population. It was a record of acts. And not even acts that could be translated into the actual information I was seeking. The Black Death had killed off so many of my fellow citizens, perhaps more than the Stones, making this endeavor nowhere near remotely useful. And that wasn’t even taking into account those that had died of natural causes.

So, while Brother Angelo’s statement was poignant in its rudimentary simplicity, it meant that I wouldn’t be able to prove my point. Since they don’t take a bean away when someone died, there was no way to get an accurate count of the city’s population. It probably wouldn’t have mattered anyway. I wouldn’t have been able to get through all the beans before my mother arrived. She was going to kill me for sneaking out, further altering the final count.

I slunk into a pew to await my fate. It was sure to arrive soon along with my mother. My mind raced, anxious to ascertain all the new chores she was sure to conjure as punishment.

But mostly, I thought about the beans.
I wondered if I would learn a better way than the beans in school. Even if I had the two numbers I wanted, I wouldn’t have been able to do much with them. The monastery, the same one we’re in right now, provided schooling to all the local boys free of charge. This was good because if they had charged, my family wouldn’t have been able to afford it. I had always been aware of the two sides of the “poor, but respectable” label my family had. More often than not it meant that people thought less of you, but didn’t want to say so to your face. My family’s poverty felt like it was emblazoned on the small of my back, something I could never scratch or rub off.

I couldn’t have been more excited for my first day. Even the rich boys went here. We were going to learn arithmetic, geometry, bookkeeping, reading, grammar, and theology. Brother Carlo taught arithmetic. He was a rotund man who looked like drawings I had seen of the Ottoman conqueror Sultan Mehmet II, the one that had just taken Constantinople.
I wanted to learn how to use an abacus. I always went shopping with my mother so I could see the merchants use them to make calculations. It was a dazzling trick, moving the beads back and forth on the rods. It looked like magic. Everyone referred to my school as an “abacco” school so I was sure it was going to be the first thing we would learn, but my eagerness got the best of me. My hand was high in the air in Brother Carlo’s classroom.

I didn’t even let him begin lecturing before I asked, “When are we going to learn the abacus?”

“We won’t. It’s an outdated tool.”

“But isn’t this an abacco school?”

“Yes. But abacco doesn’t mean abacus. It’s the modo Arabico, the Arabic style. I will teach you a method of calculation that makes the abacus redundant.”

I could feel the rich boys looking at me. Surely, they had already known this. My blood was boiling, ready to explode like a volcano. I was so embarrassed, but Brother Carlo’s next words plugged up any potential eruption. His words were just so fascinating.

He explained that abacco allows for the addition, subtraction, multiplication, and division of Hindu-Arabic numbers (1, 2, 3 . . .) instead of Roman ones (I, II, III, IV . . .). You use a pen and paper instead of an abacus. This allows you to see and keep track of the work you’ve done, whereas with an abacus there’s no trace of the calculation made.

He explained, “God shows us His work. A tree does not just appear fully formed, strong roots, and wide branches. It grows from a seed and sapling. And we will show Him our work, too.”

This made complete sense to me, but I tended not to follow the dictum. Brother Carlo routinely marked me down for not
showing my work. We hadn’t progressed far in our education at the time and were still doing simple, double-digit addition and subtraction. I didn’t need to show my work. I didn’t need a pen and paper to make the calculation. I could do it in my head.

He had us play a game. He would list a problem and we, the pupils, would calculate it. The first person with the correct answer won. Now, this was a game I could win. One day, he became particularly irate with me. The rules were that you had to show the work of the calculation before giving the answer, but that wasn’t a rule I tended to follow, often merely making a show of putting pen to paper. On this day though, I made no such show, never even lowering my head or blotting my page with ink. I merely proclaimed the answer a second or two after he read the problem. And I was very smug about it.

So, Brother Carlo, in his wisdom, announced that we would have only one more problem that day. He said that the person who solved it by the end of class would never have to come back again. Clearly, they wouldn’t have anything to learn.

This proposition excited me. Not because I wanted to be let out of arithmetic. I rather liked the class. It was the only place where I felt truly confident and it gave me an opportunity to demonstrate my burgeoning capability. I was, though, a bit too confident for how little I had actually advanced in my studies, at least in relation to my current position. Nevertheless, I told myself that I’d be gracious in victory. I would not embarrass Brother Carlo by not attending his lectures. I would remain a fixture in the class and continue to hold court there amongst the other students.

Then Brother Carlo recited the problem: 225,851,433,717 divided by 14,503.

I was fortunate that he read it aloud twice since my hand
wasn’t even on my pen when he began. I had to scramble and barely got the numbers down the second time around. But that didn’t do me much good. We had not yet learned division. I was aware of the basic concept, but these numbers were far beyond my grasp.

I didn’t even know where to start. I didn’t know how to divide. I only knew addition and subtraction. How could I be expected to solve this problem? But then I realized: I know addition and subtraction. And in a way, division is kind of like addition, only backwards and multiplied. While I didn’t yet know how to actually go about dividing numbers, I thought maybe . . . if I added up 14,503 enough times I would get to the answer that way. Since it was my only course of action, I launched into it with zeal.

But I was so out of practice showing my work for addition that I struggled and sweated my way through the first few rounds. I could have sworn there was more sweat on that page than ink. And time was running out. The numbers simply weren’t adding up fast enough to reach 225,851,433,717.

And then class was over. I looked around and noticed that I was the only student who had seriously attempted to solve it. Or perhaps they had known all along that I was the sole intended recipient of this rebuke. Either way, my fellow students filed out of the room expressing their disappointment at having to attend another arithmetic class.

Brother Carlo came over to my desk. He could see my disappointment, both on my face and on my page. I had drawn large, dramatic Xs over the work I had done. I expected his scorn, but was blessed with his scrutiny.

“Trying to solve the problem using addition. Hm. Impressive.”
For a moment I thought I would be praised, but I was soon corrected.

“It wouldn’t have worked, but it might have under the right conditions. But let’s say it did. Could you have done this in your head?”

“No.”

“Even if you could, if you made a mistake in your calculation, would you know how to fix it?”

“No.”

“You wouldn’t know where the problem was so you’d have to start all over again. You have to put in the work to achieve the result. Sometimes we face big problems and the only way we can get around them is to figure out where we went wrong and change course.”

I knew he was talking about the problem, but also about my behavior. I was humiliated, but it had the desired result. I always showed my work from then on. My ego took a bruising that day, but it was soon massaged when I kept winning the calculation game.

Brother Carlo was a great teacher. But the friars taught no subject better than theology. I loved hearing about St. Francis, who had mentored a young Brother Angelo into the Order in Sansepolcro. This was, of course, a different Brother Angelo than the one who showed me the beans since it was over two hundred years ago. Or else Brother Angelo had decided not to be bound by the ravages of time.

St. Francis taught Brother Angelo a lesson about underestimating people. Three out-of-luck brigands sought bread at the monastery and Brother Angelo turned them away. But when St. Francis offered them bread, the men used the opportunity to redeem themselves and convert to Christianity.
I was about to be confirmed in the Church myself and so this story was a revelation. I wanted to change the course of my life, too, and shed my “poor, but respectable” label, the one that others automatically placed upon me without giving me a chance to redeem myself. The Church and my education could help me do that. I just had to put in the work.
Not far into my education I would be able to divide those numbers. We learned division, multiplication, algebra and geometry. Each was a new language and my tongue was wet to learn them. I enjoyed school greatly. I could solve problems I’d never even heard of before.

School was my solace when the Black Death came to Sansepulcro. Its call was so deafening my parents had no choice but to answer it. As soon as my mother ran a fever she bid me stay away from them. But it was my father who died first. Mother held on for a couple days and then passed away. I was twelve years old. I don’t know why, but I didn’t inform my older brother Piero or the local authorities of their passing for a few days. I spent almost a week roaming about the house. I stuck to my chore schedule. The floors had never been shinier. The dishes were almost scraped clean of their luster. I didn’t want anyone to know. I thought it might pass, like a bad dream. But it did not. It was a nightmare that still rouses me from my sleep to this
day, despite the fact that it happened years ago. It was a painful occasion that I’d prefer not to speak about in further detail.

Piero tried to take my mind off the matters at hand by focusing my attention on my upcoming apprenticeship. While I didn’t have to pay the friars anything for my education, it was common for an apprentice’s family to pay a master for their training. But since my parents left neither Piero nor I with an inheritance, I could not afford to do so.

Luckily, Piero was able to arrange an apprenticeship with the businessman Folco de Belfolci. I would live with Folco, too. Since I couldn’t pay him anything up front, I would not receive any wages later on in my training, as I would have customarily. It was a necessary arrangement, but one that also taught me perhaps the most important lesson of my apprenticeship: “Free” is not all that it seems.

Folco sold fabric, buttons, and thread. On my first visit it was apparent that both he and his shop had seen better times. Although the shop was not large, maybe the size of my family’s old living room, it was sparsely populated with product. There were only a handful of reams lining the walls and they looked as decrepit as their shop owner. The musty smell of mold and death seemed to permeate everything, including Folco’s breath. I still remember that first morning when Folco showed me how he kept his books. He used a daybook ledger, recording everything that happened in the shop. It was a simple narrative in a single column. This happened, then this happened, etc.

I was surprised to see that he still used Roman numerals. “Why?” I asked him.

“I don’t trust those Arab numbers. The Turks ruined Constantinople. They destroyed Christendom and they took away
my fabrics. You see how many reams I have left? No alum means no fabric, means no business.”

Later, I would learn that alum was a mordant for dyes. Alum makes it so that dyes don’t run when the fabric is washed. The only alum mines were near Constantinople so when the city fell, the fabric trade in Italy was ruined. And Folco along with it.

Folco went on. “Those Arabs are shifty and so are their numbers.”

Folco grabbed the daybook ledger. He wrote the numeral six in it and then added a small line, turning the six into an eight. He was proud of his example. Even though it was my first day, I couldn’t help but interject.

“You can easily notice that the number’s been changed since it’s not two circles, one on top of the other. The proportion is off.”

Folco began grumbling. It was the sort of low, indecipherable grumble that I would come to ignore. He grabbed the daybook ledger and changed a numeral one to a numeral four. I could have given it to him. It was a great example. But I didn’t.

“You can manipulate roman ones, too.”

I changed a III to an VIII in the daybook ledger, and then a X to an IX.

“And that’s only the beginning,” I said in the smug tone I still hadn’t learned to get rid of.

At this, Folco became frustrated and changed the subject. “We need to count the buttons! Here, you can use this.”

He handed me an abacus.

“I’m sorry. I don’t know how to use an abacus.”

“What do they teach you at that school? First they forgot to teach you manners and now math.”

“I promise I can add up any sum you can muster.”
Folco grumbled. He gave me a list. He had already counted the numbers of the various types of buttons he had. He wanted to know how many buttons total were in his shop. It was a relatively straightforward problem. I did it in my head quickly, but put the calculation in the ledger to show my work since I had long been in the habit of doing so.

I displayed my work to Folco.

“This way you can see all the numbers and how they add up. And, if you make a mistake it’s easier to see where you went wrong. With an abacus you wouldn’t know what happened.”

“You’re my apprentice! Your job is to learn the way I do things! Watch closely.”

Folco seized his abacus and grumbled his way through the calculation. He tried to pause at certain intervals to explain why he was doing what he was doing, but he used no words to explain his actions, his face simultaneously galled at having to show me such a simple procedure and fretful that he might lose his place in the calculation.

It may, therefore, come as no surprise that Folco’s calculation was wrong. But as is the case with many such people, he didn’t know that. He proudly announced the difference between our results as proof that I had no idea what I was doing and made me update the ledger to his erroneous outcome. Later, I updated the ledger to the correct number when he wasn’t looking. I was nothing if not a dedicated apprentice. I didn’t want his failings to lead to that of the shop and ultimately my own.

In a way, I remember that day fondly. I finally got to learn how to use an abacus. Despite Folco’s inept teaching style, I got the gist of how to move the beads back and forth on the rods, breaking the rows into multiples of 10. It was elegant in its own, outmoded way. I’d go into more detail about how to use an aba-
cus, but this once revolutionary tool has no place in mathematics going forward. It is, and should remain, a relic.

Even at that young age, the irony of counting buttons with an abacus was not lost on me. If I squinted, the shiny buttons looked just like the beads of the abacus themselves. I’d have to count the buttons, one at a time, and then use the abacus to count them again, moving one bead at a time, more than doubling the amount of work needed to reach the same result since adding up the Hindu-Arabic numerals would have been so much easier.

But Folco was a stubborn man. He wanted his business run his way, the only way he knew how.

Not long into my apprenticeship I asked him, “How much does an apprentice normally pay a master?”

“100 florins.”

But I didn’t see that amount anywhere in the daybook.

“I’m sorry. Where’s that in the ledger?”

Folco was incredulous. “Why would I put that in the ledger?”

“Since I would have received wages later in my apprenticeship, how will you know when I’ve worked enough to pay you back?”

Folco grumbled loudly and walked away.

I would like to tell you my relationship with Folco got better from there, but many things I would like to tell you about this world simply aren’t true.

I wondered if there was a way to keep track of this type of thing over time. I had heard stories about big merchants in Venice trading goods all over the world. Surely, they had systems to keep track of wages and things like this, making sure that all debts were paid.

Before he died, my godfather had made a name for himself
dredging the canals there. I hoped to make a name for myself one day, too. But I was still only twelve, much too young to travel to a city like Venice on my own with no prospects. If only my godfather were still alive, perhaps I could have gone and lived with him, but I was stuck in Folco’s shop for the foreseeable future.