Measuring Unfair Inequality: Reconciling Equality of Opportunity and Freedom from Poverty

Paul Hufe*, Ravi Kanbur† & Andreas Peichl
(*University of Munich; †Cornell University)

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Let $Y^e = \{y_1^e, y_2^e, \ldots, y_n^e\}$ be the empirical distribution of income.

Let the mean of the distribution be $\mu$.

Consider standard measures of inequality:

$$G = \frac{1}{n} \left( n + 1 - 2 \frac{\sum_{i=1}^{n}(n+1-i) y_i^e}{\sum_{i=1}^{n} y_i^e} \right)$$

$$A(\epsilon) = \begin{cases} 
1, & \epsilon = 0 \\
1 - \frac{1}{\mu} \left( \prod_{i=1}^{n} y_i^e \right)^{1/n}, & \epsilon = 1 \\
1 - \frac{1}{\mu} \left( \frac{1}{n} \sum_{i=1}^{n} (y_i^e)^{1-\epsilon} \right)^{1/(1-\epsilon)}, & \text{otherwise.} 
\end{cases}$$

$$\text{MLD} = \frac{1}{n} \sum_{i=1}^{n} \ln \frac{\mu}{y_i^e}$$
Each of these measures can in turn be seen as a divergence metric between the vector of observed incomes

\[ Y^e = \{y_1^e, y_2^e, ..., y_n^e\}, \quad (1) \]

and the vector where each element is \( \mu \)

\[ M = \{\mu, \mu, ..., \mu\} \quad (2) \]

In other words, the vector where total income is distributed equally.
**Figure:** Lorenz-Curve Representation

![Lorenz-Curve Graph](image-url)
In the conventional inequality measurement literature, all the action resides in the properties of this divergence metric.

Desirable properties for this metric include:

- Scale Independence
- Principle of Populations
- Pigou-Dalton Principle of Transfers
An additional property often used is sub-group decomposability. This property, with a few other assumptions leads to the **Generalized Entropy class** of inequality measures:

\[
GE(\alpha) = \begin{cases} 
\frac{1}{n} \sum_{i=1}^{n} \ln \left( \frac{\mu}{y_i^e} \right), & \alpha = 0 \\
\frac{1}{n} \sum_{i=1}^{n} \left( \frac{y_i^e}{\mu} \right) \ln \left( \frac{y_i^e}{\mu} \right), & \alpha = 1 \\
\frac{1}{n} \frac{1}{\alpha(\alpha-1)} \sum_{i=1}^{n} \left[ \left( \frac{y_i^e}{\mu} \right)^{\alpha} - 1 \right], & \text{otherwise.}
\end{cases}
\]

Note that with \( \alpha = 0 \) we have the MLD measure.
Note that this is all to do with measuring inequality. It is a pure distributional question.

Of course if we move to redistribute then there will be incentive effects and the mean will be affected. This leads to the large literature on optimum taxation, going back to Mirrlees (1971).

This will NOT be the focus of this talk.
All of the above is to do with the divergence metric between the observed distribution $Y^e$ and the reference distribution $M$, the perfect equality distribution.

But a resurgent, perhaps insurgent, part of the literature argues that what is at issue is not so much the metric of divergence of the actual from the reference vector, but the reference distribution itself.

Why should we take equality of outcomes as the reference, or the norm, or, in effect, the ideal? Surely the process whereby the outcomes came to be, matters as well?
The general problem is then posed as the divergence between the observed distribution $Y^e$ and a reference or a norm distribution

$$Y^r = \{y_1^r, y_2^r, \ldots, y_n^r\}$$ (4)

$Y^r$ has the same mean as the observed distribution but is not necessarily $M$, the perfect equality distribution.
Our Contribution

We will focus on two well-established principles of distributive justice (Konow, 2003; Konow and Schwettmann, 2016), namely...

- **Equality of Opportunity** (EOp)
- **Freedom from Poverty** (FfP)

... to derive a new empirical measure for unfair inequalities.
Outline

1 Normative Principles

2 Norm-based Inequality Measurement

3 Empirical Application

4 Summary
Outline

1 Normative Principles
2 Norm-based Inequality Measurement
3 Empirical Application
4 Summary
Equality of Opportunity
The primary question concerns the construction of the norm vector. This is where the insurgency in the inequality measurement literature has come in recent years. The insurgency’s premise is that what matters normatively is not equality of outcome, but equality of opportunity. This insurgency has deep roots in an older and esteemed philosophical literature.
Metaphors associated with this view are “leveling the playing field” and “starting gate equality”.

Main philosophical accounts:

In general, **Equality of Opportunity** pre-supposes that all determinants of individual outcomes are the result of two sets of factors:

1. **Circumstances**, $C \in \Omega$: Factors beyond individual control.  
   $\rightarrow$ Unfair

2. **Efforts**, $E \in \Theta$: Factors within the control of individuals.  
   $\rightarrow$ Fair
Figure: Chetty et al. (2014)
Figure: Chetty et al. (2018)
Based on these circumstances we can partition the population into types:

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<td>Type 2</td>
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→ Inequality between circumstance types is morally objectionable.
The equality of opportunity principle is reflected in distributional preferences:

- Lab Experiments: Cappelen et al. (2007); Krawczyk (2010); Mollerstrom et al. (2015).
Freedom from Poverty
Are *ex-post inequalities* a matter of indifference for fairness evaluations?

Some answers:

- Anderson (1999) argues against pure opportunity egalitarians based on a number of examples -> *abandonment of negligent victims*.
Based on realized outcomes we can partition the population into groups where \( P = \{ i : y_i \leq y_{\min} \} \) and \( R = \{ i : y_i > y_{\min} \} \):

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<td>L</td>
<td>( y_i \leq \mu_P )</td>
<td>( y_i \leq \mu_R )</td>
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<tr>
<td>H</td>
<td>( y_j &gt; \mu_P )</td>
<td>( y_i &gt; \mu_R )</td>
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→ Inequalities are objectionable (i) among individuals in \( P \) and (ii) to the extent that \( \mu_P < y_{\min} \).
The freedom from poverty principle is reflected in distributional preferences:

- Lab Experiments: Cappelen et al. (2013).
Recent evidence from Germany (Sep 2018):

**Figure:** Eisnecker et al. 2018
Outline

1. Normative Principles
2. Norm-based Inequality Measurement
3. Empirical Application
4. Summary
Figure: Norm-Based Inequality Measurement
**Figure:** Norm-Based Inequality Measurement
Norm Vector
Consider the following restrictions on the set of all possible income distributions $D$:

- **Constant Resources:**

  \[ D^1 = \left\{ D : \sum_i y_i^r = \sum_i y_i^e \right\} \]  
  \[ (5) \]

- **Equality of Opportunity:**

  \[ D^2 = \{ D : \mu_t^r = \mu \ \forall \ t \in T \} \]  
  \[ (6) \]
Consider the following restrictions on the set of all possible income distributions $D$:

- **Constant Resources:**
  \[
  D^1 = \left\{ D : \sum_{i} y^r_i = \sum_{i} y^e_i \right\} \tag{5}
  \]

- **Equality of Opportunity:**
  \[
  D^2 = \{ D : \mu^r_t = \mu \ \forall \ t \in T \} \tag{6}
  \]
Freedom from Poverty:

\[ D^3 = \{ D : y_i^r = y_{\min} \ \forall \ i \in P \} \]  

(7)

Financing I:

\[ D^4 = \{ D : y_i^r \geq y_{\min} \ \forall \ i \in R \} \]  

(8)

Financing II:

\[ D^5 = \left\{ D : \forall t \in T, \frac{y_i^r - y_{\min}}{y_j^r - y_{\min}} = \frac{y_i^{e} - y_{\min}}{y_j^{e} - y_{\min}} \ \forall \ i, j \in t \cap R \right\} \]  

(9)
Freedom from Poverty:

\[ D^3 = \{ D : y^f_i = y_{\text{min}} \, \forall \, i \in P \} \quad (7) \]

Financing I:

\[ D^4 = \{ D : y^f_i \geq y_{\text{min}} \, \forall \, i \in R \} \quad (8) \]

Financing II:

\[ D^5 = \left\{ D : \forall t \in T, \, \frac{y^f_i - y_{\text{min}}}{y^f_j - y_{\text{min}}} = \frac{y^e_i - y_{\text{min}}}{y^e_j - y_{\text{min}}} \, \forall \, i, j \in t \cap R \right\} \quad (9) \]
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The intersection $\cap_{s=1}^{5} D^s$ yields a singleton:

$$y^r_i = \begin{cases} 
  y_{\text{min}}, & \text{if } y^e_i < y_{\text{min}} \\
  y^e_i \left[1 - \tilde{y}_i (\tau^{\text{FfP}} + \tau^{\text{EOp}} (1 - \tau^{\text{FfP}}))\right], & \text{otherwise.}
\end{cases} \quad (10)$$

where

$$\tilde{y}_i = \left(\frac{y^e_i - y_{\text{min}}}{y_{\text{min}}}\right),$$

$$\tau^{\text{FfP}} = \frac{N_P (y_{\text{min}} - \mu^e_P)}{N_R (\mu^e_R - y_{\text{min}})},$$

$$\tau^{\text{EOp}}_t = \frac{\mu^e_t + \frac{N_P \cap t}{N_t} (y_{\text{min}} - \mu^e_{P \cap t}) - \tau^{\text{FfP}} \left(\frac{N_R \cap t}{N_t} (\mu^e_{R \cap t} - y_{\text{min}})\right) - \mu}{\mu^e_t + \frac{N_P \cap t}{N_t} (y_{\text{min}} - \mu^e_{P \cap t}) - \tau^{\text{FfP}} \left(\frac{N_R \cap t}{N_t} (\mu^e_{R \cap t} - y_{\text{min}})\right) - y_{\text{min}}}$$
Divergence Measure
Unfair inequality is then measured as the divergence $D(Y^\text{e}||Y^r)$ between the observed and the norm income distribution.

Various divergence measures have been proposed in the literature: Almås et al. (2011); Cowell (1985); Magdalou and Nock (2011).

We rely on a generalization of the generalized entropy class proposed by Magdalou and Nock (2011) with $\alpha = 0$:

$$D(Y^\text{e}||Y^r) = \frac{1}{N} \sum_i \left[ \ln \frac{y_i^r}{y_i^e} + \frac{y_i^e}{y_i^r} - 1 \right].$$  (11)
Properties
Imagine we are indifferent to FfP. Then, the norm vector simplifies to:

\[
y_i^r = \begin{cases} 
y_{\text{min}}, & \text{if } y_i^e < y_{\text{min}} \\
y_i^e \left[1 - \tilde{y}_i \left(\tau_{FfP}^e + \tau_{EOp}^e \left(1 - \tau_{FfP}^e\right)\right)\right], & \text{otherwise.} 
\end{cases}
\]

\[
y_i^e \left[1 - \left(\frac{\mu_{t} - \mu}{\mu_{t}^e}\right)\right] = y_i^e \left[\frac{\mu}{\mu_{t}^e}\right]
\]
Using $y_i^e \left[ \frac{\mu^e}{\mu_t^e} \right]$ in the measure of distributional change gives:

$$D(Y^e || Y_{EOp}^r) = \frac{1}{N} \sum_i \left[ \ln \frac{y_i^r}{y_i^e} + \frac{y_i^e}{y_i^r} - 1 \right]$$

$$= \frac{1}{N} \sum_i \ln \frac{\mu^e}{\mu_t^e}.$$ 

This is a summary statistic of the distribution of type income means: the mean log deviation.
Imagine we are indifferent to EOp. Then, the norm vector simplifies to:

$$y_i^r = \begin{cases} 
  y_{\min}, & \text{if } y_i^e < y_{\min} \\
  y_i^e \left[ 1 - \tilde{y}_i (\tau^{FfP} + \tau^{EOp} (1 - \tau^{FfP})) \right], & \text{otherwise.}
\end{cases}$$

$$= \begin{cases} 
  y_{\min}, & \text{if } y_i^e < y_{\min} \\
  y_i^e \left[ 1 - \tilde{y}_i \tau^{FfP} \right], & \text{otherwise.}
\end{cases}$$
Using this norm vector in the measure of distributional change gives:

\[
D(Y^e || Y^r_{FfP}) = \frac{1}{N} \sum_{i \in P} \ln \frac{y_{\min}}{y_i^e} - \frac{1}{N} \sum_{i \in P} \left( \frac{y_{\min} - y_i^e}{y_{\min}} \right) + \frac{1}{N} \sum_{i \in R} \ln \frac{y_i^r}{y_i^e} + \left( \frac{y_i^e}{y_i^r} - 1 \right).
\]

This is incorporates two widely used poverty measures, the Watts Index and the Poverty Gap ratio.
Outline

1. Normative Principles
2. Norm-based Inequality Measurement
3. Empirical Application
4. Summary
Methods and Data
Data:
- Cross-sectional: EU-SILC 2011.

Type Partition:
- Circumstances: Sex, Occupation Parents, Education Parents, Immigration Background (Race) (36 types).

Income Concept:
- Equivalized disposable HH income (OECD equivalence scale).

Poverty Measure:
- At-Risk-Of-Poverty-Rate (60% of median income).
Data:

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Results
Figure: Unfair Inequality by Country (Europe)
Figure: Unfair Inequality over Time (USA)
**Figure:** Decomposition by Country (Europe)

- **Legend:**
  - $L_{FIP}$
  - $UB_{EOP}$
  - Total Inequality

*Hufe EOp and FfP*
Figure: Decomposition over Time (USA)
Sensitivity Checks
1. Varying poverty thresholds. 
2. Alternations in normative assumptions. 
Outline

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The extent of unfairness/inequity in observed inequality is either overstated (standard inequality measures) or understated (EOp measures).

We recognize the multiplicity of fairness ideals by drawing onto the principles of EOp and FfP.

Combining different normative principles, i.e. EOp and FfP, yields strong upwards corrections of the unfair share of inequality.

The framework may be fruitfully complemented by further ideals of fairness.
Thank you!

sk145@cornell.edu
hufe@ifo.de
References I


Figure: Alternative Poverty Thresholds by Country (Europe)
Figure: Alternative Poverty Thresholds over Time (USA)
Figure: Alternative Norm Distributions by Country (Europe)
Figure: Alternative Norm Distributions over Time (USA)
**Table:** Rank Correlation of Measures by Country (Europe)

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### Table: Rank Correlation of Measures over Time (USA)

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