Trickle-down housing economics

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Abstract

I estimate the welfare effects of building different types of housing in the Boston metropolitan area. Both luxury and low-quality construction improve the welfare of poor households without a college degree. Low-quality construction is twice as effective but hurts rich, educated households due to negative effects on wages and amenities. Construction is worse for these households when cross-metro mobility is high, demand for amenities rises with education, and gentrification occurs at the neighborhood rather than metro level. Tripling the construction intensity in Boston would eliminate its affordability crisis, while instituting rent control would exacerbate it.

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Since 1980, real house prices have significantly risen in many large metropolitan areas. In response, poor households and those without a college degree have migrated away (Gyourko, Mayer and Sinai, 2013; Diamond, 2016; Ganong and Shoag, 2017). Policymakers call this situation an "affordability crisis" (e.g., White House, 2016). According to economists, easing permitting rules would allow new construction and bring down house prices (Glaeser, Gyourko and Saks, 2005; Herkenhoff, Ohanian and Prescott, 2018; Hsieh and Moretti, 2019). Some people, however, worry that new construction does not benefit poor residents (Been, Ellen and O'Regan, 2019). New housing is larger and more expensive than existing housing (Rosenthal, 2014; Molloy, Nathanson and Paciorek, 2019), and migrants from other metropolitan areas occupy many new units (Mast, 2019). Furthermore, rich migrants might cause gentrification, which could cause further outmigration of the poor (Guerrieri, Hartley and Hurst, 2013; Brummet and Reed, 2019).

Estimating the effect of construction on a metropolitan area's population is difficult. A single new building constitutes a small share of a metropolitan area, so its causal effect on total migration is hard to detect. Other research has calculated the effect of construction on local neighborhoods (Schwartz et al., 2006; Baum-Snow and Marion, 2009; Diamond and McQuade, 2017; González-Pampillón, 2019; Singh, 2019), but neighborhood effects do not determine changes at the metro level because of within-metro moves.

To solve this problem, I develop a structural model in which households choose a metropolitan area in which to work and live. The model features differentiated housing, gentrification, and skill-intensive labor markets. I fit this model to Census data from the Boston metropolitan area in 2016, matching the joint distribution of income, house prices, and education. I calibrate other key parameters using previous work in urban and labor economics. While this approach certainly has limitations—no model can capture everything—it includes several important channels and enables precise counterfactual statements about the effect of construction on migration.

According to my model, luxury construction stimulates no additional migration of rich households into the metropolitan area. Instead, it induces upgrading of existing residents into nicer housing units. This chain of within-metro moves frees up lower quality units, allowing poorer households to in-migrate. For every 100 new luxury units, 47 households in the lower quartile of the metro income distribution arrive. Another way to state this result: 100 luxury units prevent 47 poor households from leaving the metropolitan area.

The benefit to the poor depends critically on how I model metro amenities, such as school quality and the absence of crime. According to Bayer, Ferreira and McMillan (2007) and Diamond

(2016), preferences for these amenities rise with education, and metropolitan areas with a higher share of educated households offer higher amenities. My model contains both channels. When I shut either down, poor households benefit two to ten times less from luxury construction because it induces much more in-migration of rich households with college degrees.

Although luxury development helps the poor, low-quality development helps them twice as much on a unit-for-unit basis. This result is my second main finding, and it is robust to all extensions I consider that allow cross-metro migration. It echoes Sweeney (1974*b*) and Braid (1981), who show that low-quality construction lowers house prices for the poor more than luxury construction in the presence of cross-metro migration. In contrast to these papers, low-quality construction makes rich households *worse* off in my framework. The in-migration of poor, non-college households lowers the wages and amenities that rich households enjoy. This effect rises when migration is more responsive and when amenity spillovers occur at the neighborhood level.

Finally, I study rent control and new construction as policy responses to the affordability crisis, which I generate in my model with a productivity shock that favors college-educated workers in Boston. While rent control succeeds at slowing down house price growth, it accelerates the out-migration of poor, low-education households from Boston. In contrast, expanding the housing stock by 1.6% annually eliminates this out-migration. Raising the construction intensity to 1.6% from its current value of 0.55% helps the poor but hurts the rich.

The paper proceeds as follows. Section 1 relates this work in more detail to the previous literature. Section 2 lays out the economic environment, defines equilibrium, and characterizes equilibrium house prices. Section 3 theoretically analyzes the equilibrium effects of construction and productivity shocks. Section 4 discusses the strategy for estimating the model and describes the data I use. Section 5 presents the quantitative results, and Section 6 concludes. Additional results appear in an included Appendix and an Internet Appendix.

1 Relation to literature

This paper's main contribution is to incorporate urban spillovers into a quantitative spatial model of heterogeneous housing quality. Spillovers distinguish this paper from an older theoretical literature building on Sweeney (1974*b*), who theoretically analyzes the effects of constructing different qualities of indivisible housing on house prices (see Arnott, 1987 for a literature review). Braid (1981), for instance, corresponds to the special case of my indivisible model without spillovers

and mobility. In these older models, neither incomes nor amenities depend on the composition of households. These spillovers drive my key theoretical and quantitative results.

As I prove in Section 3, the richness of my results arises from cross-metro household mobility and the indivisibility of housing. When households are immobile, construction makes all local households better off, and building a unit of the highest quality improves welfare more than building a unit of any other quality. The same holds when housing is divisible—households can costlessly divide any housing unit into two new units of lesser quality—for a subset of the spillovers I consider. Papers have studied indivisibility with immobility (Braid, 1981; Määttänen and Terviö, 2014; Landvoigt, Piazzesi and Schneider, 2015; Couture et al., 2019) and divisibility with mobility (Henderson, 1974; Glaeser and Gottlieb, 2009). The *combination* of mobility and indivisibility breaks the trickle-down mechanism in these frameworks.[1]

Another contribution is to explain why certain households oppose construction in their city of residence. According to my estimates, high-income and high-education households oppose low-quality construction because it attracts low-education households, who lower amenities and labor prices.[2] This mechanism differs from that in Hilber and Robert-Nicoud (2013) and Ortalo-Magné and Prat (2014). These papers formalize the "homevoter hypothesis" of Fischel (2001) with models in which homeowners oppose construction in order to increase their home equity wealth. This incentive is absent from my paper's static model. My results might explain why the restrictiveness of zoning correlates more closely with demographics than with homeownership in multiple empirical studies (Gyourko and Molloy, 2015).

Several empirical papers estimate the effect of subsidies for low-income housing on housing supply and house prices. Schwartz et al. (2006), Baum-Snow and Marion (2009), and Diamond and McQuade (2017) find that these subsidies increase the supply of low-income housing in the neighborhoods they target and increase the value of surrounding homes in low-income and declining neighborhoods. Unlike these studies, my paper estimates the effect of low-quality housing on an entire metropolitan area. This distinction matters, as many of these subsidies fail to increase the supply of low-income housing at the metropolitan area level (Eriksen and Rosenthal, 2010; Schuetz, Meltzer and Been, 2011). As a result, existing subsidy programs may not identify

[1] Davis and Dingel (2018) also incorporate spillovers into a spatial equilibrium model with heterogeneous and indivisible housing quality. However, their focus is on the sorting of households and firms across cities. They do not analyze increases to housing supply.

[2] Albouy et al. (2019) also present a model in which local residents oppose new construction due to the negative effect of migration on local amenities. Relative to their model, my innovation is to introduce heterogeneity in the demand for and provision of amenities by education. This heterogeneity leads to disagreement among existing residents about whether to allow construction and to disagreement about which type of housing to allow.

the effect of constructing low-quality housing on a metropolitan area. My structural approach does.

Another paper estimating the effect of construction on house prices with a structural approach is Anenberg and Kung (2018). My paper differs in how I treat the cross-metro migration that occurs in response to construction. Anenberg and Kung (2018) assume that households from outside the metropolitan area occupy all newly built housing, irrespective of any adjustment to house prices. In contrast, migration demand in my model is an endogenous, downward-sloping function of house prices. Anenberg and Kung (2018) estimate that building high-quality housing has no effect on the prices of other types of housing, while I find a large effect of high-quality construction on low-quality house prices.

My theoretical results on the effect of high-quality construction correspond to the empirical findings of Mast (2019). Using micro data on address history, he finds that luxury development frees up housing in poor neighborhoods by inducing a series of moves within a metropolitan area. This channel exists in my model, and its quantitative size is similar to what Mast (2019) finds empirically.

Several papers analyze the distributional consequences of rent control, which I examine as a policy response to house price growth. Diamond, McQuade and Qian (2018) find empirically that rent control increases the population of high income households relative to poor households. I find this effect in the model, although the mechanism is different. In the data, rent control leads to housing supply changes that favor the rich, whereas rent control favors the rich in the model without any housing supply changes. Favilukis, Mabille and Van Nieuwerburgh (2019) present a theoretical model of rent control and other policies that try to address affordability. They focus on the risk-sharing and distortionary effects of these policies with a lifecycle model featuring endogenous housing supply and location choices within the New York City metropolitan area. While risk and endogenous supply are absent from my model, I have endogenous cross-metro mobility, local amenities, and wages. Therefore, my paper and Favilukis, Mabille and Van Nieuwerburgh (2019) present complementary analyses of a similar problem.

My framework does not directly model filtering, the process through which high-quality construction eventually houses low-income households after years of depreciation (Rosenthal, 2014). Despite this omission, there is a simple way to map my framework into a filtering model. Filtering models admit a unique steady-state distribution for housing quality that depends on the intensity of construction of each type of housing (Sweeney, 1974*a*). The stock of housing in my static model

corresponds to the steady-state distribution in a filtering model. Increasing the housing stock in my framework maps to the change in construction intensity in a filtering model that would cause the corresponding shift to the steady-state distribution of housing quality.

2 Environment and equilibrium

2.1 Housing supply

The economy consists of T metropolitan areas indexed by t. In t, available housing qualities are $q_{j,t} > 0$, where $j \in \mathcal{J}_t = \{0, ..., J_t\}$ and $q_{j,t}$ strictly increases in j. Quality is a composite of the physical characteristics of the property as well as the value of amenities that differ within the metropolitan area, like distance to the urban center.[3] The measure of housing of quality $q_{j,t}$ is $h_{j,t} > 0$, and this housing trades in competitive markets at a price $p_{j,t}$.

There are two types of agents: households and rentiers. Rentiers are endowed with the entire housing stock and have utility that is a linear function of a composite non-housing consumption good c, whose price I normalize to 1. They take house prices as given and choose how much housing to sell and how much c to consume subject to a budget constraint.

2.2 The distribution of households

Households differ in their education, $e \in \{L, H\}$, labor endowment, $z > 0$, and taste for each metropolitan area t, ϵ_t. Across households and metropolitan areas, ϵ_t is distributed independently as a Gumbel distribution (McFadden, 1973). The distribution of z among households of education e equals $\widetilde{n}_e(z)$, an atomless distribution with full support on $(0, \infty)$.[4]

Each household lives in one metropolitan area. The household population in t is N_t. $N_{e,t}$ and $Z_{e,t}$ represent the total number and labor endowment, respectively, of households of education e in t. The measure of households of education e with labor endowment z living in t is $n_{e,t}(z)$. I restrict attention to allocations of households across cities in which $N_{e,t} > 0$ for each e and t.

[3] Landvoigt, Piazzesi and Schneider (2015) and Epple, Quintero and Sieg (2019) similarly model housing quality.

[4] I assume an unbounded support only for notational convenience—none of the results require this feature. The existence of households with arbitrarily small z is necessary for the existence and uniqueness of equilibrium, as I discuss in the proof of Lemma 2.

2.3 Household preferences and constraints

Households have preferences over four goods—composite non-housing consumption c, housing quality q, metro amenities a, and an idiosyncratic metro taste ϵ—represented by the utility function

$$u_e(c,q,a,\epsilon) = c^{\beta_{c,e}} q^{\beta_{q,e}} a^{\beta_{a,e}} \exp(\beta_{\epsilon,e}\epsilon), \tag{1}$$

where $\beta_{c,e}, \beta_{q,e}, \beta_{a,e}, \beta_{\epsilon,e} > 0$ for each e. Without loss of generality, I normalize $\beta_{\epsilon,e} = 1$.

Cobb-Douglas preferences over housing and non-housing consumption, such as in (1), appear in many equilibrium models of location choices (e.g., Glaeser and Gottlieb, 2009; Gennaioli et al., 2013; Diamond, 2016), and are consistent with the stability of housing expenditure as a share of income across places and time (Davis and Ortalo-Magné, 2011). The term involving ϵ is present in some recent work (Kline and Moretti, 2014; Hsieh and Moretti, 2019) and limits household mobility across metropolitan areas in response to changes in utility coming from c, q, and a. As in Diamond (2016), preferences may differ across education groups.

Amenities in each metro are non-rival and non-excludable to local households. Amenities depend on exogenous local characteristics as well as the relative population of households with education H:

$$a_t = \widetilde{a}_t \left(\frac{N_{H,t}}{N_{L,t}}\right)^{\gamma_a}, \tag{2}$$

where $\gamma_a \geq 0$ and $\widetilde{a}_t > 0$ for each t. When $\gamma_a > 0$, metro amenities increase when more high education households arrive. Households may simply enjoy meeting high-education households. Alternatively, consumption by high-education households may produce non-excludable benefits to other households, as is the case with philanthropy. Diamond (2016) assumes the same amenity function and discusses it further. I consider alternate formulations for amenities in Section 5.

Labor in each metropolitan area trades in competitive markets, and I denote the price of labor of education e in t by $w_{e,t}$. A household's income then equals $w_{e,t}z$, which I denote $y_{e,t}(z)$. Each household takes house prices, labor prices, and amenities as given and chooses a metropolitan area t, non-housing consumption c, and a housing quality $q_{j,t}$ subject to the budget constraint, $c + p_{j,t} \leq w_{e,t}z$. Housing is *indivisible* because households can consume only one unit of one type of house. I explore the importance of this assumption in Section 3.

2.4 Firms

Firms in each metropolitan area t combine low-education and high-education labor to produce the non-housing consumption good c according to the production function

$$F_t(Z_L, Z_H) = \left((A_{L,t}Z_L)^\rho + (A_{H,t}Z_H)^\rho\right)^{\frac{1}{\rho}}, \tag{3}$$

where Z_e is the quantity of labor of education e a firm uses, and $0 < \rho \leq 1$. A large literature in labor economics adopts (3) to explain the evolution of wages for workers with and without a college degree (Goldin and Katz, 2008; Card, 2009). Firms in t take $A_{L,t}$ and $A_{H,t}$ as given and choose labor inputs Z_L and Z_H. The resulting profits accrue to the local rentiers.

The only differences in production technology across cities come from variation in $A_{L,t}$ and $A_{H,t}$, which govern the productivity of each type of labor. As with amenities, productivity depends on exogenous characteristics as well as the metropolitan area's population:

$$A_{e,t} = \widetilde{A}_{e,t} N_t^{\gamma_N} \left(\frac{N_{H,t}}{N_t}\right)^{\gamma_H}, \tag{4}$$

where $\gamma_N \geq 0$, $\gamma_H \geq 0$, and $\widetilde{A}_{e,t} > 0$ for each t. When $\gamma_N > 0$, productivity increases when the metro population goes up and the relative share of each education group remains constant. Labor productivity is indeed higher in denser areas, and an extensive literature in urban economics finds that part of this phenomenon is a causal effect of population on productivity (Combes and Gobillon, 2015). When $\gamma_H > 0$, productivity increases when the metro share of high-education households rises. The functional form of this effect matches that in Lucas (1988), who posits a constant elasticity of productivity spillover with respect to the average human capital in the population.

2.5 Equilibrium definitions

Equilibrium consists of house prices $p_{j,t}$, labor prices $w_{e,t}$, amenity levels a_t, productivity levels $A_{e,t}$ and populations $n_{e,t}(z)$ for each e, j, and t such that households choose t and j to maximize utility, household demand for each $q_{j,t}$ equals the amount of that housing that rentiers optimally choose to sell, firms in t maximize profits by demanding the quantities of labor households supply, and (2) and (4) hold. Given populations $n_{L,t}(z)$ and $n_{H,t}(z)$, *local equilibrium* holds when house prices and wages clear labor and housing markets while households, rentiers, and firms in t optimize.

2.6 Equilibrium characterization

I first characterize equilibrium metro choices. To do so, I define indirect utility, $v_{e,t}(z)$, as follows. If $w_{e,t}z < \min_{j \in \mathcal{J}_t} p_{j,t}$, then a household is too poor to live in t, so I set $v_{e,t}(z) = 0$. Otherwise,

$$v_{e,t}(z) = \max_{j \in \mathcal{J}_t} u_e\left(w_{e,t}z - p_{j,t}, q_{j,t}, a_t, 0\right), \tag{5}$$

the maximized utility of a household in t whose idiosyncratic taste for that metropolitan area is zero. Lemma 1 solves for the equilibrium allocation of households across metropolitan areas in terms of indirect utility.

Lemma 1. *In equilibrium,*

$$n_{e,t}(z) = \frac{\widetilde{n}_e(z) v_{e,t}(z)}{\sum_{t'=1}^{T} v_{e,t'}(z)}. \tag{6}$$

Proof. Appendix A.1. □

Metro population rises in the utility households can achieve relative to other metropolitan areas.

To finish characterizing equilibrium, I solve for indirect utilities in each t, $v_{e,t}$, as functions of the distributions of metro households, $n_{e,t}$. These population distributions pin down the aggregates $Z_{e,t}$, $N_{e,t}$, and N_t. The latter two determine amenities via (2) and productivity via (4). Due to firm profit maximization, equilibrium labor prices coincide with marginal products:

$$w_{e,t} = \left((A_{L,t}Z_{L,t})^{\rho} + (A_{H,t}Z_{H,t})^{\rho}\right)^{\frac{1}{\rho}-1} A_{e,t}^{\rho} Z_{e,t}^{\rho-1} \tag{7}$$

for each e and t.

The last step is to characterize house prices, $p_{j,t}$, and the housing quality that each household chooses. Households in t differ in their income and education, and the equilibrium assigns them to qualities of housing depending on these differences. Proposition 1 describes this assignment and the equilibrium prices that yield it.

Proposition 1. *In equilibrium, $j_{0,t} = \sup(j \in \mathcal{J}_t \mid \sum_{j'=j}^{J_t} h_{j',t} \geq N_t)$ exists, $p_{j,t}$ strictly increases over $j \geq j_{0,t}$ and equals zero if $\sum_{j'=j}^{J_t} h_{j',t} > N_t$, and housing demand equals zero for $j < j_{0,t}$ and $h_{j,t}$ for $j > j_{0,t}$. The quality chosen by a household of education e and labor endowment z weakly grows in z for each e.*

Proof. Appendix A.2. □

According to Proposition 1, households live in the N_t highest quality units in t, and house prices strictly increase in quality among these occupied units. Households positively sort on income within each education group.[5]

Proposition 1 pins down the price of the lowest quality occupied unit, $p_{j_{0,t},t}$, when the housing stock exceeds the metro population: if $\sum_{j\in\mathcal{J}_t} h_{j,t} > N_t$, then $p_{j_{0,t},t} = 0$. The prices of higher quality units solve the system that equates household demand for these units to the rentiers' endowments. This system of equations admits a unique solution:

Lemma 2. *If $\sum_{j\in\mathcal{J}_t} h_{j,t} > N_t$, then a unique local equilibrium exists.*

Proof. Internet Appendix I.1. □

I characterize this local equilibrium when a positive measure of households from each education group choose each type of occupied housing. Such equilibria are the focus of the comparative statics analysis in Section 3 and the estimation in Section 4. The other case, in which a positive measure of only one education group chooses some types of housing, complicates the comparative statics analysis and does not hold for the data I analyze in Section 5.

For each chosen quality level—that is, for each $j \geq j_{0,t}$—I define $z_{e,j,t}$ to be the greatest lower bound of labor endowments z among households of education e choosing $q_{j,t}$. When $j > j_{0,t}$, $z_{e,j,t}$ also equals the least upper bound of labor endowments among households of education e choosing the quality one step below, $q_{j-1,t}$, because of sorting and because the support of $n_{e,t}$ is convex. A household with this endowment and education level is indifferent between $q_{j,t}$ and $q_{j-1,t}$:

$$\left(w_{e,t}z_{e,j,t} - p_{j,t}\right)^{\beta_{c,e}} q_{j,t}^{\beta_{q,e}} = \left(w_{e,t}z_{e,j,t} - p_{j-1,t}\right)^{\beta_{c,e}} q_{j-1,t}^{\beta_{q,e}} \tag{8}$$

for each $j > j_{0,t}$ and e. By Proposition 1, the measure of households choosing each such quality level coincides with the total housing stock available:

$$h_{j,t} = \sum_{e\in\{L,H\}} \int_{z_{e,j,t}}^{z_{e,j+1,t}} n_{e,t}(z)dz \tag{9}$$

for each $j > j_{0,t}$, where $z_{e,J_t+1,t} = \infty$. As I show in Appendix I.2, (8) and (9) admit a unique solution for the endowment cutoffs and house prices, and prices in this solution strictly increase in quality. When the endowment cutoffs also strictly increase in quality, this solution delivers the unique

[5] Määttänen and Terviö (2014) show that for more general utility functions, this sorting condition holds whenever the marginal rate of substitution from housing to non-housing consumption increases in non-housing consumption.

local equilibrium in t. The j that maximizes utility for a household is the one for which $z \in (z_{e,j,t}, z_{e,j+1,t}]$. This j pins down indirect utility via (5).

3 Equilibrium effects of construction

This section studies the effect of constructing housing in a single metropolitan area, t^*, on equilibrium house prices and welfare there. Constructing housing of quality q_{j,t^*} corresponds to increasing h_{j,t^*}. I interpret such construction as the outcome of a relaxation of permitting rules for housing of this quality. House prices exceed the marginal costs of land and structure in many metropolitan areas (Glaeser and Gyourko, 2003), suggesting that easier permitting would lead developers to build more housing.

In Section 5, I also explore the equilibrium effect of exogenous changes to labor productivity. To encompass these changes as well, I consider the equilibrium effect of marginally increasing each h_{j,t^*} by $\delta_{h,j}$ and each $\log \widetilde{A}_{e,t^*}$ by $\delta_{A,e}$. The equilibrium effect of these combined changes on outcomes is ∂.

3.1 Equilibrium assumptions

I make two assumptions about the equilibrium around which I compute comparative statics.

Assumption 1. *Households of each education choose each occupied quality of housing in t^*.*

Assumption 2. *Some of the lowest quality occupied housing, q_{j_{0,t^*},t^*}, remains vacant.*

Due to Assumption 1, (8) and (9) characterize the local equilibrium in t^*. Due to Assumption 2, $p_{j_{0,t^*},t^*} = 0$ in any perturbation to the equilibrium. These assumptions rule out edge cases in which equilibrium allocations are insufficient to determine comparative statics.[6] Sections 4 and 5 defend these assumptions in light of the data.

3.2 Local approximation

An exact solution for comparative statics in t^* necessitates computing comparative statics in all other metropolitan areas as well. This interconnectedness is apparent from (6), which shows

[6]For instance, suppose that $j_{0,t^*} > 0$ and that households occupy all h_{j_{0,t^*},t^*} units of quality q_{j_{0,t^*},t^*} (which violates Assumption 2). If the metro population rises, then quality $q_{j_{0,t^*}-1,t^*}$ becomes the lowest occupied quality, and p_{j_{0,t^*},t^*} rises above zero. The extent to which this price rises depends on $q_{j_{0,t^*}-1,t^*}$, but this quality does not affect the initial equilibrium and is therefore unobservable.

that the population distributions in t^* depend on indirect utilities in all metropolitan areas in the economy. Computing comparative statics in every metropolitan area substantially complicates both the theoretical and quantitative analysis. To avoid these complications, I propose an approximation under which comparative statics in t^* depend only on the *local* equilibrium:

Assumption 3. *For each e and z, $\partial \log v_{e,t^*}(z) \gg \partial \log\left(\sum_{t=1}^{T} v_{e,t}(z)\right)$.*

Assumption 3 seems likely to hold when T is large, both because v_{e,t^*} constitutes a small share of the sum and because population flows into and out of t^* do not affect any one of the other metropolitan areas much. An analogous approximation in urban models with perfect mobility is that reservation utility remains constant as primitives in one city change.[7] Applying Assumption 3 to (6) yields

$$\partial \log n_{e,t^*}(z) = \partial \log v_{e,t^*}(z), \tag{10}$$

meaning that population rises one-for-one with indirect utility. Using (10), I can solve for comparative statics using only the local equilibrium, so for ease of notation, I drop t^* from subscripts.

3.3 Derivatives of local equilibrium conditions

The welfare change for an inframarginal household—one who remains in the metropolitan area in the same house—comes from differentiating (5):

$$\partial \log v_e(z) = \left(\frac{\beta_{c,e} y_e(z)}{y_e(z) - p_j}\right) \partial \log w_e - \left(\frac{\beta_{c,e}}{y_e(z) - p_j}\right) \partial p_j + \beta_{a,e} \partial \log a, \tag{11}$$

where $z \in (z_{e,j}, z_{e,j+1})$.[8] Welfare rises in wages and amenities but falls in house prices. Due to (10), the migration response is proportional to this welfare change. In particular, the net migration of households of education e into housing of quality q_j is

$$\partial \log N_{e,j}^{ext} = \beta_{c,e} E_{e,j} \left(\frac{y}{y - p_j}\right) \partial \log w_e - \beta_{c,e} E_{e,j} \left(\frac{1}{y - p_j}\right) \partial p_j + \beta_{a,e} \partial \log a, \tag{12}$$

[7] In Roback (1982), the utility workers can achieve is constant across cities. She assumes that this reservation utility remains constant when amenities in one city changes, allowing her to calculate the effect of amenities on wages and rents in the same city. In the equilibrium of my model of imperfect mobility, $\exp(E(\log u_e \mid e, z, t)) = \exp(\gamma) \sum_{t'=1}^{T} v_{e,t'}(z)$, where γ is Euler's constant. Therefore, Assumption 3 makes the analogous assumption that the average utility a type of household can achieve in any metropolitan area remains constant.

[8] The change for marginal households depends on whether the corresponding endowment cutoff increases or decreases. If $\partial \log z_{e,j} > 0$ (resp. < 0), then marginal households choose the lower (resp. higher) quality level as a result of the changes to primitives, so the relevant house price for them in (11) is p_{j-1} (resp. p_j).

where $E_{e,j}(\cdot)$ denotes the expected value given that $y \in (y_{e,j}, y_{e,j+1})$, with $y_{e,j} = y_e(z_{e,j})$. I refer to this change as the *extensive* margin. The elasticities $\beta_{c,e}$ and $\beta_{a,e}$ govern the strength of the migration response to changes in wages, house prices, and amenities.

The *intensive* margin—population changes arising from switching of existing residents between adjacent qualities of housing—comes from differentiating the limits of integration in (9):

$$\partial N_{e,j}^{int} = y_{e,j+1} f_e(y_{e,j+1}) \partial \log z_{e,j+1} - y_{e,j} f_e(y_{e,j}) \partial \log z_{e,j}, \tag{13}$$

where f_e is the probability density of income among education e households. Differentiating (8) yields the changes in the endowment cutoffs:

$$\partial \log z_{e,j} = \left(\frac{y_{e,j} - p_{j-1}}{y_{e,j}(p_j - p_{j-1})} \right) \partial p_j - \left(\frac{y_{e,j} - p_j}{y_{e,j}(p_j - p_{j-1})} \right) \partial p_{j-1} - \partial \log w_e. \tag{14}$$

As (13) and (14) show, movements in house prices can induce intensive margin population changes. If p_j increases, then $z_{e,j}$ increases and $z_{e,j+1}$ decreases, increasing $N_{e,j+1}^{int}$ and $N_{e,j-1}^{int}$ at the expense of $N_{e,j}^{int}$. These intensive margin adjustments connect housing submarkets.

The housing market must clear, so

$$\delta_{h,j} = \sum_{e \in \{L,H\}} \left(\partial N_{e,j}^{int} + \partial N_{e,j}^{ext} \right), \tag{15}$$

which follows from fully differentiating (9). Differentiating (2), (4), and (7) gives

$$\partial \log a = \gamma_a \partial \log N_H - \gamma_a \partial \log N_L \tag{16}$$

$$\partial \log A_e = \delta_{A,e} + \left(\gamma_N \frac{N_H}{N} + \gamma_H \frac{N_L}{N} \right) \partial \log N_H + (\gamma_N - \gamma_H) \frac{N_L}{N} \partial \log N_L \tag{17}$$

$$\partial \log w_e = \partial \log A_e + (1-\rho) \frac{Y_{\sim e}}{Y} \partial \log \left(\frac{Z_{\sim e}}{Z_e} \right) + (1-\rho) \frac{Y_{\sim e}}{Y} \partial \log \left(\frac{A_{\sim e}}{A_e} \right), \tag{18}$$

the changes in amenities, productivities, and labor prices.[9] Changes to metro population totals come from aggregating the extensive-margin changes to each bin population: $\partial N_e = \sum_j \partial N_{e,j}^{ext}$. Changes to total labor endowments aggregate bin changes: $\partial \log Z_e = \sum_j (Y_{e,j}/Y_j) \partial \log Z_{e,j}^{ext}$, where

$$\partial \log Z_{e,j}^{ext} = \beta_{c,e} E_{e,j}^{y} \left(\frac{y}{y - p_j} \right) \partial \log w_e - \beta_{c,e} E_{e,j}^{y} \left(\frac{1}{y - p_j} \right) \partial p_j + \beta_{a,e} \partial \log a, \tag{19}$$

[9] $Y_{e,j}$, Y_e, Y_j, and Y are income totals for education e households choosing q_j, all education e households, all households choosing q_j, and all households, respectively; $\sim e$ denotes the opposite education group.

and $E^{y}_{e,j}(\cdot)$ is the income-weighted expected value over $(y_{e,j}, y_{e,j+1})$. These equations deliver a linear system whose coefficients are either elasticities or measurable outcomes of the local equilibrium. I estimate these coefficients from the data and prior literature.

3.4 When trickle-down economics works

Before estimating the coefficients, I present two variants of the model in which trickle-down economics works. That is, construction improves the welfare of all metro households, and a single new housing unit improves welfare most when that unit has the highest quality, q_J. These properties fail in the main model, so these variants illustrate the features of the model that drive this failure. In these variants, $\delta_{A,L} = \delta_{A,H} = 0$ in order to focus on the effect of construction.

The first variant shuts down migration into or out of the metropolitan area. In particular, $\partial \log n_e(z) = 0$, leading to constant labor prices and amenities. Only house prices may change, with (13)–(15) solely determining these adjustments. Proposition 2 characterizes welfare changes from construction.

Proposition 2. *Suppose $\delta_{h,j} \geq 0$ for $j > j_0$, with at least one strict inequality, in the no-migration variant of the model. For each e and $z \geq z_{e,j_0+1}$, $\partial \log v_e(z)/\partial \delta_{h,j'}$ is positive, increases over $j' \in \{j_0 + 1, \ldots, j\}$, and stays constant over $j' \in \{j, \ldots, J\}$. If $z < z_{e,j_0+1}$, then $\partial \log v_e(z) = 0$.*

Proof. Appendix A.3. □

An analogous result appears in Section 3C of Braid (1981).

The benefits of constructing high-quality housing trickle down to lower income households (with the exception of non-marginal households choosing the lowest unoccupied quality, who are indifferent). Welfare growth is largest when construction occurs at the highest quality, q_J. Therefore, the strict Pareto optimum for constructing a set amount of housing is to build all of it at this highest quality. Building any type of housing makes no one worse off. Intuitively, house prices must fall to induce households to move up the quality distribution into new supply, and declines in house prices make households better off.

The second variant maintains cross-metro migration but relaxes the assumption of housing indivisibility. Households can now choose continuous amounts of all housing qualities in their metropolitan area. The quality entering their utility function equals the total quality of the housing they consume. Internet Appendix II characterizes equilibrium in this divisible variant and

discusses changes to the comparative static equations. Proposition 3 summarizes the effect of construction on welfare.

Proposition 3. *In the divisible model variant, for each e and z,*

$$\partial \log v_e(z) \propto \sum_{j \in \mathcal{J}} q_j \delta_{h,j}, \tag{20}$$

where $\propto$ denotes proportionality that is positive if the equilibrium is stable and $\gamma_N = 0$.

Proof. Internet Appendix II.5. □

The equilibrium is *stable* when perturbations to the metro population raise the welfare of departing households or decrease the welfare of arriving households. A formal definition appears in Appendix II.3. According to the proposition, construction can only make a household worse off when stability fails, as long as $\gamma_N = 0$.[10] When the conditions of Proposition 3 hold, constructing high-quality housing improves the welfare of lower income households, and building the highest quality housing is again the strict Pareto optimum.

As Propositions 2 and 3 show, both cross-metro migration and housing indivisibility are necessary for trickle-down economics to fail. While cross-metro migration is self-evident empirically, the extent to which housing is indivisible is less apparent. Although households can split time between different locations, only 2.5% of the housing stock inside metropolitan areas in 2016 was held off the market for occasional use or temporarily occupied by persons with usual residence elsewhere, according to the U.S. Census Housing Vacancy Survey. Homeowners can divide housing through renovations, but local regulations like minimum lot sizes and maximum occupancy constraints may constrain this activity (Gyourko, Saiz and Summers, 2008; Glaeser and Ward, 2009). An additional cost to dividing housing is congestion dissipating local amenities that the quality index subsumes. The main model is more appropriate to the extent that divisibility costs are large.

[10] When this equation holds, spillovers depend only on the *relative* population of households with different education. The proof of Proposition 3 exploits this symmetry.

4 Data and estimation

4.1 American Community Survey

Household-level data come from the American Community Survey (ACS), which the U.S. Census Bureau has conducted annually since 2005 to provide current economic information about the United States (see U.S. Census Bureau, 2014 for recently available documentation). I use the public use microdata sample that is part of the Integrated Public Use Microdata Series (Ruggles et al., 2018). The data are a weighted random sample of the U.S. population. I limit the sample to the 2016 Boston-Cambridge-Newton, MA-NH metropolitan area, one of the largest and most expensive in the United States.[11]

I aggregate all persons in a housing unit into a single household observation, i, by summing total personal incomes and by assigning $e_i = 1$ when the householder has a bachelor's degree and 0 otherwise.[12] I calculate the annual price of the housing unit, p_i, as 12 times the rent for renters and ϕ times the self-reported value of the house for owner-occupants, where ϕ is a user cost of housing specified below.[13] I drop a small number of renters (about 3%) who do not pay cash rent. To capture households living in q_{j_0} (the lowest occupied quality in excess supply), I use a subsample of "group quarters" persons who likely live in homeless shelters.[14] Assumption 2 holds as long as there remain unoccupied outdoor locations in the Boston metropolitan area where households could feasibly reside, which seems likely. Education and income data are available for these persons, while I assign $p_i = 0$ based on the model. Finally, I designate units as new if their build year is 2015, the year before the survey. To capture new units that are still unoccupied, I include vacant units in the process of renting or selling when I measure new construction.

Table 1 lists summary statistics, with means calculated using household weights available in the data. In the estimation sample, owner-occupants have the highest incomes and education while the homeless have the least. Rent and home values for new construction are nearly double

[11] New York-Newark-Jersey City, NY-NJ-PA and San Francisco-Oakland-Hayward, CA may be larger or more expensive than the Boston metro area, but New York and California allow rent control while Massachusetts and New Hampshire prohibit it. The model assumes that a competitive market determines rents.

[12] Income is available for all persons at least 15 years old. The survey respondent designates someone living in the housing unit as the "householder."

[13] The Census Bureau top-codes rent and home values above thresholds with the average value above that threshold. Top-coding is not a problem because fewer than 2% of the observations in the estimation sample are top-coded and because I bin house prices using averages.

[14] Homeless shelters are one of many categories of group quarters, but the Census Bureau does not disclose whether a respondent is in a homeless shelter. I rule out many alternative group quarters by keeping only non-institutionalized persons who are non-student, non-employed adults. For a taxonomy of group quarters, see U.S. Census Bureau (2012).

TABLE 1
SUMMARY STATISTICS

	Estimation sample			New construction	
	Homeless	Renters	Owners	Renters	Owners
Income	$7,530	$64,961	$138,223	-	-
Education	0.128	0.397	0.554	-	-
Rent	-	$1,284	-	$2,101	-
Home value	-	-	$491,724	-	$851,353
Weighted observations	15,486	678,917	1,120,455	4,814	5,126
Unweighted observations	430	5,946	11,893	36	45

Notes: Data come from the 2016 American Community Survey for the Boston-Cambridge-Newtown, MA-NH metropolitan area. "Homeless" denotes non-institutionalized group quarters persons who are adults, non-students, and non-employed. "New construction" denotes units build in 2015, including vacant units, which I designate as "renters" versus "owners" based on whether the unit is for rent or for sale.

that for corresponding units in the estimation sample. The number of new units as a share of total units in the estimation sample is 0.55%.

4.2 Real Capital Analytics

Real Capital Analytics, Inc. (RCA) provides quarterly averages of the capitalization rate (i.e., the annual income return) of multifamily rental properties with at least ten units in the Boston metropolitan area that sold in 2016. RCA, through a partnership with the National Council of Real Estate Investment Fiduciaries, also provides quarterly rental revenue and net operating income per square foot for multifamily properties held by institutional investors. For each quarter in 2016, I calculate the annual rental return by multiplying the capitalization rate by the institutional rental revenue per square foot and then dividing by the institutional operating income per square foot. The average of the rental yield estimate for each quarter, 0.088, serves as the estimate for ϕ.

4.3 Estimates from other work

Several papers in labor economics estimate the inverse elasticity of substitution between college and non-college labor to be about 0.7 (see the discussion in Card, 2009). This inverse elasticity corresponds to $1-\rho$, so $\rho = 0.3$.

Combes and Gobillon (2015) find that the typical estimate in the literature of the elasticity of productivity with respect to population density lies between 0.04 and 0.07. I set $\gamma_N = 0.055$, the midpoint of this range. Moretti (2004) estimates that log output in an industry within a metropolitan area rises about 0.0055 when the college share in other industries in the same area rises by one percentage point. Interpreting this estimate as 100 times the derivative of log productivity with respect to $N_{H,t}/N_t$, I obtain $0.55 = \gamma_H N_t/N_{H,t}$. The college shares in the two years in the sample in Moretti (2004) are 0.161 and 0.191. Setting $N_{H,t}/N_H$ equal to the average gives $\gamma_H = 0.097$.

Diamond (2016) estimates $\psi = (\beta_{w,L}, \beta_{w,H}, \beta_{a,L}, \beta_{a,H}, \gamma_a)$, where $\beta_{w,e} = \beta_{c,e} + \beta_{q,e}$, using the location choices of workers with and without a college degree in the United States between 1980 and 2000. I use the estimate from her "full model," corresponding to column 3 of her Table 5, which is $\psi = (4.026, 2.116, 0.274, 1.101, 2.60)$. For reasons I discuss below, I also use the sampling distribution of ψ, available in her replication files.

4.4 Estimation strategy

The parameters f_e, p_j, $y_{e,j}$, $\beta_{c,e}$, N_e/N, and Y_e/Y determine the remaining coefficients in the linear system of Section 3. I take the population and income shares directly from the data and estimate the other parameters in three steps.

The first step assigns households, i, to quality indices, j. I partition households in housing units into 50 quantile bins according to their house price, p_i, placing (approximately) equal numbers of households in each bin.[15] Group quarters persons occupy the lowest bin, $j = 0$, so that $\mathcal{J} = \{0, \ldots, 50\}$. The price of each quality bin is the sample average: $\widehat{p_j} = \sum_i g_i \delta_{i,j} p_i / \sum_i g_i \delta_{i,j}$, where g_i is the household weight, and $\delta_{i,j}$ is a dummy for whether household i is in bin j. For $j > 0$, the housing stock estimate is $\widehat{h_j}/N = \sum_i g_i \delta_{i,j} / \sum_i g_i$, which is approximately constant by construction.

The second step estimates the income distributions, f_L and f_H, which I specify as double Pareto-lognormal, a four-parameter family that Reed (2003) and Reed and Jorgensen (2004) propose to characterize income distributions. I jointly estimate these eight distributional parameters and $\zeta = (\beta_{q,L}/\beta_{c,L})/(\beta_{q,H}/\beta_{c,H})$, which governs the relative taste for housing versus non-housing consumption across the two education groups. I denote the nine total parameters by θ.

I could calculate f_L and f_H directly in the data, but Proposition 1 sharply restricts the set of valid joint distributions of income, housing quality, and education. In particular, it rules out in-

[15] In the absence of measurement error, each distinct value p_i, corresponds to a different quality by Proposition 1. Measurement error arises for a variety of reasons, including misreporting and unmodeled search frictions leading to price dispersion for similar housing units. Binning households along p_i smooths this error.

stances in which a household with the same education and lower income than another household chooses a higher housing quality. To accommodate such occurrences, I assume that the income in the data, y_i, equals the income from the model, $y_{e_i}(z_i)$, plus noise. This noise may come from temporary fluctuations in income that, due to adjustment costs, do not cause households to change the quality of the housing they choose (Chetty and Szeidl, 2007). I proceed under the following identification assumption:

Assumption 4. $0 = E\delta_{i,j}e_i(y_i - y_{e_i}(z_i)) = E\delta_{i,j}(1-e_i)(y_i - y_{e_i}(z_i))$ *for each* $j \in \mathcal{J}$.

The noise not only has a mean of zero but also is uncorrelated with a household's education, housing choice, and the interaction of the two.

To estimate θ, I compare several conditional expectations of income and education in the model to the data. For certain θ, there exist unique, strictly increasing income cutoffs such that (8) and (9) hold at the house price and stock estimates, $\widehat{p_j}$ and $\widehat{h_j}$. Appendix A.4 defines this set, which I denote Θ. For $\theta \in \Theta$, define $\overline{y}_{e,j}(\theta)$ to be the average income among education e households choosing q_j, and denote $\overline{e}_j(\theta)$ to be the share of households choosing q_j who have education H. The following moment conditions equate empirical realizations of these conditional expectations to their model-based counterparts:

$$0 = E\delta_{i,j}(1-e_i)\left(y_i - \overline{y}_{L,j}(\theta)\right) \tag{21}$$

$$0 = E\delta_{i,j}e_i\left(y_i - \overline{y}_{H,j}(\theta)\right) \tag{22}$$

$$0 = E\delta_{i,j}\left(e_i - \overline{e}_j(\theta)\right) \tag{23}$$

for each j. These moment conditions hold due to Assumption 4. I estimate θ via two-step generalized method of moments (Hansen, 1982), using (21)–(23) as moment conditions and the diagonal inverse of the sample variances as the weighting matrix. This estimator chooses the θ that best fits the joint distribution of income, education, and housing quality in the data.

The final step allocates the estimates of $\beta_{w,e}$ from Diamond (2016) to $\beta_{q,e}$ and $\beta_{c,e}$. In Diamond (2016), the share of income spent on housing is $\beta_{q,e}/\beta_{w,e}$. Davis and Ortalo-Magné (2011) find that renters in the U.S. between 1980 and 2000 (the same location and time in Diamond, 2016) spend 24% of their income on housing, yielding the equation $0.76 = (1-e^{rent})(\beta_{c,L}/\beta_{w,L}) + e^{rent}\beta_{c,H}/\beta_{w,H}$, where e^{rent} is the share of renters with a college degree, which I calculate in IPUMS to be 0.18. Together with the one defining ζ, this equation uniquely determines $\beta_{c,e}$ and $\beta_{q,e}$ from $\beta_{w,e}$.

4.5 Stability

I restrict attention to stable equilibria to guarantee that instabilities do not affect comparative statics. To impose stability, I rule out self-reinforcing perturbations to house prices, wages, and amenities as follows. Such perturbations induce population changes via (10) and (11). These migration responses then alter the equations determining house prices, wages, and amenities: (15), (16), and (18). Combining these steps, I obtain a matrix mapping the initial perturbations to deviations in the corresponding equations driven by migration responses. I require that this matrix is stable, meaning that all of its eigenvalues have negative real parts.[16]

Equilibrium is unstable under the point estimates from Diamond (2016).[17] To guarantee stability, I draw 10,000 times from her sampling distribution for ψ and use the mean of the estimates under which the equilibrium is stable. This procedure alters $\beta_{c,e}$, $\beta_{a,e}$, and γ_a, but it does not change any other parameters that affect coefficients in the linear system of comparative statics.

5 Quantitative results

5.1 Estimation

Table 2 reports estimates of model parameters as well as equilibrium outcomes. Households split roughly evenly between college and non-college. College households earn about double what non-college households earn. The β estimates imply that college households value amenities versus non-housing consumption 8.5 times more than non-college households do. My amenity spillover elasticity is $\gamma_a = 1.082$, less than half the 2.6 estimate in Diamond (2016) due to the stability criterion I impose.

Figure 1 displays the average incomes and education shares of households choosing each quality of housing, in both the model and the data. As Panel A shows, the model matches the empirical income averages quite well. Conditional on housing quality, the incomes of college households exceed the incomes of non-college households in both the model and the data. This outcome is consistent with the estimated value of $\zeta = 1.597$, which indicates that non-college households

[16] Samuelson (1941) and Mas-Collel, Whinston and Green (1995) discuss the link between comparative statics and this definition of stability. Internet Appendix II.3 compares this definition to the one in the divisible model.

[17] Because our models differ, instability here does not imply instability in Diamond (2016). In particular, Assumption 3 assumes away economy-wide changes in utility levels from migrations into and out of Boston. Diamond (2016) estimates her model for the entire United States, meaning that such migrations change economy-wide utility levels. As a result, re-allocations of households that cause instability in my approximate model may not do so in her more complete framework.

TABLE 2
Estimates of model parameters

Description	Name	Value
College population share	N_H/N	0.491
College income share	Y_H/Y	0.667
Non-college migration elasticity to consumption	$\beta_{c,L}$	3.288
Non-college migration elasticity to amenities	$\beta_{a,L}$	0.242
College migration elasticity to consumption	$\beta_{c,H}$	1.666
College migration elasticity to amenities	$\beta_{a,H}$	1.044
Relative housing taste, non-college over college	ζ	1.597
Amenity spillover elasticity	γ_a	1.082

value housing versus non-housing consumption relatively more than college households. Panel B shows that the model closely fits the increasing relation between education shares and housing quality. Only about 10% of households choosing the lowest quality levels have a college degree, while more than 80% of the households choosing the highest quality levels do. Assumption 1 holds because these shares never equal 0% or 100%.

5.2 Baseline construction effect

I estimate the welfare effects of low- and high-quality construction by increasing the housing stocks in bins 10 and 40, representing the 20th and 80th percentiles of the occupied housing quality distribution.[18] In these experiments, I expand the metro housing stock by 0.55%, the construction intensity in the data, solely by building in the chosen bin. The welfare effect for an inframarginal household of type e and z is $\partial \log v_e(z)$. I aggregate this welfare effect across education groups and metro income quartiles. I also report the welfare effect on local rentiers, assuming that local rentiers exclusively own the metro housing stock.

Panel A of Table 3 displays the results. Three key take-aways emerge. First, luxury construction makes poor households better off—for instance, the welfare of non-college households in the lowest income quartile rises by 1.17%. Second, low-quality construction helps these households almost twice as much, as the welfare change becomes 2.03%. Finally, low-quality construction is significantly worse for rich households, both with and without a college degree. Such households

[18]Respectively, 11% and 66% of new construction in the data are at or below bins 10 and 40. Although subsidies may support low-quality construction, the prevalence of such construction in the data suggests that developers can feasibly supply low-quality housing.

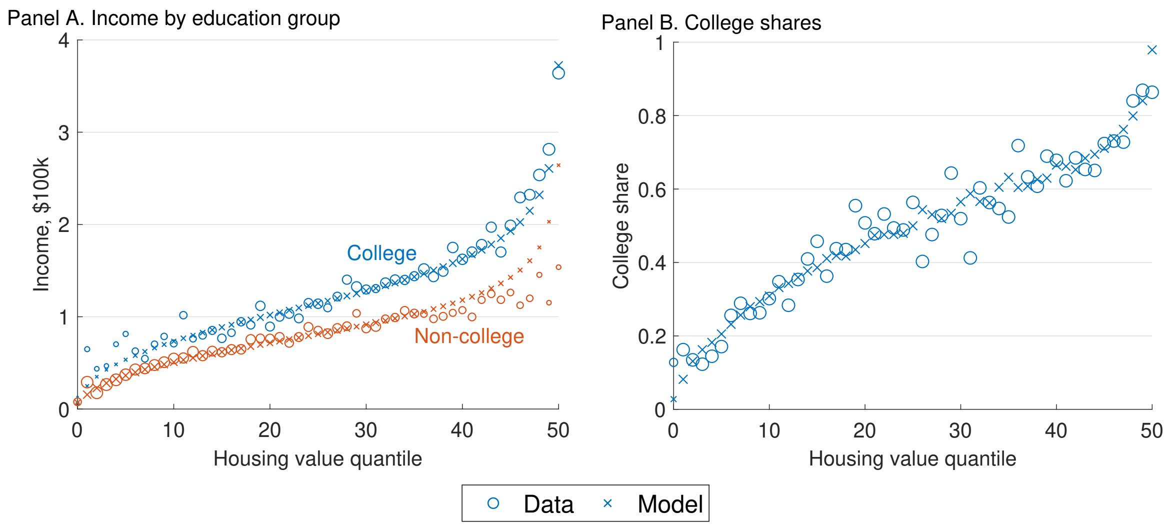

FIGURE 1. GOODNESS OF FIT

Notes: The two-dimensional size of each marker is proportional to the number of housing units in each housing value and education bin.

even suffer to a lesser degree from high-quality construction.

To clarify the mechanism behind these results, I display other outcomes in the baseline model and conduct a number of robustness checks. Figure 2 decomposes the welfare effect of luxury construction for two extreme groups from Table 3: the poorest non-college households and the richest college households. Poor non-college households benefit entirely because the price of their housing falls. In contrast, rich college households suffer solely because metro amenities decline. These responses reinforce each other in equilibrium. Few rich households move to the metropolitan area in response to the luxury construction, meaning that existing households move up the quality ladder to occupy it. As a result, housing becomes cheaper throughout the quality distribution, improving the welfare of even the poorest households. More of them move to the city. Because most poor households lack a college degree, metro amenities decline, which justifies the initial lackluster migration of rich college households, as they strongly value amenities.

How much low-quality housing does high-quality construction make available? High-quality construction raises the metro population by 0.40%. Because the housing stock expands by 0.55%, 100 new luxury units bring 73 new households into the city, freeing up new units for 27 existing residents on net. As a result, many poor households move up to higher quality housing. In aggregate, the measure of existing residents in the bottom quartile and bottom half of the non-homeless housing distribution who move to a nicer house equals 0.36% and 0.49% of the total

TABLE 3

Effect of construction on resident welfare (%)

	Non-college				College				
Income quartile:	1	2	3	4	1	2	3	4	Ren-tiers
Panel A. Baseline									
20th	2.03	0.49	−0.45	−1.25	0.28	0.06	−0.27	−0.50	−1.02
80th	1.17	0.60	0.31	−0.44	0.25	0.17	0.07	−0.03	−0.24
Panel B. Exogenous amenities									
20th	1.17	0.42	0.03	−0.24	0.59	0.49	0.30	0.19	−0.09
80th	0.67	0.56	0.59	0.16	0.43	0.42	0.41	0.37	0.31
Panel C. Neighborhood amenities									
20th	2.16	0.48	−0.58	−1.50	−0.00	−0.08	−0.37	−0.54	−1.03
80th	1.31	0.63	0.18	−0.77	0.06	0.05	−0.05	−0.10	−0.30
Panel D. Exclusively-college amenities									
20th	0.75	0.38	0.26	0.26	0.73	0.70	0.58	0.52	0.35
80th	0.11	0.51	0.91	0.83	0.63	0.70	0.78	0.83	0.92
Panel E. Local services									
20th	1.96	0.47	−0.43	−1.18	0.31	0.10	−0.22	−0.44	−0.96
80th	1.12	0.59	0.33	−0.38	0.28	0.20	0.11	0.02	−0.19
Panel F. Stronger migration									
20th	3.12	0.49	−1.40	−3.18	−0.20	−0.49	−1.02	−1.41	−2.90
80th	1.21	0.46	−0.02	−0.97	0.05	−0.02	−0.15	−0.27	−0.65
Panel G. No migration									
20th	7.43	3.79	2.23	1.36	2.68	1.78	1.07	0.55	−1.46
80th	7.49	4.46	3.26	2.12	2.68	1.85	1.30	0.82	−1.30

Notes: "20th" and "80th" denote counterfactuls in which the housing stock in the Boston metropolitan area expands by 0.55% purely through construction at the 20th or 80th percentile of the quality distribution.

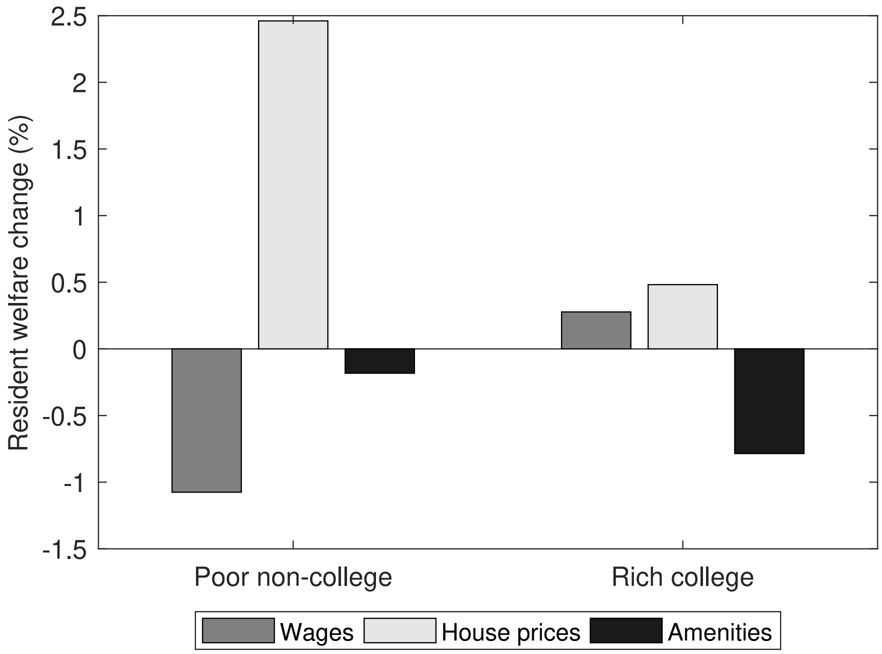

Figure 2. Components of welfare effect of 80th percentile construction

Notes: "Poor" and "rich" denote the bottom and top quartiles of the city income distribution, respectively.

metro population, corresponding to 65 and 90 vacated units. These numbers are higher but close to the 34 and 65 that Mast (2019) calculates. Of the 65 vacated units in the bottom quartile, new migrants from outside the metropolitan area occupy 47 of them (46 non-college and one college household). Previously homeless households occupy the remainder.

Figure 3 explores the effect of building low-quality housing. Why does this construction hurt the rich while helping the poor non-college households so much? These results depend critically on the model's spillovers as well as the differential preferences of college and non-college households. In Panels A and C, I turn off all spillovers by setting $\gamma_N = \gamma_H = \gamma_a = 0$ and $\rho = 1$, in which case amenities and wages no longer depend on the city population. Panels A and B eliminate preference heterogeneity by setting $\beta_{a,L}$ and $\beta_{a,H}$ (resp. $\beta_{c,L}$ and $\beta_{c,H}$) to their average value in the city before the construction, so that the migration response to amenities and non-housing consumption is identical across the two education groups.[19] The baseline appears in Panel D. In each panel, I plot the average welfare response among households of each education group in each bin. Low-quality construction benefits poor, non-college households much more in Panel D than the other panels. The rich suffer much more in this panel as well. In fact, without spillovers

[19]College and non-college households still differ in $\beta_{q,e}$, but this parameter does not govern cross-city migration by (11). The estimation forces this difference by selecting a value of $\zeta \neq 1$. When $\beta_{c,e}$ changes, I hold the ratio $(\beta_{q,L}/\beta_{c,L})/(\beta_{q,H}/\beta_{c,H})$ constant at the estimated value of ζ.

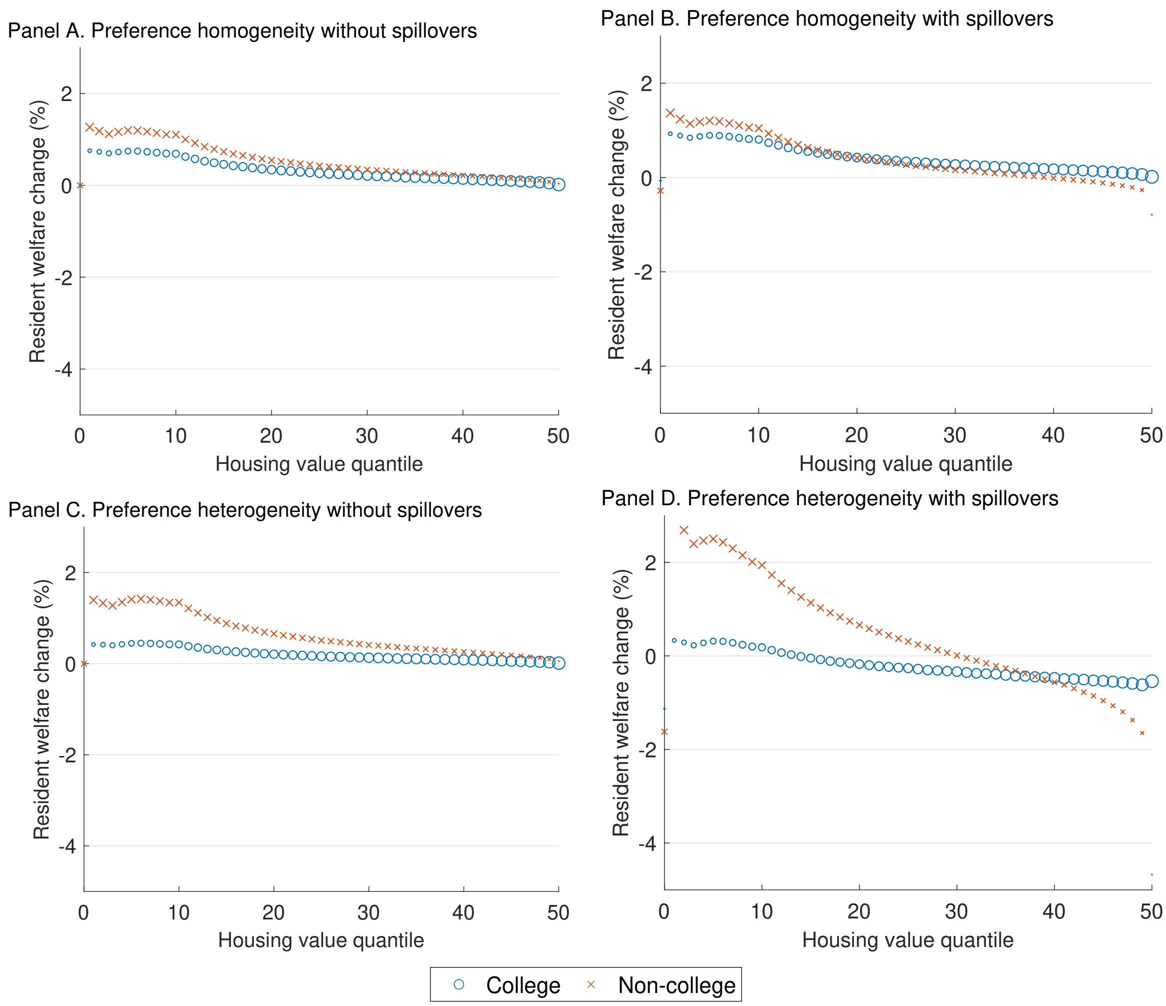

Figure 3. Effect of 20th percentile construction on resident welfare

Notes: The size of each marker is proportional to the number of housing units in each housing value and education bin.

(Panels A and C), no one suffers from construction. The strong distributional effect of low-quality construction hinges on the *combination* of urban spillovers and preference heterogeneity.

5.3 Extensions

The remaining panels in Table 3 explore welfare effects of construction under different parameters and assumptions. In Panel B, $\gamma_a = 0$ so that amenities no longer depend on the composition of city households. Luxury construction still makes poor non-college households better off, but by only half as much as before. Low-quality construction now makes college households better off. In fact, both types of construction make all four groups of college households better off than in Panel A. The endogeneity of amenities amplifies the benefit of luxury housing for the poor and is crucial in

generating the negative welfare effect on college households. In contrast, low-quality construction continues to improve poor non-college welfare more than high-quality construction. Therefore, this feature of the baseline model does not depend on endogenous amenities.

The next panel reports results from an extension in which endogenous amenities differ across housing bins. This extension captures the empirical dependence of amenities on the composition of households in one's neighborhood (Bayer, Ferreira and McMillan, 2007). Specifically, amenities for households choosing q_j are now

$$a_j = \widetilde{a}\left(\frac{N_H}{N_L}\right)^{\gamma_a-\gamma_{al}}\left(\frac{N_{H,j}}{N_{L,j}}\right)^{\gamma_{al}}, \tag{24}$$

where $N_{e,j}$ is the measure of households of education e choosing q_j. This specification re-allocates some of the metro amenity channel to each housing sub-market, which I think of as a neighborhood. Specifically, an increase in $N_{H,j}/N_{L,j}$ that is uniform across all j raises amenities the same amount under this new specification as under (2), the baseline amenity specification. I use estimates from Bayer, Ferreira and McMillan (2007) on the willingness-to-pay for an increase in college households in one's neighborhood to get a value of $\gamma_{al} = 0.028$.[20] I then re-estimate θ using this new amenity specification, as the trade-off between adjacent housing bins must now reflect the different college shares in each bin. The new θ is very close to the baseline value. I perform comparative statics with the new θ for internal consistency.

Panel C displays the results. Unsurprisingly, poor college households are worse off than under the baseline, as the migration of poor non-college households into their bins has a negative effect on local amenities. More notable is that rich households are also worse off than under the baseline, even though amenities are more localized than before. The in-migration of poor non-college households is stronger because they more easily push out poor college households. This effect is large enough to increase the overall non-college population. Because metro amenities still depend on this population (although less than in the baseline), the welfare of rich college households decreases. Rich non-college households suffer due to wage competition. The negative effect of construction on rich households is not only robust to this localization of amenities, but gets stronger.

Panel D considers another alternative to (2): $a = \widetilde{a}N_H^{\gamma_a}$. The non-college population no longer

[20] According to their Table 8, a 10 percentage point increase in the neighborhood college share increases the average willingness-to-pay of households to live in the neighborhood by \$10.46 per month. Using their summary statistics on mean income, housing costs, and college neighborhood shares, as well as my values for $\beta_{a,e}/\beta_{c,e}$, I calculate γ_{al}.

affects amenities. Both types of construction now make all groups better off, including the rentiers. However, poor non-college households benefit ten times less than they do under the baseline. They must compete more with rich and college households, whose migration is stronger in this scenario. The negative effect of non-college households on amenities helps the poorest of them benefit from construction.

I next add to the model local supply of services by non-college households. These jobs have become an increasingly important part of the U.S. labor market (Autor and Dorn, 2013). I follow Autor and Dorn (2013) in modeling the production of services using a linear production function of non-college labor. I allow services to enter the utility function, splitting the log of utility from $\beta_{c,e} \log c$ into $\beta_{cs,e} \log c_s + \beta_{cm,e} \log c_m$, with $\beta_{cs,e} + \beta_{cm,e} = \beta_{c,e}$. Here, c_s denotes consumption of services, and c_m denotes consumption of manufactured goods, which firms in the baseline model produce. Households consume only services that other households in their metro supply. Equilibrium pins down the local price of services, the wage paid to labor from L households, and the allocation of this labor across the service and manufacturing sectors in each metropolitan area.

To calibrate this extended model, I set $\beta_{cs,L} = 0$, meaning that only college households consume low-skilled services. This assumption gives the greatest chance of attenuating the negative effect of construction on college households. To estimate $\beta_{cs,H}$, I calculate the total wage and salary earnings of non-college workers in service occupations in the data. The ratio of this total to the aggregate income less housing costs of college households equals $\beta_{cs,H}$, which I calculate as 8.0%. The results appear in Panel E. As expected, construction harms college households less than in the baseline. The in-migration of poor non-college households lowers the price of services, which makes college households better off. However, the aggregate effect of low-quality construction still makes rich college households worse off.

Panel F reports results in which I increase cross-city mobility by scaling all β coefficients by 1.5 and then dividing welfare effects by 1.5 as well. This procedure is equivalent to decreasing $\beta_{\epsilon,e}$, which is normalized to 1, by 50%. The main results remain but get much stronger. Low-quality construction now makes all college groups worse off. With stronger mobility, poor non-college households move into the metropolitan area more readily, which makes others suffer under the mechanisms described above.

Finally, Panel G turns off cross-metro migration. As Proposition 2 states, construction now makes all households better off, and all households prefer high-quality construction to low-quality construction. Furthermore, all households are better off than they are in the baseline. For instance,

welfare of poor non-college households improves by more than 7%. Migration significantly attenuates the welfare gains from construction, turning them negative in some cases.

5.4 Addressing the affordability crisis

House prices have grown so much in certain cities because productivity has risen while construction has been low (Hsieh and Moretti, 2019). I study a productivity shock that raises house prices, asking how effective different policies are in addressing welfare losses.

5.4.1 Effects of a skill-biased productivity shock

The exogenous productivity shifters for L and H labor are $\delta_{A,L}$ and $\delta_{A,H}$. Diamond (2016) estimates that, for the period between 1980 and 2000, the values of these shifters are -0.314 and 0.075 for the Boston metropolitan area (see Table A.6 of her online appendix). According to these estimates, the Boston production function changed to make high-education labor more productive and low-education labor less. I annualize these numbers by setting $\delta_{A,L} = -0.0157$ and $\delta_{A,H} = 0.0038$. During this period, the real median house price in Boston doubled by growing at an annual rate of 3.2%.[21]

Figure 4 plots the house price and population changes that occur in response to this shock. House prices sharply increase relative to their initial value, as Panel A shows. The largest gains occur in the lowest quality bins. House price growth during the 2000–2005 U.S. boom also followed this pattern (Guerrieri, Hartley and Hurst, 2013; Landvoigt, Piazzesi and Schneider, 2015). The remaining panels clarify why this pattern holds in the model.

Panel C shows the net measure of households moving into each bin from outside the city. The high bins accept an influx of college households, while non-college households move out of the low bins. Therefore, the shock generates an affordability crisis: a sharp rise in prices accompanying an out-migration of poor households without a college degree. The combined effect, shown in gray, is in-migration to high bins and out-migration from low bins. Given the out-migration from low bins, why do house prices increase most sharply there?

The answer comes from changes in demand between adjacent bins. Panel D plots the net switching into each bin from households already in the city before the shock. Because the housing market clears, the combined switching effect—appearing in gray—exactly cancels the combined migration effect in Panel C. In particular, college households exert demand on the low bins by

[21] Sources: U.S. Census sample of owner-occupied properties in the Boston metropolitan area, and CPI from the BLS.

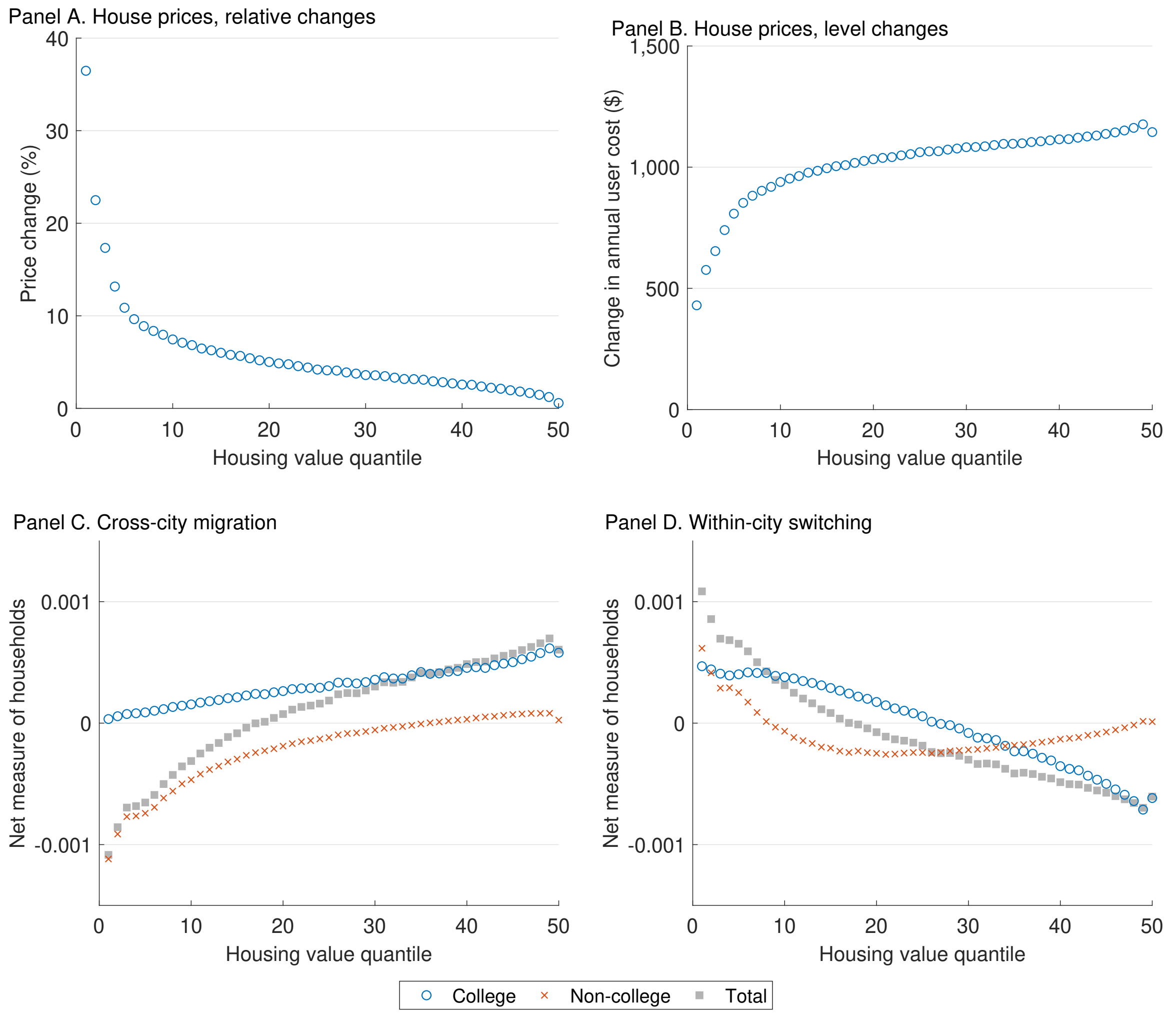

FIGURE 4. EFFECT OF SKILL-BIASED PRODUCTIVITY SHOCK ON HOUSING SUBMARKETS

Notes: Panels C and D plot the change in demand for each house type from outside and inside the city, respectively.

switching down into them. To induce this switching, the slope of house prices with respect to quality cannot increase too much, as switching households are using this slope to decide which bin to choose. Consistent with this idea, the growth in house price levels across bins is relatively constant, as Panel B shows. Only between the lowest bins does this slope increase much. The net demand to switch into these bins is very strong so the slope must rise more there.

The mechanism in Figure 4 resembles the endogenous gentrification that Guerrieri, Hartley and Hurst (2013) study. In both cases, richer households drive up low-quality house prices by pushing out lower income households. The importance of switching between adjacent housing qualities is similar to phenomenon that Landvoigt, Piazzesi and Schneider (2015) describe. However, they generate the pattern in Panel A with a housing demand shock that is strongest for poor households. In contrast, I generate the same pattern with a shock that is weakest for the poor.

5.4.2 Policy effectiveness

Table 4 analyzes the effectiveness of various policies in addressing the migratory responses to this shock. Panel A reports the population changes of non-college households in each income quartile, Panel B gives the same information for college households. Panel C summarizes house price and quantity changes from each policy.

Results in column (1) correspond to the case in Figure 4 when there is no policy response. Consistent with Panel C of that figure, the poor non-college population declines while the college population increases at all income levels. House prices rise by 5.72% on average, with a median increase of 4.11%. There is no change in the housing stock (by assumption).

Column (2) presents the case where construction matches the data for 2015. This level of construction does curtail the out-migration of poor non-college households, but it does not come close to reducing it to zero. In the lowest income quartile, the population change is −2.45% versus −3.77% in the baseline. At the same time, this construction lowers the welfare of rich college households, curtailing their in-migration from 3.52% to 3.41%. This change is small, which may explain why local governments enacted this policy in reality, as households with more education are more active in the political process (Galston, 2001; Milligan, Moretti and Oreopolous, 2004). House price growth falls to 3.80% and 2.74% for the average and median, respectively. These values are close to the empirical house price growth of 3.2% in response to the shock.

How much construction is necessary to stem the out-migration of poor non-college households? Columns (3)–(5) answer this question in different ways. I search for a set of construction

TABLE 4

Effects of skill-biased productivity shock under different policies (%)

		Consruction policies				
	None (1)	2015 (2)	Unit-minimizing optimum (3)	Cost-minimizing optimum (4)	2015 quality optimum (5)	Rent control (6)
Panel A. Non-college population change						
Income quartile						
1	−3.77	−2.45	0.08	0.07	0.12	−4.21
2	−1.42	−0.85	0.02	0.02	0.23	−1.79
3	0.29	0.35	0.02	0.02	0.47	0.53
4	1.80	1.43	0.17	0.05	0.75	0.90
Panel B. College population change						
Income quartile						
1	2.28	2.51	2.87	2.92	2.96	−1.22
2	2.68	2.77	2.84	2.90	2.95	−1.50
3	3.16	3.11	2.83	2.89	3.01	2.91
4	3.52	3.41	2.94	2.89	3.19	3.96
Panel C. Housing market changes						
Average price	5.72	3.80	0.45	0.41	0.23	1.77
Median price	4.11	2.74	0.41	0.37	0.18	1.96
Rentier welfare	3.14	2.92	2.05	1.81	2.49	1.49
Housing units	0.00	0.55	1.40	1.41	1.61	0.00
Construction cost	0.00	0.84	1.80	1.53	2.47	0.00
Construction quality	–	0	−16	−29	0	–

Notes: “Construction cost” gives the total value of new housing as a share of the value of existing housing, using pre-shock prices. “Construction quality” gives the average value of new housing relative to the average value of new housing in 2015, using pre-shock prices.

amounts, $\delta_{h,j}$, so that the combined effect of the productivity shock and construction makes no household in the city worse off. Such an outcome represents a Pareto improvement over the baseline. In particular, poor non-college households no longer leave the city.

Column (3) finds the minimal number of new housing units necessary to achieve this objective. As Panel C reports, this minimal construction expands the housing stock by 1.40%, about triple the amount in the data. Furthermore, the quality of new housing is 16% lower in column (3) than column (2). I compute construction quality as the average price of new housing in the pre-shock equilibrium. The population of all eight household groups increases, which must hold because the shock and construction response makes no one worse off.

Should governments in the Boston metropolitan area change permitting rules to implement the construction in column (3)? Doing so would make poor non-college households better off, but would harm many households relative to the status quo in column (2). In particular, the welfare of households in the upper half of the income distribution would fall. Switching from actual construction to that in column (3) is not a Pareto improvement.

In column (4), I instead minimize the value of new housing using the pre-shock equilibrium prices. This minimand addresses the possibility that it is cheaper to supply lower quality housing. The "construction cost" column reports this sum as a fraction of the initial value of the housing stock. The construction in column (4) involves a cost of only 1.53% of the housing stock, which is lower than the 1.80% cost in column (3). The construction tilts even more toward lower quality housing, with a quality that is 29% lower than the 2015 amount. While cheaper to implement, this policy further lowers the welfare of households in the top income quartile.

Column (5) checks the usefulness of changing the quality of new construction in columns (3) and (4). It requires that the quality distribution match that in column (2), while solving for the total quantity of construction that eliminates out-migration. The resulting optimum expands the housing stock by 1.61%, which is only slightly more than in columns (3) and (4). The construction cost, at 2.47% of the housing stock, is much higher than the 1.53% number from column (4). The welfare losses relative to column (2) are much smaller. Therefore, while keeping the quality distribution involves higher construction costs, many households prefer that policy over one that involves building lower-quality units.

Finally, column (6) reports an analysis of rent control. I limit all price changes to 2%. For housing bins where demand exceeds supply, I ration households randomly within each bin. To solve the system, I impose 2% growth of prices at certain indices and solve for the rationing at

those indices and the price responses at other indices. Using iteration, I find a set of indices where demand exceeds supply at the rationed indices and price growth is less than 2% at the un-rationed indices. The rationed indices are the lowest 22 occupied housing bins.[22]

Consistent with this price cap, price growth is much smaller in column (6) than under the baseline without construction or the 2015 construction policy. Both average and median price growth are between 1.5% and 2%. However, the policy fails to limit the out-migration of the poor. In fact, it accelerates it—the out-migrations in the bottom two quartiles are larger in column (6) than in column (1). By capping price growth, rent control limits the growth in the slope of prices with respect to quality at the low end of the distribution. As a result, switching down the quality distribution is stronger. Consistent with this mechanism, the populations of third quartile non-college households and top quartile college households grow under rent control. These are the households exerting the most downward pressure through switching in Panel D of Figure 4. Rent control makes these households better off even though it does not directly limit the growth of their house prices. Diamond, McQuade and Qian (2018) find empirically that rent control attracted higher income residents to San Francisco, which is consistent with these results.

6 Conclusion

This paper develops and estimates a structural model of the Boston metropolitan area housing market. The model allows me to calculate the welfare effects of varying types of construction for different households. Luxury housing makes the poor better off: 100 new luxury units prevent the outmigration of 47 poor households. This trickle-down effect is strong, however, only because metro amenities deteriorate when households without a college degree move into the metropolitan area. Without this endogenous amenity channel, the benefit to the poor from luxury construction is two to ten times smaller.

Poor households benefit more from low- rather than high-quality construction. To the extent that governments aim to help the poor, this result may explain the prevalence of subsidies to low-quality housing construction. Although such construction helps the poor, it makes the rich worse off when metro amenities depend negatively on the non-college population. The quality of construction emerges from my analysis as a point of contention between different groups of metro

[22] In the data, 71% of the housing in those bins are rentals, while only 11% of the housing in the non-controlled bins are rentals. Therefore, the units subject to rent control in this model exercise are highly correlated with the actual rental housing stock.

residents.

My analysis considers only positive construction amounts, but the model applies equally to negative construction, such as teardowns. For instance, the effect of building one luxury unit and demolishing one low-quality unit equals the welfare effect of the former minus that of the latter. Poor non-college households would suffer from this policy because they benefit more from low-quality construction. An important caveat, then, is that luxury construction might not benefit the poor if it involves tearing down low-quality housing. Quantifying this aspect of the housing production process is an important task for future research.

I evaluate policy responses to an affordability crisis I generate within my model. Rent control exacerbates the crisis by driving even more poor households out of the metropolitan area. In contrast, tripling the construction intensity in Boston eliminates the outmigration of the poor by slowing down house price growth. By granting relatively more permits for low-quality housing, Boston achieves this objective with slightly fewer total permits. In either case, tripling construction makes rich households worse off and lowers the welfare of rentiers endowed with the housing stock. Construction is a solution to the affordability crisis with winners and losers.

A Appendix

A.1 Proof of Lemma 1

The denominator in (6) is well-defined for all e and z. For a contradiction, suppose not. Then for some e and z, $v_{e,t}(z) = 0$ for all t, implying that $w_{e,t}z \leq \min_{j\in\mathcal{J}_t} p_{j,t}$ for all t. There are households of education e with $z' < z$ because the support of each $\widetilde{n}_e$ is $(0,\infty)$. For such households, $w_{e,t}z' < w_{e,t}z \leq \min_{j\in\mathcal{J}_t} p_{j,t}$ for all t, contradicting the budget constraint.

To prove (6), consider households of education e with endowment z. No such household chooses t if $v_{e,t}(z)\exp(\epsilon_t) < v_{e,t'}(z)\exp(\epsilon_{t'})$ for any $t' \neq t$. If $v_{e,t}(z) = 0$, then this inequality holds because $v_{e,t'}(z) > 0$ for some t', as proved above. In this case, $n_{e,t}(z) = 0$, validating (6). If $v_{e,t}(z) > 0$, then the measure of such households choosing t equals the measure for whom $\epsilon_t - \epsilon_{t'} > \log v_{e,t'}(z) - \log v_{e,t}(z)$ for all $t' \neq t$ such that $v_{e,t'}(z) > 0$. Train (2009), on pages 36 and 74–75, shows that the probability that independent draws from a Gumbel distribution satisfy this inequality is $v_{e,t}(z)/\sum_{t'|v_{e,t'}(z)>0} v_{e,t'}(z)$. Because the total measure of e and z households is $\widetilde{n}_e(z)$, (6) follows.

A.2 Proof of Proposition 1

In equilibrium, the housing market clears, so the measure of households must be no greater than the total measure of housing rentiers may sell: $\sum_{j\in\mathcal{J}_t} h_{j,t} \geq N_t$. It follows that $j_{0,t}$ exists.

I next prove the statements about housing demand, which I denote $x_{j,t}$. Because rentiers optimize, they sell $h_{j,t}$ units of housing of quality $q_{j,t}$ when $p_{j,t} > 0$, they sell any amount of such housing when $p_{j,t} = 0$, and $p_{j,t} < 0$ is impossible. Consider $j < j_{0,t}$ such that $x_{j,t} > 0$. Because $\sum_{j'=j_{0,t}}^{J_t} h_{j',t} \geq N$, there exists $j' \geq j_{0,t}$ such that $x_{j',t} < h_{j',t}$. It follows that $p_{j',t} = 0$, which implies that households choosing $q_{j,t}$ are failing to optimize because $q_{j',t} > q_{j,t}$ and $0 = p_{j',t} \leq p_{j,t}$. This contradiction implies that no such j exist, so $x_{j,t} = 0$ when $j < j_{0,t}$. Similarly, consider j such that $j > j_{0,t}$ and $x_{j,t} < h_{j,t}$. Then $p_{j,t} = 0$. Furthermore, because $\sum_{j'=j_{0,t}+1}^{J_t} h_{j',t} < N_t$, there exists $j' \leq j_{0,t}$ such that $x_{j',t} > 0$. Households choosing $q_{j',t}$ cannot be optimizing, as $p_{j,t} \leq p_{j',t}$ and $q_{j,t} > q_{j',t}$. This contradiction implies that there are no such j, proving that $x_{j,t} = h_{j,t}$ when $j > j_{0,t}$.

To prove the statements about house prices, first consider j such that $\sum_{j'=j}^{J_t} h_{j',t} > N_t$ and $h_{j,t} > 0$. By the definition of $j_{0,t}$, $j \leq j_{0,t}$. If $j < j_{0,t}$, then $0 = x_{j,t} < h_{j,t}$, so $p_{j,t} = 0$. If $j = j_{0,t}$, then $x_{j,t} = N_t - \sum_{j'=j_{0,t}+1}^{J_t} h_{j',t} < h_{j_{0,t},t} = h_{j,t}$, again implying $p_{j,t} = 0$. Next, consider j, j' such that $h_{j,t}, h_{j',t} > 0$ and $j_{0,t} \leq j < j'$. Because $x_{j,t} = h_{j,t} > 0$, there exist households choosing $q_{j,t}$. If $p_{j,t} \geq p_{j',t}$, these households are failing to optimize because $q_{j',t} > q_{j,t}$. It follows that $p_{j,t} < p_{j',t}$.

I prove the final statement by contradiction. Consider two households with education e and respective labor endowments $z < z'$ and quality choices $q_{j,t} > q_{j',t}$. Optimality for each household implies the inequalities $(w_{e,t}z - p_{j,t})^{\beta_{c,e}} q_{j,t}^{\beta_{q,e}} \geq (w_{e,t}z - p_{j',t})^{\beta_{c,e}} q_{j',t}^{\beta_{q,e}}$ and $(w_{e,t}z' - p_{j',t})^{\beta_{c,e}} q_{j',t}^{\beta_{q,e}} \geq (w_{e,t}z' - p_{j,t})^{\beta_{c,e}} q_{j,t}^{\beta_{q,e}}$. If $w_{e,t}z = p_{j,t}$, then the first optimality inequality is violated because $p_{j',t} < p_{j,t}$, which implies that $w_{e,t}z - p_{j',t} > w_{e,t}z - p_{j,t} = 0$. By the budget constraint, $w_{e,t}z \geq p_{j,t}$, so $w_{e,t}z > p_{j,t}$. Because $z < z'$, $w_{e,t}z < w_{e,t}z'$, so $w_{e,t}z' - p_{j,t} > w_{e,t}z - p_{j,t} > 0$. Rearranging the optimality inequalities therefore produces

$$\frac{w_{e,t}z - p_{j',t}}{w_{e,t}z - p_{j,t}} \leq \left(\frac{q_{j,t}}{q_{j',t}}\right)^{\frac{\beta_{q,e}}{\beta_{c,e}}} \leq \frac{w_{e,t}z' - p_{j',t}}{w_{e,t}z' - p_{j,t}}. \tag{A1}$$

Cross-multiplying the outer terms yields the contradiction $(p_{j,t} - p_{j',t})(z' - z) \leq 0$.

A.3 Proof of Proposition 2

Solving for $z_{e,j}$ using (8) gives

$$z_{e,j} = \frac{q_j^{\frac{\beta_{q,e}}{\beta_{c,e}}} p_j - q_{j-1}^{\frac{\beta_{q,e}}{\beta_{c,e}}} p_{j-1}}{\left(q_j^{\frac{\beta_{q,e}}{\beta_{c,e}}} - q_{j-1}^{\frac{\beta_{q,e}}{\beta_{c,e}}}\right) w_e} \tag{A2}$$

for $j > 0$. Cumulatively summing (9) yields

$$\sum_{j'=j}^{J} h_j = \sum_{e \in \{L,H\}} \int_{z_{e,j}}^{\infty} n_{e,t}(z) dz. \tag{A3}$$

Substituting (A2) into (A3), differentiating, and solving for ∂p_j produce

$$\partial p_j \propto \chi_j \partial p_{j-1} - \sum_{j'=j}^{J} \delta_{h,j'}, \tag{A4}$$

where $\chi_j > 0$. By way of this equation, induction on j' proves that $\partial p_j / \partial \delta_{h,j'}$ is negative for $j, j' \in \{j_0 + 1, ..., J\}$, strictly decreases over $j' \in \{j_0 + 1, ..., j\}$, and remains constant over $j' \in \{j, ..., J\}$. The claim is immediate for $j' = j_0 + 1$ because $\partial p_{j_0} = 0$. The inductive step follows immediately as well.

Because $\partial \log n_e(z) = 0$ by assumption, $\partial \log a = \partial \log w_e = 0$, as metro population and labor endowment totals remain constant. Therefore, (11) indicates that $\partial \log v_e(z) = -\beta_{c,e} \partial p_j / (y_e(z) - p_j)$ when $z \in (z_{e,j}, z_{e,j+1})$. This equation holds for $z = z_{e,j}$ because $\partial \log z_{e,j} \leq 0$, which comes from differentiating (A3) and applying the assumption that $\delta_{h,j'} \geq 0$ for $j' > j_0$. Proposition 2 then follows from the earlier statements proved about ∂p_j.

A.4 Definition of Θ

I show that θ uniquely determines $y_{e,j}$ if

$$\sum_{j'=j}^{50} \frac{\widehat{h}_j}{N} \leq \sum_{e \in \{L,H\}} \frac{N_e}{N} \int_{\widehat{p}_j}^{\infty} f_e(y) dy \tag{A5}$$

for each j. Equating the two solutions to (8) for q_j / q_{j-1} and then re-arranging terms gives

$$y_{H,j} = \widehat{p}_{j-1} + \frac{\widehat{p}_j - \widehat{p}_{j-1}}{1 - \left(1 - \frac{\widehat{p}_j - \widehat{p}_{j-1}}{y_{L,j} - \widehat{p}_{j-1}}\right)^{\zeta}} \tag{A6}$$

for each $j > 0$. The term in parentheses strictly increases from zero to one in $y_{L,j} \in [\widehat{p}_j, \infty)$. Therefore, as a function of $y_{L,j}$, $y_{H,j}$ strictly increases from $\widehat{p}_j$ to ∞ for $y_{L,j} \in [\widehat{p}_j, \infty)$. Dividing (9) by N, changing variables from z to y, and taking cumulative sums yields

$$\sum_{j'=j}^{50} \frac{\widehat{h}_j}{N} = \frac{N_L}{N} \int_{y_{L,j}}^{\infty} f_L(y) dy + \frac{N_H}{N} \int_{y_{H,j}(y_{L,j})}^{\infty} f_H(y) dy \tag{A7}$$

for each $j > 0$. The right side is defined for $y_{L,j} \geq \widehat{p_j}$ and strictly increases in $y_{L,j}$. When (A5) holds, the intermediate value theorem implies the existence of a unique solution for $y_{L,j}$, which then delivers a unique $y_{H,j}$.

If $\zeta = 1$, then $y_{H,j} = y_{L,j}$. The solutions to (A7) then strictly increase in j because the left side does. A valid definition of Θ, then, is the set of θ for which (A5) holds and ζ is sufficiently close to 1. I verify numerically that Θ is non-empty.

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I Equilibrium proofs

I.1 Proof of Lemma 2

First, I prove technical conditions about equilibrium $n_{e,t}$ that are necessary for the proof.

Lemma IA1. *In equilibrium, $n_{e,t}$ is an atomless distribution with convex support. If $\sum_{j\in\mathcal{J}_t} h_{j,t} > N_t$, then the greatest lower bound of the support is zero.*

Proof. The denominator of (6) is positive (see proof in Appendix A.1), and $v_{e,t}(z)$ is zero or positive. Therefore, $n_{e,t}$ has a mass point only if $\widetilde{n}_e$ does, which Section 2.2 rules out by assumption. For convexity, consider $z' < z''$ such that $n_{e,t}(z'), n_{e,t}(z'') > 0$. By (6), $\widetilde{n}_e(z'), \widetilde{n}_e(z''), v_{e,t}(z') > 0$. If $z \in (z', z'')$, then $\widetilde{n}_e(z) > 0$ because Section 2.2 assumes that $\widetilde{n}_e$ is convex. Furthermore, $v_{e,t}(z) > v_{e,t}(z')$ because $w_{e,t}z > w_{e,t}z'$, meaning that a household with endowment z can choose the same housing bundle and greater non-housing consumption as a household with endowment z'. It follows that $n_{e,t}(z) > 0$, implying that the support of $n_{e,t}$ is convex.

For the final statement, $z^{lb}_{e,t} = \inf\{z \mid n_{e,t}(z) > 0\}$ exists because $N_{e,t} > 0$ due to the assumption in Section 2.2. By another assumption in that section, $\inf\{z \mid \widetilde{n}_e(z) > 0\} = 0$. By (6), $n_{e,t}(z) > 0$ if $v_{e,t}(z) > 0$ and $\widetilde{n}_e(z) > 0$. It follows that $z^{lb}_{e,t} = \inf\{z \mid v_{e,t}(z) > 0\} = w^{-1}_{e,t}\min_{j\in\mathcal{J}_t} p_{j,t}$, which equals zero because $p_{j_{0,t},t} = 0$ by Proposition 1 when $\sum_{j\in\mathcal{J}_t} h_{j,t} > N_t$. □

Next, I derive housing demand as a function of price vectors $p_t = (p_{0,t}, ..., p_{J_t,t})$ in the set $\mathcal{P}_t = \{p_t \mid 0 \le p_{0,t} \le ... \le p_{J_t,t}\}$. For $j, j' \in \mathcal{J}_t$ distinct, a household of education e and endowment z prefers and can afford $q_{j,t}$ over $q_{j',t}$ only if two conditions hold. First, $w_{e,t}z \ge p_{j,t}$. Second, either $w_{e,t}z < p_{j',t}$ or both $w_{e,t}z \ge p_{j',t}$ and

$$\left(w_{e,t}z - p_{j,t}\right)^{\beta_{c,e}} q_{j,t}^{\beta_{q,e}} \ge \left(w_{e,t}z - p_{j',t}\right)^{\beta_{c,e}} q_{j',t}^{\beta_{q,e}}, \tag{IA1}$$

with strict preference in the case of strict inequality. The point where (IA1) reaches equality is

$$z_{e,j,j',t}(p_t) = \frac{q_{j,t}^{\frac{\beta_{q,e}}{\beta_{c,e}}} p_{j,t} - q_{j',t}^{\frac{\beta_{q,e}}{\beta_{c,e}}} p_{j',t}}{\left(q_{j,t}^{\frac{\beta_{q,e}}{\beta_{c,e}}} - q_{j',t}^{\frac{\beta_{q,e}}{\beta_{c,e}}}\right) w_{e,t}}. \tag{IA2}$$

If $j > j'$, then (IA1) holds if and only if $z \ge z_{e,j,j',t}(p_t)$. Furthermore, because $p_{j',t} \le p_{j,t}$, $z_{e,j,j',t}(p_t) \ge w^{-1}_{e,t}p_{j,t} \ge w^{-1}_{e,t}p_{j',t}$. Therefore, if $j > j'$, the household prefers and can afford $q_{j,t}$ over $q_{j',t}$ only if $z_{e,j,j',t}(p_t)$, with strict preference when $z > z_{e,j,j',t}(p_t)$. By symmetry, when $j < j'$, these preference relations hold when $z \le z_{e,j',j,t}(p_t) = z_{e,j,j',t}(p_t)$, strictly with strict inequality. Let $z^{min}_{e,j,t}(p_t) = \max\{z_{e,j,j',t}(p_t) \mid j > j'\}$ for $j > 0$ and zero for $j = 0$. Let $z^{max}_{e,j,t}(p_t) = \min\{z_{e,j,j',t}(p_t) \mid j < j'\}$ for $j < J_t$ and infinity for $j = J_t$. Demand for $q_{j,t}$ equals

$$x_{j,t}(p_t) = \sum_{e\in\{L,H\}} x_{e,j,t}(p_t), \tag{IA3}$$

where

$$x_{e,j,t}(p_t) = \int_{z^{min}_{e,j,t}(p_t)}^{z^{max}_{e,j,t}(p_t)} n_{e,t}(z)dz. \tag{IA4}$$

The following lemma collects useful facts about demand.

Lemma IA2. *For each $j \in \mathcal{J}_t$, $x_{j,t}$ is continuous. Let $p_t, p_t' \in \mathcal{P}_t$. If $p_{j,t}' \geq p_{j,t}$ and $p_{j',t}' \leq p_{j',t}$ for $j' \neq j$, then $x_{j,t}(p_t') \leq x_{j,t}(p_t)$, with strict inequality if $p_{j,t}' < p_{j,t}$ and $x_{j,t}(p_t) > 0$. Finally,*

$$\overline{p}_t = \inf\left\{ \widetilde{p} \geq 0 \;\middle|\; \sum_{e \in \{L,H\}} \int_{w_{e,t}^{-1}\widetilde{p}}^{\infty} n_{e,t}(z)dz < h_{J_t,t} \right\} \tag{IA5}$$

exists, and if $p_{J_t+1,t} = \overline{p}_t$, then $x_{j,t}(p_t) \leq h_{j,t}$ if $p_{j,t} = p_{j+1,t}$.

Proof. I prove the first two sentences for each $x_{e,j,t}$; they then hold for $x_{j,t}$ immediately. Both $z_{e,j,t}^{min}$ and $z_{e,j,t}^{max}$ (the latter only for $j < J_t$), are continuous functions of p_t. Because $n_{e,t}$ is atomless by Lemma IA1, and because the composition of continuous functions is continuous, the fundamental theorem of calculus implies that $x_{e,j,t}$ is continuous.

From (IA2), $z_{e,j,j',t}(p_t)$ strictly increases in $p_{j,t}$ and decreases in $p_{j',t}$ if $j > j'$ and strictly decreases in $p_{j,t}$ and increases in $p_{j',t}$ if $j < j'$. Therefore, $z_{e,j,t}^{min}(p_t') \geq z_{e,j,t}^{min}(p_t)$ and $z_{e,j,t}^{max}(p_t') \leq z_{e,j,t}^{max}(p_t)$. It follows that $x_{e,j,t}(p_t') \leq x_{e,j,t}(p_t)$. Now suppose that $x_{e,j,t}(p_t) > 0$ and that $p_{j,t}' < p_{j,t}$. We claim that $0 \leq z_{e,j,t}^{min}(p_t) < \sup\{z \mid n_{e,t}(z) > 0\}$. The first inequality follows because for $j > j'$, $z_{e,j,j',t}(p_t) \geq w_{e,t}^{-1} p_{j,t} \geq 0$. If the second inequality fails, then $[z_{e,j,t}^{min}(p_t), z_{e,j,t}^{max}(p_t)] \cap \{z \mid n_{e,t}(z) > 0\}$ is either $\emptyset$ or $\{\sup\{z \mid n_{e,t}(z) > 0\}\}$, both of which have measure zero under $n_{e,t}$, contradicting $x_{e,j,t}(p_t) > 0$. Then $\inf\{z \mid n_{e,t}(z) > 0\} = 0 \leq z_{e,j,t}^{min}(p_t) < \sup\{z \mid n_{e,t}(z) > 0\}$. Therefore, because $z_{e,j,t}^{min}(p_t) < z_{e,j,t}^{min}(p_t')$, the convexity of the support of $n_{e,t}$ (from Lemma IA1) implies that $x_{e,j,t}(p_t') < x_{e,j,t}(p_t)$, as desired.

Let $j < J_t$. If $p_{j,t} = p_{j+1,t}$, then $z_{e,j,j+1,t}(p_t) = w_{e,t}^{-1} p_{j,t}$. A household of education e can afford $q_{j,t}$ only if $z \geq w_{e,t}^{-1} p_{j,t}$ and prefers $q_{j+1,t}$ over $q_{j,t}$ only if $z \leq z_{e,j,j+1,t}(p_t) = w_{e,t}^{-1} p_{j,t}$. Therefore, only those with $z = w_{e,t}^{-1} p_{j,t}$ may choose $q_{j,t}$, and the measure of such households equals zero, so $x_{j,t}(p_t) = 0 \leq h_{j,t}$. I turn to the case $j = J_t$. The limit of the integral in (IA5) as $\widetilde{p} \to \infty$ equals zero, so $\overline{p}_t$ exists. A household chooses $q_{J_t,t}$ only if $z \geq w_{e,t} p_{J_t,t}$, so the summation in (IA5) provides an upper bound on $x_{J_t,t}(p_t)$ when $\widetilde{p} = p_{j,t}$. Therefore, $x_{J_t,t}(p_t) \leq h_{J_t,t}$ when $p_{J_t,t} \leq \overline{p}_t = p_{J_t+1,t}$. □

I prove local equilibrium existence by constructing a sequence $p_t^i \in \mathcal{P}_t$ whose limit provides a local equilibrium. For all $i \geq 0$, set $p_{J_t+1,t} = \overline{p}_t$. Set $p_t^0 = 0$. If $i + j \equiv 0 \pmod{J_t + 1}$ and $x_{j,t}(p_t^i) > h_{j,t}$, then $p_{j,t}^i$ is the unique solution to $h_{j,t} = x_{j,t}(p_{j,t}^i; p_{-j,t}^{i-1})$. Uniqueness and existence follow from the intermediate value theorem and Lemma IA2 because $x_{j,t}(p_{j,t}^{i-1}; p_{-j,t}^{i-1}) > h_{j,t} \geq x_{j,t}(p_{j+1,t}^{i-1}; p_{-j,t}^{i-1})$ and $h_{j,t} > 0$. Otherwise, set $p_{j,t}^i = p_{j,t}^{i-1}$. At each step, $p_{j,t}^i \in [p_{j,t}^{i-1}, p_{j+1,t}^{i-1}]$, so $p^i \in \mathcal{P}_t$ for all $i \geq 0$. Furthermore, $p_{j,t}^i$ weakly increases in i for each $j \in \{0, \ldots, J_t\}$, and $p_{j,t}^i \leq p_{J_t+1,t}^i = \overline{p}$ for all such j and all $i \geq 0$. By the monotone converge theorem, p_t^i converges. I denote the limit p_t^*.

Consider j such that $p_{j,t}^* > 0$. Let i_j be the first i such that $p_{j,t}^i > 0$. I claim that $x_{j,t}(p_t^i) \geq h_{j,t}$ for $i \geq i_j$. When $i = i_j$, $p_{j,t}^i > p_{j,t}^{i-1} = 0$, so $x_{j,t}(p_t^i) = h_{j,t}$. I proceed by induction. For each $i > i_j$, $p_{j',t}^i \geq p_{j',t}^{i-1}$ for $j' \neq j$. If $p_{j,t}^i = p_{j,t}^{i-1}$, then by Lemma IA2, $x_{j,t}(p_t^i) \geq x_{j,t}(p_t^{i-1}) \geq h_{j,t}$. If $p_{j,t}^i > p_{j,t}^{i-1}$, then $x_{j,t}(p_t^i) = h_{j,t}$. Therefore, $x_{j,t}(p_t^i) \geq h_{j,t}$ for all $i \geq i_j$, as claimed. It follows that $x_{j,t}(p_t^i) = h_{j,t}$ for all $i \equiv -j \pmod{J_t + 1}$. Because $x_{j,t}$ is continuous by Lemma IA2, $x_{j,t}(p_t^i)$ converges because p_t^i does. This limit must equal $h_{j,t}$ because it appears infinitely often in the sequence. Therefore, $x_{j,t}(p_t^*) = \lim_{i \to \infty} x_{j,t}(p_t^i) = h_{j,t}$.

Consider j such that $p^*_{j,t} = 0$. For all $i > 0$ such that $i \equiv -j \pmod{J_t+1}$, $x_{j,t}(p^i_t) \le h_{j,t}$. Because $x_{j,t}$ is continuous by Lemma IA2, $x_{j,t}(p^i_t)$ converges. This limit cannot exceed $h_{j,t}$ because $x_{j,t}(p^i_t) \le h_{j,t}$ for infinitely many i. Therefore, $x_{j,t}(p^*_t) = \lim_{i\to\infty} x_{j,t}(p^i_t) \le h_{j,t}$.

Given that $w_{L,t}$ and $w_{H,t}$ clear the labor market when (7) holds, local equilibrium holds if and only if prices clear the housing market. Rentiers sell all housing at positive prices and any amount of housing at zero prices, so the market clears at p^*_t if and only if $x_{j,t}(p^*_t) = h_{j,t}$ when $p^*_{j,t} > 0$ and $x_{j,t}(p^*_t) \le h_{j,t}$ when $p^*_{j,t} = 0$. Therefore, p^*_t provides local equilibrium house prices.

Finally, I prove that this local equilibrium is unique. Consider two local equilibrium price vectors p_t and p'_t. For a contradiction, suppose that $p_t \ne p'_t$. By Proposition 1, $p_{j_{0,t},t} = 0$ because $\sum_{j\in\mathcal{J}_t} h_{j,t} > N_t$. Proposition 1 then further implies that $p_{j,t} = p'_{j,t} = 0$ for $j \le j_{0,t}$, $p_{j,t}, p'_{j,t} > 0$ for $j > j_{0,t}$, and $p_t, p'_t \in \mathcal{P}_t$. Furthermore, for $j > j_{0,t}$, $x_{j,t}(p_t) = x_{j,t}(p'_t) = h_{j,t}$. Without loss of generality, suppose that $p_{j,t} < p'_{j,t}$ for some $j > j_{0,t}$. Let $\mathcal{J}'_t = \{j \in \{0,\ldots,J_t\} \mid p_{j,t} < p'_{j,t}\}$. Given p_t, a household of education e and endowment z prefers and can afford $q_{j,t}$ over all $q_{j',t}$, where $j \in \mathcal{J}'_t$ and $j' \notin \mathcal{J}'_t$, only if $\max\{z_{e,j,j',t}(p_t) \mid j > j' \notin \mathcal{J}'_t\} \le z \le \min\{z_{e,j,j',t}(p_t) \mid j < j' \notin \mathcal{J}'_t\}$. Therefore, total demand for qualities $q_{j,t}$, where $j \in \mathcal{J}'_t$, equals

$$\sum_{j\in\mathcal{J}'_t} h_{j,t} = \sum_{j\in\mathcal{J}'_t} x_{j,t}(p_t) = \sum_{e\in\{L,H\}} \int_{\mathcal{Z}_{e,t}(p_t)} n_{e,t}(z)dz, \tag{IA6}$$

where

$$\mathcal{Z}_{e,t}(p_t) = \bigcup_{j\in\mathcal{J}'_t} \left[\max\left\{z_{e,j,j',t}(p_t) \mid j > j' \notin \mathcal{J}'_t\right\}, \min\left\{z_{e,j,j',t}(p_t) \mid j < j' \notin \mathcal{J}'_t\right\}\right]. \tag{IA7}$$

I define $\mathcal{Z}_{e,t}(p'_t)$ similarly. Because $p_{j,t} < p'_{j,t}$ for $j \in \mathcal{J}'_t$ and $p_{j,t} \ge p'_{j,t}$ for $j \notin \mathcal{J}'_t$, $z_{e,j,j',t}(p_t) < z_{e,j,j',t}(p'_t)$ if $\mathcal{J}'_t \ni j > j' \notin \mathcal{J}'_t$ and $z_{e,j,j',t}(p_t) > z_{e,j,j',t}(p'_t)$ if $\mathcal{J}'_t \ni j < j' \notin \mathcal{J}'_t$. Therefore, $\mathcal{Z}_{e,t}(p'_t) \subset \mathcal{Z}_{e,t}(p_t)$. Furthermore, $z_{e,j,j',t}(p'_t) > 0$ when $\mathcal{J}'_t \ni j > j'$ because $p'_{j,t} > 0$ and $p'_t \in \mathcal{P}_t$. It follows that $\min \mathcal{Z}_{e,t}(p_t) > \min \mathcal{Z}_{e,t}(p'_t) > 0$. Because the greatest lower bound of the support of $n_{e,t}$ equals zero (Lemma IA1),

$$\int_{\mathcal{Z}_{e,t}(p'_t)} n_{e,t}(z)dz \le \int_{\mathcal{Z}_{e,t}(p_t)} n_{e,t}(z)dz, \tag{IA8}$$

with strict inequality if $\mathcal{Z}_{e,t}(z) \ne \emptyset$, which holds for some e due to (IA6). Therefore, by (IA6), $\sum_{j\in\mathcal{J}'_t} h_{j,t} = \sum_{j\in\mathcal{J}'_t} x_{j,t}(p'_t) < \sum_{j\in\mathcal{J}'_t} x_{j,t}(p_t) = \sum_{j\in\mathcal{J}'_t} h_{j,t}$, a contradiction.

I.2 Solution to (8) and (9)

For each $j \in \{j_{0,t}+1,\ldots,J_t\}$, I rewrite (8) and (9) as

$$N_t - \sum_{j'=j}^{J_t} h_{j',t} = \sum_{e\in\{L,H\}} \int_0^{z_{e,j,t}(p_{j,t},p_{j-1,t})} n_{e,t}(z)dz, \tag{IA9}$$

where $z_{e,j,t}(p_{j,t},p_{j-1,t}) = z_{e,j,j-1,t}(p_t)$. By the definition of $j_{0,t}$, the minimum value of the left side of (IA9) is positive. The maximal value is less than N_t because $h_{J_t,t} > 0$. Therefore, the left side of (IA9) lies in $(0,N_t)$ for each $j \in \{j_{0,t}+1,\ldots,J_t\}$. Because $z_{e,j,t}(\cdot,\cdot)$ is an increasing linear function of its first argument, and because the support of each $n_{e,t}$ is convex, the right side of (IA9) strictly and continuously increases in $p_{j,t}$ over the range $[0,N_t)$. Therefore, given $p_{j-1,t}$, a unique value of $p_{j,t}$ solves (IA9) for each $j \in \{j_{0,t}+1,\ldots,J_t\}$. Because $p_{j_{0,t},t} = 0$, induction shows that unique values of $p_{j,t}$

solve (IA9).

I now prove that this unique solution for $p_{j,t}$ strictly increases over $j \in \{j_{0,t}, \ldots, J_t\}$. Because $z_{e,j_{0,t}+1,t}(p_{j_{0,t},t}, p_{j_{0,t},t}) = 0$ and the left of (IA9) exceeds zero, $p_{j_{0,t}+1,t} > 0 = p_{j_{0,t},t}$. Proceeding inductively, I note that

$$z_{e,j,t}(p_{j-1,t}, p_{j-1,t}) = w_{e,t}^{-1} p_{j-1,t} = z_{e,j-1,t}(p_{j-1,t}, p_{j-1,t}) < z_{e,j-1,t}(p_{j-1,t}, p_{j-2,t}) \tag{IA10}$$

as $z_{e,j-1,t}(\cdot,\cdot)$ strictly falls in its second argument. The left of (IA9) strictly rises in j, so $p_{j,t} > p_{j-1,t}$.

II Divisible housing

II.1 Household constraints

Households choose c, x_j, and t to maximize $u_e(c, q, a_t, \epsilon_t)$ subject to $q = \sum_{j \in \mathcal{J}_t} x_j q_{j,t}$, $c + \sum_{j \in \mathcal{J}_t} p_{j,t} x_j \leq w_{e,t} z$, and $x_j \geq 0$. The main model further imposes $(x_j)_{j \in \mathcal{J}_t} \in \{0,1\}^{J_t+1}$ and $\sum_{j \in \mathcal{J}_t} x_j = 1$.

II.2 Equilibrium characterization

Equilibrium definitions remain the same, as do (6) and (7). Lemma IA3 characterizes house prices.

Lemma IA3. *In equilibrium, $p_{j,t} = \mu_t q_{j,t}$ for each $j \in \mathcal{J}_t$, where*

$$\mu_t = \frac{\sum_{e \in \{L,H\}} \left(\beta_{c,e} + \beta_{q,e}\right)^{-1} \beta_{q,e} w_{e,t} Z_{e,t}}{\sum_{j' \in \mathcal{J}_t} h_{j',t} q_{j',t}}. \tag{IA11}$$

Proof. Each household's first-order condition with respect to x_j is

$$\frac{\partial u_e / \partial q}{\partial u_e / \partial c} \leq \frac{p_{j,t}}{q_{j,t}}, \tag{IA12}$$

with consumption of x_j only in the case of equality. If $p_{j,t}/q_{j,t}$ exceeds $p_{j',t}/q_{j',t}$ for any $j' \neq j$, then no household chooses quality $q_{j,t}$. This situation cannot hold in equilibrium because the market for that type of housing must clear. Therefore, $p_{j,t}/q_{j,t} = \mu_t$ for all $j \in \mathcal{J}_t$ for some μ_t.

To solve for μ_t, I re-write the budget constraint as $c + \mu_t q \leq w_{e,t} z$. Given t, maximizing (1) is equivalent to maximizing the Cobb-Douglas utility function $c^{1-\alpha} q^{\alpha}$ given income $w_{e,t} z$ and prices one and μ_t for c and q, where $\alpha = \beta_{q,e}(\beta_{c,e} + \beta_{q,e})^{-1}$. If $\mu_t \leq 0$, then households cannot optimize, so $\mu_t > 0$ in equilibrium. As shown on pages 55–56 of Mas-Collel et al. (1995), the optimal consumption bundle in this environment consists of expending a share α of income on q, so that $q = \beta_{q,e}(\beta_{c,e} + \beta_{q,e})^{-1} \mu_t^{-1} w_{e,t} z$. Given that $p_{j,t} > 0$ for all $j \in \mathcal{J}_t$, rentiers maximize utility by selling all of their housing. Equating the total housing quality rentiers sell to the total quality households demand produces (IA11). $\square$

Substituting a household's q and c choices into (1) yields

$$v_{e,t}(z) = \beta_c^{\beta_c} \beta_q^{\beta_q} \left(\beta_c + \beta_q\right)^{-(\beta_c + \beta_q)} (w_{e,t} z)^{\beta_{c,e} + \beta_{q,e}} \mu_t^{-\beta_{q,e}} a_t^{\beta_{a,e}}. \tag{IA13}$$

II.3 Stability

In any equilibrium, $n_{e,t}(z)/N_{e,t} = \widetilde{n}_e(z)/\widetilde{N}_e$, where $\widetilde{N}_e$ equals the mass of households of education e in the economy. This relation follows from substituting (IA13) into (6). As a result, $Z_{e,t} = \int_0^\infty z n_{e,t}(z)dz = N_{e,t}\int_0^\infty z\widetilde{N}_e^{-1}\widetilde{n}_e(z)dz = N_{e,t}\overline{z}_e$, where $\overline{z}_e$ equals the average z among households of education e. Therefore, in equilibrium, $N_{L,t}$ and $N_{H,t}$ determine $Z_{L,t}$ and $Z_{H,t}$. Because amenities, productivities, and prices in t are all functions of $N_{L,t}$, $N_{H,t}$, $Z_{L,t}$, and $Z_{H,t}$, the populations $N_{L,t}$ and $N_{H,t}$ pin down the local equilibrium in any equilibrium. By (IA13), there exist functions $v^N_{e,t}(N_{L,t}, N_{H,t})$ such that $v_{e,t}(z) = z^{\beta_{c,e}+\beta_{q,e}} v^N_{e,t}(N_{L,t}, N_{H,t})$ in any equilibrium.

An equilibrium is *stable* in t if the Jacobian of $v^N_t = (v^N_{L,t}, v^N_{H,t})$ at $(N_{L,t}, N_{H,t})$ is Volterra-Lyapunov stable (Cross, 1978). Equivalently for a two-by-two Jacobian, the diagonal is negative and its determinant is positive (Cross, 1978).

Because the diagonal is negative, perturbing $N_{e,t}$ moves equilibrium welfare of households of education e in the opposite direction when local stability holds. Furthermore, perturbing both $N_{L,t}$ and $N_{H,t}$ moves equilibrium welfare of at least one of the education groups in the opposite direction. If not, then there exist $dN_{L,t}, dN_{H,t} \neq 0$ and a non-negative diagonal matrix D^N_t such that $(Dv^N_t)(dN_{L,t}, dN_{H,t}) = D^N_t(dN_{L,t}, dN_{H,t})$. As a result, $Dv^N_t - D^N_t$ has an eigenvalue of zero, meaning that dv^N_t is not strongly stable and hence is not Volterra-Lyapunov stable, a contradiction (Cross, 1978).

In the indivisible model, $N_{L,t}$ and $N_{H,t}$ do not determine local equilibrium, so $v^N_{e,t}$ does not exist. For that reason, Section 4.5 proposes an alternate stability definition using perturbations to house prices, wages, and amenities instead of $N_{L,t}$ and $N_{H,t}$. This definition rules out instabilities from perturbations to $n_{L,t}$ and $n_{H,t}$ that arise from perturbations to house prices, wages, and amenities, the outcomes households take as given when making metro choices.

II.4 Comparative statics

I still make Assumption 3, so (10) holds, permitting me to drop t^* subscripts as in Section 3. Differentiating (IA13) yields

$$\partial \log v_e(z) = (\beta_{c,e} + \beta_{q,e})\partial \log w_e - \beta_{q,e}\partial \log \mu + \beta_{a,e}\partial \log a. \tag{IA14}$$

Differentiating (IA11) gives

$$\partial \log \mu = -\frac{\sum_{j\in\mathcal{J}} \delta_{h,j} q_j}{\sum_{j\in\mathcal{J}} h_j q_j} + \sum_{e\in\{L,H\}} \frac{(\beta_{c,e}+\beta_{q,e})^{-1}\beta_{q,e} Y_e(\partial \log w_e + \partial \log Z_e)}{(\beta_{c,L}+\beta_{q,L})^{-1}\beta_{q,L} Y_L + (\beta_{c,H}+\beta_{q,H})^{-1}\beta_{q,H} Y_H}. \tag{IA15}$$

Equations (16)–(18) continue to hold. Because $\partial \log v_e(z)$ does not depend on z, neither does $\partial \log n_e(z)$, so $\partial \log N_e = \partial \log Z_e = \partial \log v_e$.

II.5 Proof of Proposition 3

Each $\delta_{h,j}$ enters the derivatives of the equilibrium conditions only in (IA15), where it appears with a coefficient proportional to q_j. Due to the linearity of the system of equations, $\partial \log v_e(z)$ is proportional to $\sum_{j\in\mathcal{J}} q_j \delta_{h,j}$, as claimed.

Suppose that $\gamma_N = 0$ and that the equilibrium is locally stable in t^*. Define $Q = \sum_{j\in\mathcal{J}} q_j h_j$. To prove positive proportionality in (20), I assume without loss of generality that $\partial Q > 0$. For a contradiction, suppose that $\partial \log v_{e'}(z) \leq 0$ for some e' and z. By (IA14), $\partial \log v_{e'}(z') = \partial \log v_{e'}(z)$ for all

z', which, by (10), implies that $\partial N_{e'} \leq 0$. Consider the following perturbation to the equilibrium:

$$d\log N_e = \partial \log N_e - \begin{cases} 0, & \partial \log N_{\sim e'} \in (-\infty, 0] \\ \partial \log N_{\sim e'}, & \partial \log N_{\sim e'} \in (0, \partial \log Q] \\ \partial \log Q, & \partial \log N_{\sim e'} \in (\partial \log Q, \infty) \end{cases} \tag{IA16}$$

for each e, where $\sim$ denotes the opposite education group. In equilibrium, $Z_e = \overline{z}_e N_e$, so $Z_L/Z_H = N_L/N_H$. As a result, N_L/N_H determines labor prices and amenities in t^* in equilibrium when $\gamma_N = 0$. Because $d\log(N_L/N_H) = \partial \log(N_L/N_H)$, $da = \partial a$ and $dw_e = \partial w_e$. Combining (IA15) with (IA16) yields

$$d\log \mu = \partial \log \mu + \begin{cases} \partial \log Q, & \partial \log N_{\sim e'} \in (-\infty, 0] \\ \partial \log Q - \partial \log N_{\sim e'}, & \partial \log N_{\sim e'} \in (0, \partial \log Q] \\ 0, & \partial \log N_{\sim e'} \in (\partial \log Q, \infty). \end{cases} \tag{IA17}$$

When $\partial \log N_{\sim e'} \leq 0$, $d\log v_e^N < \partial \log v_e^N \leq 0$ for each e. These inequalities contradict local stability because $d\log N_e \leq 0$ for each e. When $0 < \partial \log N_{\sim e'} \leq \partial \log Q$, $d\log v_{e'}^N < \partial \log v_{e'}^N \leq 0$. This inequality contradicts local stability because $d\log N_{e'} \leq 0$ and $d\log N_{\sim e'} = 0$. When $\partial \log N_{\sim e'} > \partial \log Q$, $d\log N_{e'} < \partial \log N_{e'} \leq 0$ and $d\log N_{\sim e'} > 0$. These inequalities contradict local stability because $d\log v_{e'}^N = \partial \log v_{e'}^N \leq 0$ and $d\log v_{\sim e'}^N = \partial \log v_{\sim e'}^N > 0$.

References

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