

# Experiment 4

## Addition and Resolution of Vectors: The Force Table

### OBJECTIVES

The objectives of this experiment are to investigate how quantities that have both magnitude and direction can be represented by vectors or by sums of vectors. Vector addition and resolution will be investigated experimentally using the force table. The results of the graphical method for addition of force vectors will be compared to experimental results for addition obtained from a force table. The experimental arrangements of forces (vectors) will physically illustrate the principle of the methods of vector addition and resolution.

### EQUIPMENT

Force table<sup>1</sup> with three pulleys, protractor, three weight hangers, metric ruler, set of slotted weights (masses) including three 50-g and three 100-g masses, level, three sheets of paper, string, double-meter stick, demonstration protractor, cardboard cutouts for rulers; campus maps.

### SAFETY

Handle mechanical apparatus and masses with care to avoid personal injury.

### INTRODUCTION

Whether at rest or in motion, every object in our world has forces acting upon it. Under the influence of the force of gravity, an apple falls from a tree until it meets an opposing force from the ground. A motorist presses on an accelerator, setting in motion a series of forces that propel the vehicle forward. Water rushes along a river, eroding the rock underneath. A batter hits a baseball, which is stopped by a glove. There are far too many examples to name here, and you can probably already think of a few examples that aren't on this list.

In fact, you'd be amazed to learn all the ways in which we are obliged to consider the forces in our daily lives with extreme care. Imagine the care with which an architect must balance forces from steel beams, wood, and concrete in the construction of buildings weighing thousands of tons. Consider the attention to detail an Olympic trainer practices in instructing a gymnast in the optimal muscle coordination to achieve astounding height and rotation on a vault. Our mastery of forces allows us to live in safety and comfort, and to achieve high levels of human performance.

Mathematics and laws of physics were discovered and developed to describe the details of how forces work. In this experiment, you will explore a few problems involving forces in the laboratory with an apparatus called a **force table** and analyze force problems mathematically using **vectors**.

A **vector** is a mathematical concept used to represent quantities such as velocity, acceleration, and force. What these three quantities have in common is that they all have both *magnitude* and *direction*. Because vectors have a specific direction, they do not combine in the same way as simple numbers (scalars) in arithmetic.

In this experiment, you will learn to use the force table and **graphical methods** involving vectors to study forces. Although many, many forces may be operating on a body at the same time, this experiment will be limited to the case of finding the combination of a few forces (from hanging masses)

connected with strings to a small, stationary ring; in physics, this type of problem is called a study of coplanar, concurrent, static tensile forces in equilibrium.

### Graphical Methods

Vectors are represented graphically by arrows. The length of the arrow is proportional to the magnitude of the vector and the arrow points in the direction of the vector.

To add two vectors  $F_1 + F_2$  or to determine the **resultant** or the **vector sum** of vectors, the vector  $F_1$  is drawn and its head becomes the starting point for drawing vector  $F_2$ , as shown in Fig 4.1a. The vectors must be drawn at the same scale. An arrow for the vector to be added or subtracted may be drawn at any location on the graph, as long as its length and direction are not changed. This **head-to-tail** graphical method gives the resultant vector  $F_R$  whose magnitude and direction are given by a vector drawn from the tail of  $F_1$  to the head of  $F_2$ .

The magnitude (length)  $F_R$  and the orientation angle of the resultant vector  $F_R$  can be measured directly from the vector diagram using a ruler and protractor. For example, suppose that the vectors  $F_1$  and  $F_2$  represent two forces and the scale of the vector diagram is 400 g-dynes per centimeter of length on the graph. If the length of  $F_R$  is 6.50 cm, then Newton's second law of motion gives the magnitude  $F_R$  as

$$F_R = mg = (400 \text{ g-dynes/cm}) (6.50 \text{ cm}) = 2600 \text{ g-dynes} = 2.60 \times 10^3 \text{ g-dynes}$$

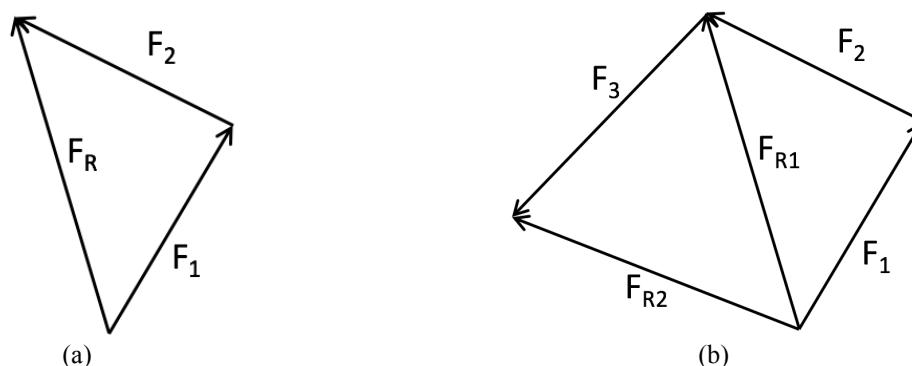
where  $m$  is mass and where  $g$  is the acceleration due to gravity ( $980 \text{ cm/s}^2$ ). For example,

$$F = mg = 100 \text{ g-dynes}$$

where  $m = 100 \text{ g}$ . This use of the acceleration due to gravity,  $g$ , in symbol form allows the expression of  $F$  in terms of mass, thus avoiding large numbers as in

$$F = (200 \text{ g}) (980 \text{ cm/s}^2) = 196,000 \text{ dynes}$$

Addition of more than two vectors with the head-to-tail method forms a polygon. As shown in Fig. 4-1b, the resultant ( $F_{R2} = F_1 + F_2 + F_3$ ) is the vector from the tail of  $F_1$  to the head of  $F_3$ . The length (magnitude) and the angle of orientation of  $F_{R2}$  are measured from the vector diagram. Note that this is equivalent to two successive applications of the head-to-tail method:  $F_1$  and  $F_2$  are added to give  $F_{R1}$  and then  $F_3$  is added to  $F_{R1}$  to give  $F_{R2}$ .



**Fig. 4-1. (a) Head-to-tail method:  $F_R = F_1 + F_2$ ; (b) Polygon method  $F_{R1} = F_1 + F_2$ ,  $F_{R2} = F_{R1} + F_3$**

A change in position of an object can also be described with vectors. If an object moves from point A to point B, it has experienced a **displacement** between the two positions. Displacements can be described with vectors, since they are also quantities that have both magnitude and direction.

Consider the following practical application of vectors: barring any obstructions, if a student walks 50 m due south and then an additional 50 m due south of that position, what is the student's resulting location, or displacement? If this student now walks another 45 m due north, what is the student's second displacement? If the student begins another trek and travels 50 m due north and then 50 m at  $135^\circ$  west, what is the total displacement for the entire trek? The graphical head-to-tail method described earlier is one of several methods useful in determining these displacements.

### Resolution of Vectors

A vector pointing in any direction can also be analyzed by decomposing it into its  $x$  and  $y$  components in a Cartesian coordinate system. In Fig. 4-2 notice that the components  $F_x$  and  $F_y$  of vector  $F_R$  are separated by an angle of  $90^\circ$ . Thus, any two vectors separated by  $90^\circ$  represent the  $x$  and  $y$  components of the resultant vector.

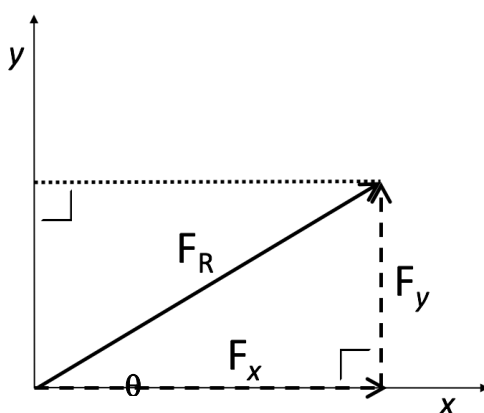


Fig. 4-2. Resolution of vector  $F_R$  into  $F_x + F_y$ , the sum of its  $x$  and  $y$  components.  $F_x = F_R \cos \theta$  ;  $F_y = F_R \sin \theta$

### The Force Table

The force table, shown in Fig. 4-3, is an apparatus that allows the experimental determination of the resultant force produced by a combination of forces. The rim of the circular table is calibrated in degrees. Weight forces are applied to a central ring by means of strings running over pulleys and attached to mass hangers. Adding or removing slotted masses varies the magnitude of a force and changing the angle of the pulley varies the direction. Each pulley with its suspended mass represents a vector.

On a force table, the resultant of two or more forces (vectors) is found by balancing the forces by adding weight to the hangers and by changing the angles of the pulleys so that the ring becomes centered at the central pin. The balancing force is not the **resultant**  $R$ , but rather its opposite, called the **equilibrant** vector, a force of equal magnitude to  $F_R$  but in the *opposite* direction, as shown in Fig. 4-4.



Fig. 4-3. Force Table. Courtesy of AMEP.

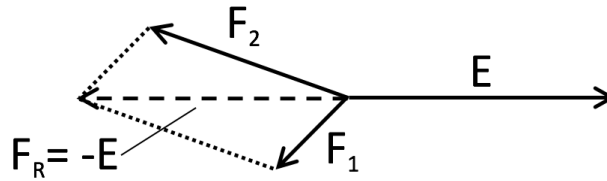


Fig. 4-4. Relationship of the equilibrant ( $E$ ) and resultant ( $F_R$ ) vectors:  $F_R = -E$ .

## PROCEDURE

### A. Graphical Method of Vector Addition

Given two vectors  $F_1 = 250$  g-dynes at  $60^\circ$  and  $F_2 = 250$  g-dynes at  $150^\circ$ , find their vector sum or resultant  $F_R = F_1 + F_2$  by the following procedures.

1. Using the head-to-tail method of vector addition, draw a vector diagram for Part A. Use a scale for which the finished vector diagram fills about half a sheet of graph paper.
2. Measure the magnitude and direction of the resultant with a ruler and protractor; record the results in the data table.
3. Use your scale to convert the magnitude of the vector in centimeters to a force in units of g-dynes.

### B. Addition of Two Forces on the Force Table

Find the resultant vector  $F_R = F_1 + F_2$  of Part A.

1. Check to see if the force table is level. Ask your instructor for a level if this has not been done. Make any necessary adjustments by means of the leveling screws in the tripod base of the table. Orientation angles of vectors are measured from the  $0^\circ$  reference line or x-axis.
2. On the force table, clamp two pulleys at  $60^\circ$  and  $150^\circ$ .
3. Make sure the strings will move freely on the ring and allow the strings to pull directly away from the center.
4. Add enough weights to each weight hanger to total 250 g, to give weight forces of  $F_1 = F_2 = 250$  g-dynes in these directions. (Weigh the mass of each hanger. Weight hangers usually have masses of 50 g.)
5. Add weights to a third pulley positioned opposite the first two until the central ring is centered around the center pin. These weights are the vector sum, or resultant.
6. Record the magnitude and direction of the resultant in the data table. Remember that the resultant has the same magnitude as the equilibrant but is in the opposite direction.
7. When the forces are balanced, the ring is centered around the central pin. To confirm its position, the pin should be carefully removed to see if the ring is centered exactly around the central hole. Make adjustments if necessary. Record the results in Table 4-1.

- Determine the *percent difference* of the experimental and graphical results in Parts A and B for the magnitude and direction of the resultant and record the value at the bottom of Table 4-1.

### C. Resolution of a Force into Its Components $F_x$ and $F_y$ on the Force Table

Given a force vector of  $F = 300$  g-dynes at  $40^\circ$ , resolve the vector into its  $x$  and  $y$  components of  $F_x$  and  $F_y$ .

- Given a force vector of  $F = 300$  g-dynes at  $40^\circ$ , resolve the vector into its  $x$  and  $y$  components of  $F_x$  and  $F_y$ . Record the results in the data table.
- Clamp pulleys at  $220^\circ$ ,  $90^\circ$  and  $0^\circ$  on the force table.
- Use a weight hanger to place a total of 300 g on the  $220^\circ$  pulley string. This force is the equilibrant of  $F = 300$  g-dynes at  $40^\circ$ , which must be used on the force table rather than the force itself.
- Add weights to the  $0^\circ$  and  $90^\circ$  hangers until the system is in equilibrium. The  $0^\circ$  and  $90^\circ$  forces are then the  $F_x$  and  $F_y$  components, respectively, of  $F$ .
- Record their magnitudes in Table 4-1.

### D. Vectors in Real Life: Physical Displacements on Campus Studied by the Graphical Method

*How far do you walk in one day on campus? How does that compare to the distance between your initial and final locations? Let's map the vectors between your destinations on campus.*

- Get a map of your campus from your instructor.
- Place a small circle or "o" at the location of your dormitory. If you live off campus, place the "x" at the point where you enter the campus (bus stop or train station). This is your starting position.
- Draw a straight arrow with a dashed line representing a vector to your first destination. Mark the destination as "1."
- Draw another straight arrow with a dashed line, head-to-tail with the first one, representing a vector from your first to your second destination. Mark this destination as "2."
- Repeat this process for all of the consecutive destinations you have during the day on campus.
- Record the latitude, longitude, and  $x$  and  $y$  coordinates for each of the points in Table 4-2.
- Resolve the vectors drawn in steps 3-5 into  $x$  and  $y$  components that match your actual walking or driving path as closely as possible. Draw solid lines representing these vectors.
- Use the cardboard cutouts provided to construct a ruler based on the scale of the campus map.
- A vector drawn from the initial position to the final position represents the total vector sum or resultant displacement. Draw an arrow representing the total displacement from your initial to your final position.

### E. Cleanup

Follow instructions provided by your instructor as to how the apparatus should be disassembled and the equipment and supplies should be returned to their original locations.

#### **FURTHER READING**

1. [https://www.amep.com/index.php?route=product/product&product\\_id=3260&search=Force+table+](https://www.amep.com/index.php?route=product/product&product_id=3260&search=Force+table+) (last accessed January 2019)
2. <http://www.physicsclassroom.com/class/vectors/Lesson-3/Addition-of-Forces> (last accessed January 2019)
3. <http://phys.eng.ankara.edu.tr/wp-content/uploads/sites/611/2016/09/ForceTable.pdf> (last accessed January 2019)
4. [https://www.youtube.com/watch?v=1B\\_PtO7cNE8](https://www.youtube.com/watch?v=1B_PtO7cNE8) (last accessed January 2019)

**NOTES**





4. How will you know when the forces on a force table are balanced?

5. Compare and contrast the graphical method with the displacement method.

**REPORT ON EXPERIMENT 4**  
**Addition and Resolution of Vectors: The Force Table**

**DATA****Table 4-1. Addition of Two Forces on the Force Table and Graphical Method of Vector Addition**

	Force (g-dynes) Displacement (m)	Resultant/Component Vector (magnitude and direction)	
		Graphical	Experimental (Force Table)
Part A: Addition	$F_1 = 250, \theta_1 = 60^\circ$ $F_2 = 250, \theta_2 = 150^\circ$		
Part B: Resolution	$F_1 = 250, \theta_1 = 50^\circ$		$F_x =$ $F_y =$
Part C: Real Displacement	(List the displacement vectors in Table 4-2)		

**Percent Difference Results:**

Magnitude: \_\_\_\_\_ Direction: \_\_\_\_\_

**Table 4-2. Vectors in Real Life: Physical Displacements on Campus Studied by the Graphical Method**

Position	Coordinates:			
	Latitude	Longitude	$x$	$y$
Starting Position				
1				
2				
3				
4				
5				
6				

Total displacement from starting position to final position (feet): \_\_\_\_\_

Total displacement from starting position to final position (meters): \_\_\_\_\_

