The Derivation Rules of SD+

All the Derivation Rules of *SD* and Rules of Inference

Modus Tollens (MT)

$\mathbf{P} \supset \mathbf{Q}$
$\sim \mathbf{Q}$
$\triangleright \sim \mathbf{P}$

Hypothetical Syllogism (HS)

$\mathbf{P} \supset \mathbf{Q}$
$\mathbf{Q} \supset \mathbf{R}$
$\triangleright \mathbf{P} \supset \mathbf{R}$

Disjunctive Syllogism (DS)

$\mathbf{P} \vee \mathbf{Q}$
$\sim \mathbf{P}$
$\triangleright \mathbf{Q}$

or

$\mathbf{P} \vee \mathbf{Q}$
$\sim \mathbf{Q}$
$\triangleright \mathbf{P}$

Rules of Replacement

Commutation (Comm)

$\mathbf{P} \,\&\, \mathbf{Q} \lhd\rhd \mathbf{Q} \,\&\, \mathbf{P}$
$\mathbf{P} \vee \mathbf{Q} \lhd\rhd \mathbf{Q} \vee \mathbf{P}$

Association (Assoc)

$\mathbf{P} \,\&\, (\mathbf{Q} \,\&\, \mathbf{R}) \lhd\rhd (\mathbf{P} \,\&\, \mathbf{Q}) \,\&\, \mathbf{R}$
$\mathbf{P} \vee (\mathbf{Q} \vee \mathbf{R}) \lhd\rhd (\mathbf{P} \vee \mathbf{Q}) \vee \mathbf{R}$

Implication (Impl)

$\mathbf{P} \supset \mathbf{Q} \lhd\rhd \sim \mathbf{P} \vee \mathbf{Q}$

Double Negation (DN)

$\mathbf{P} \lhd\rhd \sim \sim \mathbf{P}$

De Morgan (DeM)

$\sim (\mathbf{P} \,\&\, \mathbf{Q}) \lhd\rhd \sim \mathbf{P} \vee \sim \mathbf{Q}$
$\sim (\mathbf{P} \vee \mathbf{Q}) \lhd\rhd \sim \mathbf{P} \,\&\, \sim \mathbf{Q}$

Idempotence (Idem)

$\mathbf{P} \lhd\rhd \mathbf{P} \,\&\, \mathbf{P}$
$\mathbf{P} \lhd\rhd \mathbf{P} \vee \mathbf{P}$

Transposition (Trans)

$\mathbf{P} \supset \mathbf{Q} \lhd\rhd \sim \mathbf{Q} \supset \sim \mathbf{P}$

Exportation (Exp)

$\mathbf{P} \supset (\mathbf{Q} \supset \mathbf{R}) \lhd\rhd (\mathbf{P} \,\&\, \mathbf{Q}) \supset \mathbf{R}$

Distribution (Dist)

$\mathbf{P} \,\&\, (\mathbf{Q} \vee \mathbf{R}) \lhd\rhd (\mathbf{P} \,\&\, \mathbf{Q}) \vee (\mathbf{P} \,\&\, \mathbf{R})$
$\mathbf{P} \vee (\mathbf{Q} \,\&\, \mathbf{R}) \lhd\rhd (\mathbf{P} \vee \mathbf{Q}) \,\&\, (\mathbf{P} \vee \mathbf{R})$

Equivalence (Equiv)

$\mathbf{P} \equiv \mathbf{Q} \lhd\rhd (\mathbf{P} \supset \mathbf{Q}) \,\&\, (\mathbf{Q} \supset \mathbf{P})$
$\mathbf{P} \equiv \mathbf{Q} \lhd\rhd (\mathbf{P} \,\&\, \mathbf{Q}) \vee (\sim \mathbf{P} \,\&\, \sim \mathbf{Q})$