THE LOGIC BOOK

Sixth Edition

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Chapter 1

INTRODUCTION TO DEDUCTIVE LOGIC

1.1 INTRODUCTION

This is a text in deductive logic—more specifically, in formal or symbolic deductive logic. Chapters 1–5 are devoted to sentential logic, the branch of symbolic deductive logic that takes sentences as the fundamental units of logical analysis. Chapters 7–10 are devoted to predicate logic, the branch of symbolic deductive logic that takes predicates and individual terms as the fundamental units of logical analysis. Chapter 6 is devoted to the metatheory of sentential logic, while Chapter 11 is devoted to the metatheory of predicate logic.

The hallmark of deductive logic is **truth-preservation.** Reasoning that is acceptable by the standards of deductive logic is always truth-preserving; that is, it never takes one from truths to a falsehood. The following syllogism provides an example of such reasoning:

All mammals are vertebrates.
Some sea creatures are mammals.
___
Some sea creatures are vertebrates.

If the first and second sentences (the **premises**) are true, then the third sentence (the **conclusion**) must also be true. In deductive logic, reasoning that is truth-preserving is said to be '**valid**'.[1]

Over the centuries, a variety of systems of deductive logic have been developed. One of the oldest is Euclid's axiomatization of plane geometry, developed around 300 BCE in classical Greece. All of the truths or theorems of plane geometry can be derived from the five fundamental assumptions or axioms of Euclid's system. Many have attempted to axiomatize other areas of knowledge, including many of the sciences and many areas of mathematics. Giuseppe Peano successfully axiomatized arithmetic in 1889. Aristotle (350 BCE), a near contemporary of Euclid, developed a system of deductive logic that is known as "categorical" or "syllogistic" logic. Our earlier example of valid deductive reasoning was an Aristotelian syllogism. Aristotle's system is built around the logic of terms that identify categories of things, fish, human beings, animals, and so on. Aristotelian logic is still taught today, and the Law School Admissions Test (LSAT) usually contains questions about Aristotelian logic. However, Aristotle's system is limited in some important ways. For example, every syllogism must have exactly two premises, and the premises and conclusion of a syllogism must be structured according to very restrictive rules. Aristotelian logic cannot accommodate such obviously valid reasoning as

Either the maid or the butler killed Watson.

If it was the maid, Watson was poisoned.

Watson wasn't poisoned.

The butler killed Watson.

because there are three premises, not two, and the first and second premises have more complex forms than can be accommodated in Aristotelian logic.

The systems of deductive logic that we present in this text have their foundations in the work of Gottlob Frege, David Hilbert, Bertrand Russell, and other logicians in the late nineteenth and early twentieth centuries. Unlike axiomatic systems, which are based on a (usually) small number of axioms, the deductive systems in this text are based on a small number of reasonably intuitive rules that govern how sentences can be derived from other sentences.

There are a variety of reasons for studying deductive logic. It is a well-developed discipline that many find interesting in its own right, a discipline that has a rich history and important current research programs and practical applications. Certainly, those who plan to major or do graduate work in areas such as philosophy, mathematics, and computer science should have a solid grounding in skills that are needed for presenting and evaluating arguments in

[1]Deductive logic's requirement that good reasoning be truth-preserving sets a very high standard for acceptable reasoning. This stands in contrast to inductive logic, which sets a more modest standard for good reasoning, namely that if the claims with which one starts are true, then the claims one reaches by using inductive principles are likely to be true. A great deal of the reasoning used in the sciences and in ordinary life is judged by inductive rather than deductive standards.

any discipline. Another reason for studying symbolic logic is that, in learning to symbolize natural language sentences (in our case, English sentences) in a formal language, one becomes more aware and more appreciative of the importance of the structure and complexities of natural languages. The specific words that we use have a direct bearing on whether a piece of reasoning is valid or invalid. For example, it is essential to distinguish between 'Roberta will pass if she completes all the homework' and 'Roberta will pass only if she completes all the homework' if we want to reason well about Roberta's prospects for passing. Finally, the concepts that we explore in this text are abstract concepts. Learning to think about abstract concepts and the relations between them is an important skill that is useful in a wide range of theoretical and applied disciplines.

1.2 CORE CONCEPTS OF DEDUCTIVE LOGIC

Many—but not all—sentences of English are either true or false (this is true of any natural language). We will say that true sentences have the **truth-value T** and that false sentences have the **truth-value F.** Sentences that are true or false include

Canada is located in South America.

Beethoven composed nine symphonies.

The Boston Red Sox will win the next World Series.

On December 29, 1012, it rained in what is now Manhattan.

The first of these sentences is false and therefore has the truth-value **F**. The second sentence is true and has the truth-value **T**. We do now know whether the third and fourth sentences are true or false, but we do know that each is one or the other. Time will tell whether the third sentence has the truth-value **T** or the truth-value **F,** but we will probably never know the truth-value of the fourth sentence. However, regardless of the state of our knowledge, the fourth sentence does have either the truth-value **T** or the truth-value **F**. It is important not to confuse our inability to know which truth-value a sentence has with the sentence's lack of a truth-value. There are obviously many sentences whose truth-values we will never know but that do nevertheless have truth-values.

Examples of sentences that lack truth-values include

Do I really have to do the homework to do well in this course?

Lock the door when you leave.

Hurrah!

The first of these sentences is a question. The second is a request or command and the third is an exclamation. To have a truth-value, a sentence must assert something. These three sentences do not assert or claim anything and hence do not have truth-values. In this text, we will be concerned only with sentences that do have truth-values; when we refer to a sentence or sentences we are referring to sentences that do have truth-values.

When we are talking about expressions of English we will often use the **variables p, q, r**, and **s** to do so. We use these variables in the same way that mathematicians use **x** and **y** as variables when they are talking about positive integers, that is, the numbers 1, 2, 3, . . . For example, the claim 'If **x** is an even positive integer and **y** is an odd positive integer then **x** plus **y** is an odd positive integer' is a true claim of arithmetic. So too, where **p** and **q** are variables that take expressions of English as their values, the following is true:

> If **p** is a sentence of English and **q** is a sentence of English then
>
> Either **p** or **q**
>
> is also a sentence of English.

The use of variables provides a convenient way for us to make claims about all expressions of English of a certain sort.

We define an **argument** as follows:

> An *argument* is a set of two or more sentences, one of which is designated as the conclusion and the others as the premises.[2]

Note that this definition uses the concept of a set of sentences. Sets are abstract objects that have members (zero or more). We can specify a finite set by listing its members, separated by commas, within a set of curly brackets. Here, for example, is a set of three English sentences:

> {Helen is not very well educated if she believes there is intelligent life on Mars, Helen is very well educated, Helen does not believe there is intelligent life on Mars}

If we designate the first two members that we have listed as the premises and the third sentence as the conclusion, then we have the argument

> Helen is not very well educated if she believes there is intelligent life on Mars.
>
> Helen is very well educated.
>
> ---
>
> Helen does not believe there is intelligent life on Mars.

We adopt the convention of displaying arguments by listing the premises with a horizontal line under the last premise, followed by the conclusion. We will say that arguments displayed in this way are in **standard form.**

[2]This definition allows arguments to have *any* number of premises, including an infinite number. However, all the arguments that we use as examples in this text have only a finite number of premises.

We now have all the terminology we need to introduce the core concepts of deductive logic. The first concept is **logical validity,** a concept that applies to arguments:

> *Logically valid argument:* An argument is *logically valid* if and only if it is not possible for all the premises to be true and the conclusion false. An argument is *logically invalid* if and only if it is not logically valid.

A logically valid argument is truth-preserving. If the premises are true, then the conclusion must also be true. The previous argument about Helen is logically valid because it is impossible for the premises to be true and the conclusion false. That is, if the premises are all true, then the conclusion must be true as well. Note that to determine validity, we do not need to know whether the premises or conclusion are in fact true. All that we need to know is the logical relation between the premises and the conclusion.

The following argument is not logically valid:

If Sara receives an A in her chemistry class, she will graduate with a 3.5 grade point average.

If Sara graduates with a 3.5 grade point average, she will get into medical school.

Sara will get into medical school

Sara will receive an A in her chemistry class.

This argument is invalid because it is possible for all three premises to be true and the conclusion false. Perhaps Sara will only receive a B in her chemistry class but will nevertheless graduate with a 3.5 grade point average because her other grades are so high, or perhaps she'll get into medical school with less than a 3.5 average because her medical school admissions interview was exceptional.

An argument that is logically valid and that has true premises is said to be **logically sound:**

> *Logically sound argument:* An argument is *logically sound* if and only if it is logically valid and all of its premises are true. An argument is *logically unsound* if and only if it is not logically sound.

Obviously, all logically sound arguments are logically valid, but not all logically valid arguments are logically sound because not all logically valid arguments have premises that are all true. The following argument is logically valid but is not logically sound:

Italy is a country that is located in North America.

Every country that is located in North America uses the United States dollar as its currency.

Italy uses the United States dollar as its currency.

This argument is logically valid because if the premises were both true, the conclusion would have to be true as well. Obviously, however, the premises are not both true; in fact, they are both false (as is the conclusion), and so the argument is not logically sound, On the other hand, the following logically valid argument is also logically sound, because both of its premises are true:

> The United States is a country that is located in North America.
>
> No country that is located in North America uses the euro as its currency.
>
> ---
>
> The United States does not use the euro as its currency.

Note that if an argument is logically sound, its conclusion will also be true. This is because if the premises of a logically valid argument are true, then, because it is impossible for the argument's premises to be true and its conclusion false, the conclusion must also be true.

Identifying passages of English that contain arguments, extracting those arguments, and presenting them in standard form are important skills that must be mastered before the techniques presented in this text can be used to evaluate English arguments. The following expressions often signal that the sentence that follows is the conclusion of an argument:

therefore
thus
it follows that
so
hence
consequently
as a result

We will call these 'conclusion indicator expressions'. Similarly, expressions such as

since
for
because
on account of
inasmuch as
for the reason that

often indicate that the sentences following these expressions are the premises of an argument, and we will call these 'premise indicator expressions'.

Sometimes premise and conclusion indicators signal that what is grammatically a single sentence can reasonably be recast as an argument. For example, in the sentence

> If Ron went to the store, he'd be home by now, but he isn't home, and so we may conclude that he didn't go to the store.

the words 'so we may conclude' indicate that the sentence should be understood as an argument. We can present the argument in standard form as follows:

> If Ron went to the store, Ron would be home by now.
>
> Ron isn't home yet.
>
> ---
>
> Ron didn't go to the store.

This argument is, by the way, logically valid.

Note that we cannot assume that the premises of an argument always occur before the conclusion in natural language discourse. The following sentence can also be recast as an argument, and the argument's conclusion occurs at the beginning, rather than the end, of the sentence:

> Michael will not get the job, for the person that gets the job must have strong references, and Michael's references are not strong.

The extracted argument, which is logically valid, is

> The person that gets the job must have strong references.
>
> Michael's references are not strong.
>
> ---
>
> Michael will not get the job.

The conclusion of an argument can also occur between its premises, and an entire argument may be buried in a larger piece of prose or discourse. Here is an example:

> I've got more relatives than I know what to do with. I've got relatives in Idaho and in New Jersey, in Ireland, and in Israel. Among them are a couple of cousins, Tom and Fred Culverson. Both Tom and Fred are hardworking, and Tom is as tenacious as a bulldog. So Tom is sure to be a success, for if there is one thing I have learned in life, it is that everyone who is both hardworking and tenacious succeeds. But I'm sure success won't change Tom. He'll work just as hard after he makes his first million as he does now. He is, after all, a Culverson. And no one is as predictable as a Culverson, unless it's a Hutchings. There are lots of Hutchings on my mother's side, but I haven't had much to do with them . . .

The following explicit argument can be extracted from this passage:

> Tom and Fred are hardworking.
>
> Tom is tenacious.
>
> Everyone who is both hardworking and tenacious succeeds.
>
> ---
>
> Tom will succeed.

As frequently happens, there is a lot of information in the passage that is not relevant to the specific argument we have extracted.

We note that our formal definition of 'argument' allows for arguments in which the premise or premises provide no support whatsoever for the conclusion. Here is an example:

> Two is the smallest prime number.
>
> ---
>
> In 1967 the Green Bay Packers won the first Super Bowl.

The one premise of this argument is true, as is the conclusion. But it is certainly possible for the premise to be true and the conclusion false, so the argument is invalid. (The Kansas City Chiefs lost to the Green Bay Packers in the first super bowl game, but they might have won.) Some might object that we should not count this as an argument at all, for the premises of an argument usually have something to do with the conclusion. But it is not at all clear what kind of connections there must be between sentences before we are willing to count one of them as the conclusion and the rest as the premises of an argument. Some benighted and superstitious soul might think that the fact that a bluebird landed on his window sill in the morning is relevant to his winning the lottery and 'argue' as follows:

> This morning a bluebird landed on my bedroom window as I was getting out of bed.
>
> Therefore, this will be my lucky day and I will win today's lottery.

Although where and when bluebirds land has nothing to do with who wins lotteries and when this happens, it is prudent to count this as an argument, because we can then use the tools of deductive logic to show that it is a bad–and clearly invalid—argument.

Every sentence that has a truth-value is either logically true, logically false, or logically indeterminate:

> A sentence is *logically true* if and only if it is not possible for the sentence to be false.
>
> A sentence is *logically false* if and only if it is not possible for the sentence to be true.

A sentence is *logically indeterminate* if and only if it is neither logically true nor logically false.

The sentence 'Either June will pass Chemistry 101 or June will not pass Chemistry 101' is logically true, for it is impossible for this sentence to be false. The sentence is true by virtue of its structure alone. That is, all sentences of the form **p** or not **p**, where **p** is any sentence that has a truth-value and **not p** is a denial of **p**, is logically true. So, for example, 'Either it is raining or it is not raining' is logically true. Of course, there are other forms of logically true sentences. For example, 'If all dogs have tails, then there are no dogs that do not have tails' is logically true. Logically true sentences are uninformative; they don't tell us anything about the 'real world'. The sentence about June tells us nothing about her chances of passing Chemistry 101, and the sentence about dogs tells us nothing about whether dogs do or do not have tails. An example of a logically false sentence is

June will pass Chemistry 101 and she will not pass Chemistry 101.

Logically false sentences are false by virtue of their structure. This sentence is logically false because it is impossible for June to both pass and not pass Chemistry 101. Like logically true sentences, logically false sentences give us no information about the world. The sentence 'It rained on July 6, 1309, in what is now San Francisco' is logically indeterminate. Whether this sentence is true or false is not a matter of logic but rather depends on what in fact happened on that day in that place. Most of the sentences that we encounter in ordinary conversation, writing, and reading are logically indeterminate. Whether they are true or false depends upon how the world in fact is.

Logical equivalence is a relation between sentences:

Logically equivalent sentences: Sentences **p** and **q** are *logically equivalent* if and only if it is not possible for one of these sentences to be true while the other sentence is false.

The sentences 'Jake loves Henry' and 'Henry is loved by Jake' are logically equivalent. One cannot be true without the other being true, and if either is false the other is false. Given a pair of logically equivalent sentences, both of which are logically indeterminate, we know that either both members of the pair are true or both are false, but we don't always know which. We know the sentences 'Mary is taller than Henry' and 'Henry is shorter than Mary' are logically equivalent. Mary cannot be taller than Henry without Henry being shorter than she is, and Henry cannot be shorter than Mary without Mary being taller than he is. But the fact that the sentences are logically equivalent doesn't tell us whether they are both true or both false. Since we may replace the variables **p** and **q** with the same sentence, it follows from our definition of logical equivalence that every sentence is equivalent to itself.

Logical consistency is defined as follows:

A set of sentences is *logically consistent* if and only if it is possible for all the members of that set to be true. A set of sentences is *logically inconsistent* if and only if it is not logically consistent.

The set

{Tom is left-handed, Carol is left-handed, Mona is left-handed}

is logically consistent. Whoever Tom, Carol, and Mona are, it is logically possible that they are all left-handed. But

{Everyone in the room is left-handed, Mona is in the room, Mona is not left-handed}

is logically inconsistent. The three sentences in the set cannot all be true. If everyone in the room is left-handed and Mona is in the room, then Mona must be left-handed, so it is false that she is not left-handed.

It may seem that the notion of what is and is not logically possible, a concept we have used in defining all of the core concepts of deductive logic, is itself in need of clarification. One of the motivations for developing formal or symbolic systems of deductive logic is in fact to refine the concept of logical possibility so that there are clear criteria for what is logically possible and what is not. These criteria form the bases of our formal deductive systems in the rest of this book. In Chapter 2 we will present the symbolic language *SL* (for 'Sentential Logic'), and in Chapter 3 we will define formal sentential logic versions of the core concepts of deductive logic. In Chapter 7 we will present the far more powerful language *PL* (for 'Predicate Logic'), and in Chapter 8 we will define formal predicate logic versions of the core concepts.

The last core concept of deductive logic is that of **entailment.** This concept is closely related to, but not identical, with that of validity:

A set of sentences *logically entails* a sentence if and only if it is impossible for the members of the set to be true and that sentence false.

The set

{Henry and Joan will both receive their law degrees in June}

logically entails the sentence 'Joan will receive her law degree in June', for it is impossible for the sentence 'Joan will receive her law degree in June' to be false if the single sentence in the set is true. On the other hand, the set

{Andrew plays soccer, Siri plays tennis} does not logically entail the sentence ‘Andrew does not play tennis and Siri does not play soccer’, for it may be the case that in addition to playing soccer, Andrew plays tennis and in addition to playing tennis, Siri plays soccer.

If an argument is logically valid, then the set consisting of the premises of the argument logically entails the argument’s conclusion. However, logical entailment is a more general concept than is logical validity, for there are sentences that are entailed by the empty set, while arguments must have at least one premise. The former may seem odd, but there are good reasons for introducing the more general concept—one reason being that the concept of logical entailment facilitates reasoning about logical systems, as we will do in Chapters 6 and 11.

What sentences are entailed by the empty set? One example is the sentence ‘Either June will pass Chemistry 101 or she will not pass Chemistry 101’. This sentence is logically entailed by the empty set of sentences because it is impossible for all of the members of the empty set (there are none) to be true and ‘Either June will pass Chemistry 101 or she will not pass Chemistry 101’ to be false, precisely because this sentence cannot be false. One way that we might intuitively understand this sentence’s and all logical truths being entailed by the empty set is by noting that this sentence requires no support. It is true regardless of what the world is like. This means that reasoning that starts with no assumptions (the empty set) and reaches this sentence is truth-preserving.

1.2E EXERCISES

Note: Solutions to unstarred exercises can be accessed from the following web page: http://highered.mcgraw-hill.com/sites/007353563x/information_center_view0/

Select ‘Student Edition’ on the left-hand side of the page, then select ‘Chapter 1’ and the solutions will appear as a pdf file. If you do not have *Adobe Reader* you can download it for free from Adobe’s website.

1. For each of the following, indicate whether it is the kind of sentence that falls within the scope of this text—that is, whether it is a sentence that has a truth-value. If it is not this kind of sentence, explain why not.

a. George Washington was the second president of the United States.
*b. The next president of the United States will be a Republican.
c. Turn in your homework on time or not at all.
*d. Would that 9/11 had never happened.
e. Two is the smallest prime number.
*f. One is the smallest prime number.
g. George Bush, Senior, immediately preceded George W. Bush as president.
*h. At 3:00 pm EST on January 15, 1134, there was a snowstorm in what is now Manhattan.

2. For each of the following passages, determine whether it advances an argument. If an argument is probably being expressed, restate the argument in standard form. If the intent is probably not to express an argument, explain why not.

a. When Mike, Sharon, Sandy, and Vicky are all away from the Tacoma office, no important decisions get made. Mike is off skiing, Sharon is in Spokane, Vicky is in Olympia, and Sandy is in Seattle. So no decisions will be made today.

*b. Our press releases are always crisp and upbeat. Mike is the press officer, and if he has his way, all press releases are crisp and upbeat. Mike always has his way.

c. Shelby and Noreen are wonderful at dealing with irate students and faculty members. Stephanie is very good at managing the chancellor's very demanding schedule, and Tina keeps everything moving and cheers everyone up.

*d. Tom and Ray are our office assistants, and one of them is incompetent and the other one is lazy. So either Tom is incompetent and Ray is lazy, or Tom is lazy and Ray is incompetent.

e. We won't be able to repair the deck because to do so we need stainless steel screws. All the screws we have are in the nail and screw cabinet. The first drawer contains only galvanized nails, the second contains only ordinary nails, and the third contains only drywall screws. And the fourth and bottom drawers contain only brass screws.

*f. If the budget is to be balanced we will have to raise taxes or cut spending. If we cut spending, then entitlement programs, including Medicare and Social Security, will have to be significantly cut and the Democrats will have a fit. If we raise taxes, Republicans will go ballistic. So if the budget is balanced, either the Democrats will have a fit or the Republicans will go ballistic.

g. The weather is perfect, the view is wonderful, and we're on vacation. So why are you unhappy?

*h. The new kitchen will be finished on time only if our carpenter works over the weekend. He will work over the weekend only if he doesn't go duck hunting, but he will go duck hunting. So the new kitchen will not be finished on time.

i. If Sarah did the wiring, it was done right, and if Marcie did the plumbing, it was done right. As neither the wiring nor the plumbing was done right, Sarah didn't do the wiring and Marcie didn't do the plumbing.

*j. Sarah, John, Rita, and Bob have all worked hard, and one of them will be promoted. Their company is having a cash flow problem and is offering its employees who are over 55 a $50,000 bonus if they retire at the end of this year. Sarah, John, and Bob are all over 55 and will take early retirement. So Rita will be promoted.

k. Having cancer is a good, for whatever is required by something that is a good is itself a good. Being cured of cancer is a good, and being cured of cancer requires having cancer.

*l. Humpty Dumpty sat on a wall. Humpty Dumpty had a great fall. All the king's horses and all the king's men couldn't put Humpty together again. So they made him into an omelet and had a great lunch.[3]

1.3 SPECIAL CASES OF LOGICAL CONCEPTS

In this section we point out and explain some special cases of our logical concepts. These may seem counterintuitive, but we shall explain why they follow from our definitions.

[3] With apologies to Lewis Carroll.

The first special case is that of an argument whose conclusion is logically true. Such an argument is logically valid no matter what premises it has. Here is such an argument:

> The Philadelphia Phillies is the best team in the National League.
>
> ---
>
> Either the next president of the United States will be a woman or the next president will not be a woman.

The conclusion of this argument is logically true. No matter who wins the next presidential election, that person either will or will not be a woman. Because the conclusion is logically true, it is impossible for the argument's premise to be true and the conclusion false, because it is impossible for the conclusion to be false. So, the argument is logically valid; it will never lead us from truths to a falsehood. The second, and related, special case is that of every logically true sentence being entailed by every set of sentences, including the empty set, because it is impossible for a logically true sentence to be false and hence impossible for the members of a set, any set, all to be true and that logically true sentence false.

The third special case concerns arguments whose premises form logically inconsistent sets. Here is an example:

> Albert is brighter than all his sisters.
>
> Sally is Albert's sister.
>
> Sally is brighter than all her brothers.
>
> ---
>
> *Tyrannosaurus rex* was the fiercest of all dinosaurs.

The premises form a logically inconsistent set because they cannot all be true. If the first and second premises are both true, for example, the third premise cannot be true. And if the premises cannot all be true, it is impossible for the premises to be true and the conclusion false. The argument is therefore logically valid. Although every argument whose premises form an inconsistent set is logically valid, of course no such argument can be logically sound. Note that every argument that has at least one logically false premise is an instance of this special case, because any set of sentences that contains a logically false sentence is logically inconsistent.

Our fourth and fifth special cases concern logical equivalence, which we have defined as follows:

> Sentences **p** and **q** are *logically equivalent* if and only if it is not possible for one of the sentences to be true while the other sentence is false.

By this definition, all logically true sentences **p** and **q** are logically equivalent. Because it is not possible for any logically true sentence to be false, it is impossible for logically true sentences **p** and **q** to be such that one is true while the other is false. It follows that all logically true sentences are logically equivalent. Similar reasoning shows that all logically false sentences are logically equivalent. Of course, not all logically indeterminate sentences are logically equivalent.

1.3E EXERCISES

1. Which of the following are true, and which are false? Explain your answers, giving examples where appropriate.

a. Every argument that has 'Whatever will be, will be' as a conclusion is logically valid.

*b. Any argument that includes among its premises 'Everyone is a scoundrel' and 'I'm no scoundrel' is logically valid.

c. Every argument that has 'Everyone is a scoundrel and I'm no scoundrel' as a conclusion is logically invalid.

*d. The premises of a logically valid argument always provide support for the conclusion of the argument.

2. Answer each of the following.

a. Suppose that an argument has a premise that is logically true. Must the argument be logically valid? Explain.

*b. Suppose that an argument has a premise that is logically equivalent to a logically false sentence. Must the argument be logically valid? Explain.

c. Suppose that an argument has a logically true sentence as its conclusion. Explain why the argument must be valid no matter what its premises are. Explain why some such arguments are sound and some are not.

*d. Suppose that the premises of an argument form a logically inconsistent set. Explain why the argument must be logically valid but unsound.

GLOSSARY

ARGUMENT: An argument is a set of two or more sentences, one of which is designated as the conclusion and the others as the premises.

LOGICAL VALIDITY: An argument is *logically valid* if and only if it is not possible for the premises to be true and the conclusion false. An argument is *logically invalid* if and only if it is not logically valid.

LOGICAL SOUNDNESS: An argument is *logically sound* if and only if it is logically valid and all its premises are true. An argument is *logically unsound* if and only if it is not logically sound.

LOGICAL TRUTH: A sentence is *logically true* if and only if it is not possible for the sentence to be false.

LOGICAL FALSITY: A sentence is *logically false* if and only if it is not possible for the sentence to be true.

LOGICAL INDETERMINACY: A sentence is *logically indeterminate* if and only if it is neither logically true nor logically false.

LOGICAL EQUIVALENCE: Sentences **p** and **q** are *logically equivalent* if and only if it is not possible for one of these sentences to be true while the other sentence is false.

LOGICAL CONSISTENCY: A set of sentences is *logically consistent* if and only if it is possible for all the members of that set to be true. A set of sentences is *logically inconsistent* if and only if it is not logically consistent.

LOGICAL ENTAILMENT: A set of sentences *logically entails* a sentence if and only if it is impossible for all the members of the set to be true and that sentence false.