
CHAPTER FIVE

Section 5.1.1E

1.a. Derive: $A \& B$

1	A	Assumption
2	A \supset B	Assumption
3	B	1, 2 \supset E
4	A & B	1, 3 &I

c. Derive: $\sim (A \equiv \sim B)$

1	$\sim (A \equiv \sim B) \equiv (\sim C \vee \sim D)$	Assumption
2	A \supset ($\sim D \& C$)	Assumption
3	D & A	Assumption
4	A	3 &E
5	$\sim D \& C$	2, 4 \supset E
6	$\sim D$	5 &E
7	$\sim C \vee \sim D$	6 \vee I
8	$\sim (A \equiv \sim B)$	1, 7 \equiv E

e. Derive: $F \supset \sim G$

1	(E \vee H) \supset (F \supset \sim G)	Assumption
2	(C \vee D) \equiv (E & \sim H)	Assumption
3	C	Assumption
4	C \vee D	3 \vee I
5	E & \sim H	2, 4 \equiv E
6	E	5 &E
7	E \vee H	6 \vee I
8	F \supset \sim G	1, 7 \supset E

g. Derive: $D \equiv \sim B$

1	(A & \sim B) \supset C	Assumption
2	(C \vee D) \supset (D \equiv \sim B)	Assumption
3	\sim B & A	Assumption
4	A	3 &E
5	\sim B	3 &E
6	A & \sim B	4, 5 &I
7	C	1, 6 \supset E
8	C \vee D	7 \vee I
9	D \equiv \sim B	2, 8 \equiv E

i. Derive: $\sim D \ \& \ \sim F$

1	(A \vee \sim B) \equiv (A $\&$ \sim F)	Assumption
2	C \equiv \sim B	Assumption
3	C $\&$ \sim D	Assumption
4	\sim D	3 $\&$ E
5	C	3 $\&$ E
6	\sim B	2, 5 \equiv E
7	A \vee \sim B	6 \vee I
8	A $\&$ \sim F	1, 7 \equiv E
9	\sim F	8 $\&$ E
10	\sim D $\&$ \sim F	4, 9 $\&$ I

2. Complete the following derivations.

a. Derive: D $\&$ \sim B

1	A $\&$ \sim B	Assumption
2	(A \vee \sim C) \supset D	Assumption
3	A	1 $\&$ E
4	A \vee \sim C	3 \vee I
5	D	2, 4 \supset E
6	\sim B	1 $\&$ E
7	D $\&$ \sim B	5, 6 $\&$ I

c. Derive: \sim D \vee E

1	A $\&$ \sim B	Assumption
2	\sim B \equiv (A \equiv \sim D)	Assumption
3	\sim B	1 $\&$ E
4	A \equiv \sim D	2, 3 \equiv E
5	A	1 $\&$ E
6	\sim D	4, 5 \equiv E
7	\sim D \vee E	6 \vee I

e. Derive: $H \& \sim I$

1	$\sim F \& \sim G$	Assumption
2	$\sim G \supset H$	Assumption
3	$(H \& \sim F) \equiv \sim I$	Assumption
<hr/>		
4	$\sim G$	1 &E
5	H	2, 4 \supset E
6	$\sim F$	1 &E
7	$H \& \sim F$	5, 6 &I
8	$\sim I$	3, 7 \equiv E
9	$H \& \sim I$	5, 8 &I

g. Derive: $F \& \sim G$

1	$(F \vee \sim G) \supset (F \& \sim H)$	SM
2	$\sim H \supset \sim G$	SM
3	$(\sim H \supset \sim G) \equiv F$	SM
<hr/>		
4	F	2, 3 \equiv E
5	G	A / \sim I
6	$F \vee \sim G$	5 \vee I
7	$F \& \sim H$	1, 6 \supset E
8	$\sim H$	7 &E
9	$\sim H$	A / \sim E
10	$\sim G$	2, 9 \supset E
11	G	5 R
12	H	9-11 \sim E
13	$\sim G$	5-12 \sim I
14	$F \& \sim G$	4, 13 &I

Section 5.1.2E

a. Derive: $(A \supset B) \ \& \ (A \supset \sim D)$

1	A \supset (B & \sim D)	Assumption
2	A	A / \supset I
3	B & \sim D	1, 2 \supset E
4	B	3 &E
5	A \supset B	2-4 \supset I
6	A	A / \supset I
7	B & \sim D	1, 6 \supset E
8	\sim D	7 &E
9	A \supset \sim D	6-8 \supset I
10	(A \supset B) & (A \supset \sim D)	5, 9 &I

c. Derive: B

1	\sim B \supset B	Assumption
2	\sim B	A / \sim E
3	B	1, 2 \supset E
4	\sim B	2 R
5	B	2-4 \sim E

e. Derive: $E \vee D$

1	A \vee (B & \sim C)	Assumption
2	A \supset D	Assumption
3	\sim C \supset E	Assumption
4	A	A / \vee E
5	D	2, 4 \supset E
6	E \vee D	5 \vee I
7	B & \sim C	A / \vee E
8	\sim C	7 &E
9	E	3, 8 \supset E
10	E \vee D	9 \vee I
11	E \vee D	1, 4-6, 7-10 \vee E

g. Derive: $F \equiv \sim G$

1	(F \supset \sim G) & (\sim G \supset F)	Assumption
2	F	A / \equiv I
3	F \supset \sim G	1 &E
4	\sim G	2, 3 \supset E
5	\sim G	A / \equiv I
6	\sim G \supset F	1 &E
7	F	5, 6 \supset E
8	F \equiv \sim G	2-4, 5-7 \equiv I

2. Complete the following derivations.

a. Derive: $A \equiv B$

1	A	Assumption
2	B	Assumption
3	A	A / \equiv I
4	B	1 R
5	B	A / \equiv I
6	A	1 R
7	A \equiv B	3-4, 5-6 \equiv I

c. Derive: A

1	~ ~ A	Assumption
2	~ A	A / ~ E
3	~ A	2 R
4	~ ~ A	1 R
5	A	2, 4 ~ E

e. Derive: B

1	~ B \supset C	Assumption
2	~ C \equiv A	Assumption
3	A	Assumption
4	~ B	A / ~ E
5	C	1, 4 \supset E
6	~ C	2, 3 \equiv E
7	B	4-6 ~ E

g. Derive: ~ H

1	H \supset I	Assumption
2	~ I	Assumption
3	H	A / ~ I
4	I	1, 3 \supset E
5	~ I	2 R
6	~ H	3-5 ~ I

i. Derive: ~ (F \vee G)

1	(F \vee G) \supset (H & I)	Assumption
2	~ H	Assumption
3	F \vee G	A / ~ I
4	H & I	1, 3 \supset E
5	H	4 &E
6	~ H	2 R
7	~ (F \vee G)	3-6 ~ I

Section 5.1.3E

1. a. Derive: $(A \& C) \vee (B \& C)$

1	(A \vee B) & C	Assumption
2	A \vee B	1 &E
3	C	1 &E
4	A	A / \vee E
5	A & C	3, 4 &I
6	(A & C) \vee (B & C)	5 \vee I
7	B	A / \vee E
8	B & C	3, 7 &I
9	(A & C) \vee (B & C)	8 \vee I
10	(A & C) \vee (B & C)	2, 4 -6, 7 -9 \vee E

c. Derive: $\sim B$

1	B \supset (A & \sim B)	Assumption
2	B	A / \sim I
3	A & \sim B	1, 2 \supset E
4	\sim B	3 & E
5	B	2 R
6	\sim B	2-5 \sim I

e. Derive: $C \supset (\sim A \& B)$

1	\sim D	Assumption
2	C \supset (A \equiv B)	Assumption
3	(D \vee B) \supset \sim A	Assumption
4	(A \equiv B) \supset (D & E)	Assumption
5	\sim B \supset D	Assumption
6	C	A / \supset I
7	A \equiv B	2, 6 \supset E
8	D & E	4, 7 \supset E
9	D	8 &E
10	D \vee B	9 \vee I
11	\sim A	3, 10 \supset E
12	\sim B	A / \sim E
13	D	5, 12 \supset E
14	\sim D	1 R
15	B	12-14 \sim E
16	\sim A & B	11, 15 &I
17	C \supset (\sim A & B)	6-16 \supset I

g. Derive: $A \equiv B$

1	$\sim A \ \& \ \sim B$	Assumption
2	A	$A / \equiv I$
3	$\sim B$	$A / \sim E$
4	$\sim A$	1 &E
5	A	2 R
6	B	3-5 $\sim E$
7	B	$A / \equiv I$
8	$\sim A$	$A / \sim E$
9	B	7 R
10	$\sim B$	1 & E
11	A	8-10 $\sim E$
12	$A \equiv B$	2-6, 7-11 $\equiv I$

Section 5.3E

1. Derivability

a. Derive: $A \supset (A \ \& \ B)$

1	$A \supset B$	Assumption
G	$A \supset (A \ \& \ B)$	$2-_ \supset I$

Derive: $A \supset (A \ \& \ B)$

1	$A \supset B$	Assumption
2	A	$A / \supset I$
3	B	1, 2 $\supset E$
4	$A \ \& \ B$	2, 3 &I
5	$A \supset (A \ \& \ B)$	2-5 $\supset I$

c. Derive: $L \equiv K$

1	(K \supset L) & (L \supset K)	Assumption
2	L	A / \equiv I
G	K	
	K	A / \equiv I
G	L	
G	L \equiv K	2- <u> </u> , <u> </u> -2 \equiv I

Derive: $L \equiv K$

1	(K \supset L) & (L \supset K)	Assumption
2	L	A / \equiv I
3	L \supset K	1 &E
4	K	2, 3 \supset E
5	K	A / \equiv I
6	K \supset L	1 &E
7	L	5, 6 \supset E
8	L \equiv K	2-4, 5-7 \equiv I

e. Derive: C

1	B & \sim B	Assumption
2	\sim C	A / \sim E
G	C	2- <u> </u> \sim E

Derive: C

1	B & ~ B	Assumption
2	~ C	A / ~ E
3	B	1 &E
4	~ B	1 &E
5	C	2-4 ~ E

g. Derive: $D \supset B$

1	$A \supset C$	Assumption
2	$(\sim A \vee C) \supset (D \supset B)$	Assumption
G	$\sim A \vee C$	
G	$D \supset B$	2, — \supset E

Derive: $D \supset B$

1	$A \supset C$	Assumption
2	$(\sim A \vee C) \supset (D \supset B)$	Assumption
3	$\sim (\sim A \vee C)$	A / ~ E
4	A	A / ~ I
5	C	1, 4 \supset E
6	$\sim A \vee C$	5 \vee I
7	$\sim (\sim A \vee C)$	3 R
8	$\sim A$	4-7 ~I
9	$\sim A \vee C$	8 \vee I
10	$\sim (\sim A \vee C)$	3 R
11	$\sim A \vee C$	3, 10 ~ E
12	$D \supset B$	2, 11 \supset E

i. Derive: B

1	A \supset B	Assumption
2	\sim (B & \sim C) \supset A	Assumption
3	\sim B	A / \sim E
G	B	3— \sim E

Derive: B

1	A \supset B	Assumption
2	\sim (B & \sim C) \supset A	Assumption
3	\sim B	A / \sim E
4	B & \sim C	A / \sim I
5	B	4 &E
6	\sim B	3 R
7	\sim (B & \sim C)	4-6 \sim I
8	A	2, 7 \supset E
9	B	1, 8 \supset E
10	\sim B	3 R
11	B	3-10 \sim E

k. Derive: B \vee \sim C

1	A \vee (B & C)	Assumption
2	C \supset \sim A	Assumption
3	A	A / \vee E
G	B \vee \sim C	
G	B \vee \sim C	A / \vee E
G	B \vee \sim C	1, 3—, —— \vee E

Derive: $B \vee \sim C$

1	$A \vee (B \& C)$	Assumption
2	$C \supset \sim A$	Assumption
3	A	$A / \vee E$
4	C	$A / \sim I$
5	$\sim A$	2, 4 $\supset E$
6	A	3 R
7	$\sim C$	4-6 $\sim I$
8	$B \vee \sim C$	7 $\vee I$
9	$B \& C$	$A / \vee E$
10	B	9 $\& E$
11	$B \vee \sim C$	10 $\vee I$
12	$B \vee \sim C$	1, 3-8, 9-11 $\vee E$

m. Derive: $D \supset (F \supset C)$

1	$(A \vee B) \supset C$	Assumption
2	$(D \vee E) \supset [(F \vee G) \supset A]$	Assumption
3	D	$A / \supset I$
G	$F \supset C$	
G	$D \supset (F \supset C)$	3- $\supset I$

Derive: $D \supset (F \supset C)$

1	$(A \vee B) \supset C$	Assumption
2	$(D \vee E) \supset [(F \vee G) \supset A]$	Assumption
3	D	$A / \supset I$
4	F	$A / \supset I$
5	$D \vee E$	3 $\vee I$
6	$(F \vee G) \supset A$	2, 5 $\supset E$
7	$F \vee G$	4 $\vee I$
8	A	6, 7 $\supset E$
9	$A \vee B$	8 $\vee I$
10	C	1, 9 $\supset E$
11	$F \supset C$	4-10 $\supset I$
12	$D \supset (F \supset C)$	3-11 $\supset I$

o. Derive: $B \supset F$

1	A \supset \sim (B \vee C)		
2	(C \vee D) \supset A		Assumption
3	\sim F \supset (D & \sim E)		Assumption
4	B		A / \supset I
G	F		
G	B \supset F		4— \supset I

Derive: $B \supset F$

1	A \supset \sim (B \vee C)		
2	(C \vee D) \supset A		Assumption
3	\sim F \supset (D & \sim E)		Assumption
4	B		A / \supset I
5	\sim F		A / \sim E
6	D & \sim E		3, 5 \supset E
7	D		6 &E
8	C \vee D		7 \vee I
9	A		2, 8 \supset E
10	\sim (B \vee C)		1, 9 \supset E
11	B \vee C		4 \vee I
12	F		5–11 \sim E
13	B \supset F		4–12 \supset I

q. Derive: H

1	F \supset (G \vee H)		
2	\sim (\sim F \vee H)		Assumption
3	\sim G		Assumption
4	\sim H		A / \sim E
G	\sim F \vee H		
G	\sim (\sim F \vee H)		2 R
G	H		4— \sim E

q. Derive: H

1	F \supset (G \vee H)		Assumption
2	\sim (\sim F \vee H)		Assumption
3	\sim G		Assumption
4	\sim H		A / \sim E
5	F		A / \sim I
6	G \vee H		1, 5 \supset E
7	G		A / \vee E
8	\sim H		A / \sim E
9	G		7 R
10	\sim G		3 R
11	H		8-10 \sim E
12	H		A / \vee E
13	H		12 R
14	H		6, 7-11, 12-13 \vee E
15	\sim H		4 R
16	\sim F		5-15 \sim E
17	\sim F \vee H		16 \vee I
18	\sim (\sim F \vee H)		2 R
19	H		4-18 \sim E

2. Validity

a. Derive: A \supset C

1	A \supset \sim B		Assumption
2	\sim B \supset C		Assumption
3	A		A / \supset I
4	\sim B		1, 3 \supset E
5	C		2, 4 \supset E
6	A \supset C		3-5 \supset I

c. Derive: \sim B

1	A \equiv B		Assumption
2	\sim A		Assumption
3	B		A / \sim I
4	A		1, 3 \equiv E
5	\sim A		2 R
6	\sim B		3-5 \sim I

e. Derive: $A \supset [B \supset (C \supset D)]$

1	<u>D</u>	Assumption
2	<u>A</u>	A / \supset I
3	<u>B</u>	A / \supset I
4	<u>C</u>	A / \supset I
5	D	1 R
6	C \supset D	4-5 \supset I
7	B \supset (C \supset D)	3-6 \supset I
8	A \supset [B \supset (C \supset D)]	2-7 \supset I

g. Derive: $A \supset (D \supset C)$

1	A \supset (B \supset C)	Assumption
2	<u>D \supset B</u>	Assumption
3	<u>A</u>	A / \supset I
4	<u>D</u>	A / \supset I
5	B \supset C	1, 3 \supset E
6	B	2, 4 \supset E
7	C	5, 6 \supset E
8	D \supset C	4-7 \supset I
9	A \supset (D \supset C)	3-8 \supset I

i. Derive: $A \supset C$

1	<u>$\sim A \vee B$</u>	Assumption
2	<u>B \supset C</u>	Assumption
3	<u>A</u>	A / \supset I
4	<u>$\sim A$</u>	A / \vee E
5	<u>$\sim C$</u>	A / \sim E
6	A	3 R
7	$\sim A$	4 R
8	C	5-7 \sim E
9	<u>B</u>	A / \vee E
10	C	2, 9 \supset E
11	C	1, 4-8, 9-10 \vee E
12	A \supset C	3-11 \supset I

k. Derive: B

1	A \supset (C \supset B)	Assumption		
2	\sim C \supset \sim A	Assumption		
3	A	Assumption		
4	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">~ B</td> <td style="padding-left: 5px;"></td> </tr> </table>	~ B		A / \sim E
~ B				
5	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">C \supset B</td> <td style="padding-left: 5px;"></td> </tr> </table>	C \supset B		1, 3 \supset E
C \supset B				
6	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">C</td> <td style="padding-left: 5px;"></td> </tr> </table>	C		A / \sim I
C				
7	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">B</td> <td style="padding-left: 5px;"></td> </tr> </table>	B		5, 6 \supset E
B				
8	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">~ B</td> <td style="padding-left: 5px;"></td> </tr> </table>	~ B		4 R
~ B				
9	~ C	6-8 \sim I		
10	~ A	2, 9 \supset E		
11	A	3 R		
12	B	4-11 \sim E		

m. Derive: F & G

1	F \equiv G	Assumption		
2	F \vee G	Assumption		
3	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">F</td> <td style="padding-left: 5px;"></td> </tr> </table>	F		A / \vee E
F				
4	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">F</td> <td style="padding-left: 5px;"></td> </tr> </table>	F		3 R
F				
5	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">G</td> <td style="padding-left: 5px;"></td> </tr> </table>	G		A / \vee E
G				
6	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">F</td> <td style="padding-left: 5px;"></td> </tr> </table>	F		1, 5 \equiv E
F				
7	F	2, 3-4, 5-6 \vee E		
8	G	1, 7 \equiv E		
9	F & G	7, 8 &I		

3. Theorems

a. Derive: A \supset (A \vee B)

1	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">A</td> <td style="padding-left: 5px;"></td> </tr> </table>	A		A / \supset I
A				
2	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">A \vee B</td> <td style="padding-left: 5px;"></td> </tr> </table>	A \vee B		1 \vee I
A \vee B				
3	A \supset (A \vee B)	1-2 \supset I		

c. Derive: A \supset [B \supset (A & B)]

1	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">A</td> <td style="padding-left: 5px;"></td> </tr> </table>	A		A / \supset I		
A						
2	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">B</td> <td style="padding-left: 5px;"></td> </tr> </table> </td> <td style="padding-left: 5px;"></td> </tr> </table>	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">B</td> <td style="padding-left: 5px;"></td> </tr> </table>	B			A / \supset I
<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">B</td> <td style="padding-left: 5px;"></td> </tr> </table>	B					
B						
3	<table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">A & B</td> <td style="padding-left: 5px;"></td> </tr> </table>	A & B		1, 2 &I		
A & B						
4	B \supset (A & B)	2-3 \supset I				
5	A \supset [B \supset (A & B)]	1-4 \supset I				

e. Derive: $(A \equiv B) \supset (A \supset B)$

1	A \equiv B	A / \supset I
2	A	A / \supset I
3	B	1, 2 \equiv E
4	A \supset B	2-3 \supset I
5	(A \equiv B) \supset (A \supset B)	1-4 \supset I

g. Derive: $(A \supset B) \supset [(C \supset A) \supset (C \supset B)]$

1	A \supset B	A / \supset I
2	C \supset A	A / \supset I
3	C	A / \supset I
4	A	2, 3 \supset E
5	B	1, 4 \supset E
6	C \supset B	3-5 \supset I
7	(C \supset A) \supset (C \supset B)	2-6 \supset I
8	(A \supset B) \supset [(C \supset A) \supset (C \supset B)]	1-7 \supset I

i. Derive: $[(A \supset B) \& \sim B] \supset \sim A$

1	(A \supset B) $\&$ \sim B	A / \supset I
2	A	A / \sim I
3	A \supset B	1 $\&$ E
4	B	2, 3 \supset E
5	\sim B	1 $\&$ E
6	\sim A	2-5 \sim I
7	[(A \supset B) $\&$ \sim B] \supset \sim A	1-6 \supset I

k. Derive: $A \supset [B \supset (A \supset B)]$

1	A	A / \supset I
2	B	A / \supset I
3	A	A / \supset I
4	B	2 R
5	A \supset B	3-4 \supset I
6	B \supset (A \supset B)	2-5 \supset I
7	A \supset [B \supset (A \supset B)]	1-6 \supset I

m. Derive: $(A \supset B) \supset [\sim B \supset \sim (A \& D)]$

1		$A \supset B$	$A / \supset I$
2			$A / \supset I$
3			$A / \sim I$
4			$3 \&E$
5			$1, 4 \supset E$
6			$2 R$
7			$3-6 \sim I$
8			$2-7 \supset I$
9			$1-8 \supset I$

4. Equivalence

a. Derive: $A \& \sim A$

1		$B \& \sim B$	Assumption
2			$A / \sim E$
3			$1 \&E$
4			$1 \&E$
5			$2-4 \sim E$

Derive: $B \& \sim B$

1		$A \& \sim A$	Assumption
2			$A / \sim E$
3			$1 \&E$
4			$1 \&E$
5			$2-4 \sim E$

c. Derive: $(A \vee B) \supset A$

1		$B \supset A$	Assumption
2			$A / \supset I$
3			$A / \vee E$
4			$3 R$
5			$A / \vee E$
6			$1, 5 \supset E$
7			$2, 3-4, 5-6 \vee E$
8			$2-7 \supset I$

Derive: $B \supset A$

1	(A \vee B) \supset A	Assumption
2	B	A / \supset I
3	A \vee B	2 \vee I
4	A	1, 3 \supset E
5	B \supset A	2-4 \supset I

e. Derive: $\sim (A \equiv B)$

1	(A & \sim B) \vee (B & \sim A)	Assumption
2	A & \sim B	A / \vee E
3	A \equiv B	A / \sim I
4	A	2 &E
5	B	3, 4 \equiv E
6	\sim B	2 &E
7	$\sim (A \equiv B)$	3-6 \sim I
8	B & \sim A	A / \vee E
9	A \equiv B	A / \sim I
10	B	8 &E
11	A	9, 10 \equiv E
12	\sim A	8 &E
13	$\sim (A \equiv B)$	9-12 \sim I
14	$\sim (A \equiv B)$	1, 2-7, 8-13 \vee E

Derive: (A & \sim B) \vee (B & \sim A)

1	$\sim (A \equiv B)$	Assumption
2	$\sim [(A \& \sim B) \vee (B \& \sim A)]$	A / \sim E
3	A	A / \equiv I
4	\sim B	A / \sim E
5	A & \sim B	3, 4 &I
6	(A & \sim B) \vee (B & \sim A)	5 \vee I
7	$\sim [(A \& \sim B) \vee (B \& \sim A)]$	2 R
8	B	4-7 \sim I
9	B	A / \equiv I
10	\sim A	A / \sim E
11	B & \sim A	9, 10 &I
12	(A & \sim B) \vee (B & \sim A)	11 \vee I
13	$\sim [(A \& \sim B) \vee (B \& \sim A)]$	2 R
14	A	10-13 \sim E
15	A \equiv B	3-8, 9-14 \equiv I
14	$\sim (A \equiv B)$	1 R
15	(A & \sim B) \vee (B & \sim A)	2-14 \sim E

5. Inconsistency

a. Derive: $A \supset A, \sim (A \supset A)$

1	$\sim (A \supset A)$	Assumption
2	A	$A / \supset I$
3	A	2 R
4	$A \supset A$	2-3 $\supset I$
5	$\sim (A \supset A)$	1 R

c. Derive: $A, \sim A$

1	$A \equiv B$	Assumption
2	$B \supset \sim A$	Assumption
3	A	Assumption
4	A	3 R
5	B	1, 4 $\equiv E$
6	$\sim A$	2, 5 $\supset E$

e. Derive: $A, \sim A$

1	$A \supset \sim A$	Assumption
2	$\sim A \supset A$	Assumption
3	A	$A / \sim I$
4	$\sim A$	1, 3 $\supset E$
5	A	3 R
6	$\sim A$	3-5 $\sim I$
7	A	2, 6 $\supset E$

g. Derive: $A \vee B, \sim (A \vee B)$

1	$\sim (A \vee B)$	Assumption
2	$C \supset A$	Assumption
3	$\sim C \supset B$	Assumption
4	C	$A / \sim I$
5	A	2, 4 $\supset E$
6	$A \vee B$	5 $\vee I$
7	$\sim (A \vee B)$	1 R
8	$\sim C$	4-7 $\sim I$
9	B	3, 8 $\supset E$
10	$A \vee B$	9 $\vee I$
11	$\sim (A \vee B)$	1 R

i. Derive: $F \vee G, \sim (F \vee G)$

1	$\sim (F \vee G) \equiv (A \supset A)$	Assumption
2	$H \supset F$	Assumption
3	$\sim H \supset F$	Assumption
4	A	A / \supset I
5	A	4 R
6	$A \supset A$	4-5 \supset I
7	$\sim (F \vee G)$	1, 6 \equiv E
8	H	A / \sim I
9	F	2, 8 \supset E
10	$F \vee G$	9 \vee I
11	$\sim (F \vee G)$	7 R
12	$\sim H$	8-11 \sim I
13	F	3, 12 \supset E
14	$F \vee G$	13 \vee I

6. Derivability

a. Derive: $A \equiv B$

1	$A \supset B$	Assumption
2	$\sim A \supset \sim B$	Assumption
3	A	A / \equiv I
4	B	1, 3 \supset E
5	B	A / \equiv I
6	$\sim A$	A / \sim E
7	$\sim B$	2, 6 \supset E
8	B	5 R
9	A	6-8 \sim E
10	$A \equiv B$	3-4, 5-9 \equiv I

c. Derive: A

1	$A \equiv (\sim B \vee C)$	Assumption
2	$B \supset C$	Assumption
3	$\sim A$	A / \sim E
4	B	A / \sim I
5	C	2, 4 \supset E
6	$\sim B \vee C$	5 \vee I
7	A	1, 6 \equiv E
8	$\sim A$	3 R
9	$\sim B$	4-8 \sim I
10	$\sim B \vee C$	9 \vee I
11	A	1, 10 \equiv E
12	$\sim A$	3 R
13	A	3-12 \sim E

e. Derive: $B \vee D$

1	$B \vee (C \vee D)$	Assumption			
2	$C \supset A$	Assumption			
3	$A \supset \sim C$	Assumption			
4	<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="width: 5%; text-align: right; vertical-align: top;"> </td> <td style="border-left: 1px solid black; padding-left: 5px; vertical-align: top;">B</td> <td style="width: 90%; vertical-align: top;">A / $\vee E$</td> </tr> </table>		B	A / $\vee E$	A / $\vee E$
	B	A / $\vee E$			
5	$B \vee D$	4 $\vee I$			
6	<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="width: 5%; text-align: right; vertical-align: top;"> </td> <td style="border-left: 1px solid black; padding-left: 5px; vertical-align: top;">$C \vee D$</td> <td style="width: 90%; vertical-align: top;">A / $\vee E$</td> </tr> </table>		$C \vee D$	A / $\vee E$	A / $\vee E$
	$C \vee D$	A / $\vee E$			
7	<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="width: 5%; text-align: right; vertical-align: top;"> </td> <td style="border-left: 1px solid black; padding-left: 5px; vertical-align: top;">C</td> <td style="width: 90%; vertical-align: top;">A / $\vee E$</td> </tr> </table>		C	A / $\vee E$	A / $\vee E$
	C	A / $\vee E$			
8	<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="width: 5%; text-align: right; vertical-align: top;"> </td> <td style="border-left: 1px solid black; padding-left: 5px; vertical-align: top;">$\sim (B \vee D)$</td> <td style="width: 90%; vertical-align: top;">A / $\sim E$</td> </tr> </table>		$\sim (B \vee D)$	A / $\sim E$	A / $\sim E$
	$\sim (B \vee D)$	A / $\sim E$			
9	<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="width: 5%; text-align: right; vertical-align: top;"> </td> <td style="border-left: 1px solid black; padding-left: 5px; vertical-align: top;">A</td> <td style="width: 90%; vertical-align: top;">2, 7 $\supset E$</td> </tr> </table>		A	2, 7 $\supset E$	2, 7 $\supset E$
	A	2, 7 $\supset E$			
10	<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="width: 5%; text-align: right; vertical-align: top;"> </td> <td style="border-left: 1px solid black; padding-left: 5px; vertical-align: top;">$\sim C$</td> <td style="width: 90%; vertical-align: top;">3, 9 $\supset E$</td> </tr> </table>		$\sim C$	3, 9 $\supset E$	3, 9 $\supset E$
	$\sim C$	3, 9 $\supset E$			
11	<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="width: 5%; text-align: right; vertical-align: top;"> </td> <td style="border-left: 1px solid black; padding-left: 5px; vertical-align: top;">C</td> <td style="width: 90%; vertical-align: top;">7 R</td> </tr> </table>		C	7 R	7 R
	C	7 R			
12	$B \vee D$	8–11 $\sim E$			
13	<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="width: 5%; text-align: right; vertical-align: top;"> </td> <td style="border-left: 1px solid black; padding-left: 5px; vertical-align: top;">D</td> <td style="width: 90%; vertical-align: top;">A / $\vee E$</td> </tr> </table>		D	A / $\vee E$	A / $\vee E$
	D	A / $\vee E$			
14	$B \vee D$	13 $\vee I$			
15	$B \vee D$	6, 7–12, 13–14 $\vee E$			
16	$B \vee D$	1, 4–5, 6–15 $\vee E$			

g. Derive: $(A \vee B) \supset \sim C$

1	$A \supset (D \& B)$	Assumption			
2	$(\sim D \equiv B) \& (C \supset A)$	Assumption			
3	<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="width: 5%; text-align: right; vertical-align: top;"> </td> <td style="border-left: 1px solid black; padding-left: 5px; vertical-align: top;">$A \vee B$</td> <td style="width: 90%; vertical-align: top;">A / $\supset I$</td> </tr> </table>		$A \vee B$	A / $\supset I$	A / $\supset I$
	$A \vee B$	A / $\supset I$			
4	<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="width: 5%; text-align: right; vertical-align: top;"> </td> <td style="border-left: 1px solid black; padding-left: 5px; vertical-align: top;">C</td> <td style="width: 90%; vertical-align: top;">A / $\sim I$</td> </tr> </table>		C	A / $\sim I$	A / $\sim I$
	C	A / $\sim I$			
5	<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="width: 5%; text-align: right; vertical-align: top;"> </td> <td style="border-left: 1px solid black; padding-left: 5px; vertical-align: top;">$C \supset A$</td> <td style="width: 90%; vertical-align: top;">2 $\&E$</td> </tr> </table>		$C \supset A$	2 $\&E$	2 $\&E$
	$C \supset A$	2 $\&E$			
6	A	4, 5 $\supset E$			
7	$D \& B$	1, 6 $\supset E$			
8	B	7 $\&E$			
9	$\sim D \equiv B$	2 $\&E$			
10	$\sim D$	8, 9 $\equiv E$			
11	D	7 $\&E$			
12	$\sim C$	4–11 $\sim I$			
13	$(A \vee B) \supset \sim C$	3–12 $\supset I$			

7. Validity

a. Derive: $\sim (C \equiv \sim A)$

1	$\sim (C \vee A)$		Assumption
2	$C \equiv \sim A$		A / \sim I
3	$\sim A$		A / \sim E
4	C		2, 3 \equiv E
5	$C \vee A$		4 \vee I
6	$\sim (C \vee A)$		1 R
7	A		3-6 \sim E
8	$C \vee A$		7 \vee I
9	$\sim (C \vee A)$		1 R
10	$\sim (C \equiv \sim A)$		2-9 \sim I

c. Derive: $A \equiv B$

1	$\sim A \ \& \ \sim B$		Assumption
2	A		A / \equiv I
3	$\sim B$		A / \sim E
4	$\sim A$		1 &E
5	A		2 R
6	B		3-5 \sim E
7	B		A / \equiv I
8	$\sim A$		A / \sim E
9	$\sim B$		1 &E
10	B		7 R
11	A		8-10 \sim E
12	$A \equiv B$		2-6, 7-11 \equiv I

e. Derive: $\sim H$

1	$H \equiv \sim (I \& \sim J)$	Assumption
2	$\sim I \equiv \sim H$	Assumption
3	$J \supset \sim I$	Assumption
4	H	A / $\sim I$
5	$\sim (I \& \sim J)$	1, 4 $\equiv E$
6	$\sim I$	A / $\sim E$
7	$\sim H$	2, 6 $\equiv E$
8	H	4 R
9	I	6-8 $\sim E$
10	J	A / $\sim I$
11	$\sim I$	3, 10 $\supset E$
12	I	9 R
13	$\sim J$	10-12 $\sim I$
14	$I \& \sim J$	9, 13 & I
15	$\sim H$	4-14 $\sim I$

g. Derive: $H \vee \sim I$

1	$(F \vee G) \vee (H \vee \sim I)$	Assumption
2	$F \supset H$	Assumption
3	$I \supset \sim G$	Assumption
4	$F \vee G$	A / $\vee E$
5	F	A / $\vee E$
6	H	2, 5 $\supset E$
7	$H \vee \sim I$	6 $\vee I$
8	G	A / $\vee E$
9	I	A / $\sim I$
10	$\sim G$	3, 9 $\supset E$
11	G	8 R
12	$\sim I$	9-11 $\sim I$
13	$H \vee \sim I$	12 $\vee I$
14	$H \vee \sim I$	4, 5-7, 8-13 $\vee E$
15	$H \vee \sim I$	A / $\vee E$
16	$H \vee \sim I$	15 R
17	$H \vee \sim I$	1, 4-14, 15-16 $\vee E$

i. Derive: $F \vee (I \& \sim G)$

1	$\sim(F \vee \sim G) \equiv \sim(H \vee I)$	Assumption
2	$F \vee I$	Assumption
3	F	$A / \vee E$
4	$F \vee (I \& \sim G)$	$3 \vee I$
5	I	$A / \vee E$
6	$\sim(F \vee \sim G)$	$A / \sim E$
7	$\sim(H \vee I)$	$1, 6 \equiv E$
8	$H \vee I$	$5 \vee I$
9	$F \vee \sim G$	$6-8 \sim E$
10	F	$A / \vee E$
11	$F \vee (I \& \sim G)$	$10 \vee I$
12	$\sim G$	$A / \vee E$
13	$I \& \sim G$	$5, 12 \& I$
14	$F \vee (I \& \sim G)$	$13 \vee I$
15	$F \vee (I \& \sim G)$	$9, 10-11, 12-14 \vee E$
16	$F \vee (I \& \sim G)$	$2, 3-4, 5-15 \vee E$

k. Derive: $(\sim A \equiv \sim C) \supset (\sim A \equiv D)$

1	$(\sim A \equiv \sim C) \equiv (B \equiv \sim D)$	Assumption
2	$\sim A \supset \sim B$	Assumption
3	$C \supset \sim D$	Assumption
4	$\sim A \equiv \sim C$	$A / \supset I$
5	$\sim A$	$A / \equiv I$
6	$\sim D$	$A / \sim E$
7	$B \equiv \sim D$	$1, 4 \supset E$
8	B	$6, 7 \equiv E$
9	$\sim B$	$2, 5 \supset E$
10	D	$6-9 \sim E$
11	D	$A / \equiv I$
12	C	$A / \sim I$
13	$\sim D$	$3, 12 \supset E$
14	D	$11 R$
15	$\sim C$	$12-14 \sim I$
16	$\sim A$	$4, 15 \equiv E$
17	$\sim A \equiv D$	$5-10, 11-16 \equiv I$
18	$(\sim A \equiv \sim C) \supset (\sim A \equiv D)$	$4-17 \supset I$

m. Derive: $\sim E$

1	$\sim (A \supset B) \& (C \& \sim D)$		Assumption
2	$(B \vee \sim A) \vee [(C \& E) \supset D]$		Assumption
3	E		A / $\sim I$
4	B \vee $\sim A$		A / $\vee E$
5	B		A / $\vee E$
6	A		A / $\supset I$
7	B		5 R
8	A \supset B		6-7 $\supset I$
9	$\sim A$		A / $\vee E$
10	A		A / $\supset I$
11	$\sim B$		A / $\sim E$
12	A		10 R
13	$\sim A$		9 R
14	B		11-13 $\sim E$
15	A \supset B		10-14 $\supset I$
16	A \supset B		4, 5-8, 9-15 $\vee E$
17	(C & E) \supset D		A / $\vee E$
18	$\sim (A \supset B)$		A / $\sim E$
19	C & $\sim D$		1 &E
20	$\sim D$		19 &E
21	C		19 &E
22	C & E		3, 21 &I
22	D		17, 22 $\supset E$
23	A \supset B		18-22 $\sim E$
24	A \supset B		2, 4-16, 17-23 $\vee E$
25	$\sim (A \supset B)$		1 &E
26	$\sim E$		3-25 $\sim I$

8. Theorems

a. Derive: $\sim (A \supset B) \supset \sim (A \equiv B)$

1	$\sim (A \supset B)$		A / $\supset I$
2	A \equiv B		A / $\sim I$
3	A		A / $\supset I$
4	B		2, 3 $\equiv E$
5	A \supset B		3-4 $\supset I$
6	$\sim (A \supset B)$		1 R
7	$\sim (A \equiv B)$		2-6 $\sim I$
8	$\sim (A \supset B) \supset \sim (A \equiv B)$		1-7 $\supset I$

c. Derive: $(A \supset B) \vee (B \supset A)$

1		$\sim [(A \supset B) \vee (B \supset A)]$	A / \sim E
2		B	A / \sim I
3		A	A / \supset I
4		B	2 R
5		A \supset B	3-4 \supset I
6		$(A \supset B) \vee (B \supset A)$	5 \vee I
7		$\sim [(A \supset B) \vee (B \supset A)]$	1 R
8		\sim B	2-7 \sim I
9		B	A / \supset I
10		\sim A	A / \sim E
11		B	9 R
12		\sim B	8 R
13		A	10-12 \sim E
14		B \supset A	9-13 \supset I
15		$(A \supset B) \vee (B \supset A)$	14 \vee I
16		$\sim [(A \supset B) \vee (B \supset A)]$	1 R
17		$(A \supset B) \vee (B \supset A)$	1-16 \sim E

e. Derive: $[(A \vee B) \supset C] \equiv [(A \supset C) \& (B \supset C)]$

1		$(A \vee B) \supset C$	A / \equiv I
2		A	A / \supset I
3		A \vee B	2 \vee I
4		C	1, 3 \supset E
5		A \supset C	2-4 \supset I
6		B	A / \supset I
7		A \vee B	6 \vee I
8		C	1, 7 \supset E
9		B \supset C	6-8 \supset I
10		$(A \supset C) \& (B \supset C)$	5, 9 $\&$ I
11		$(A \supset C) \& (B \supset C)$	A / \equiv I
12		A \vee B	A / \supset I
13		A	A / \vee E
14		A \supset C	11 $\&$ E
15		C	13, 14 \supset E
16		B	A / \vee E
17		B \supset C	11 $\&$ E
18		C	16, 17 \supset E
19		C	12, 13-15, 16-18 \vee E
20		$(A \vee B) \supset C$	12-19 \supset I
21		$[(A \vee B) \supset C] \equiv [(A \supset C) \& (B \supset C)]$	1-10, 11-20 \equiv I

g. Derive: $\sim (A \equiv B) \equiv (A \equiv \sim B)$

1	$\sim (A \equiv B)$	$A / \equiv I$
2	A	$A / \equiv I$
3	B	$A / \sim I$
4	A	$A / \equiv I$
5	B	3 R
6	B	$A / \equiv I$
7	A	2 R
8	A \equiv B	4-5, 6-7 $\equiv I$
9	$\sim (A \equiv B)$	1 R
10	$\sim B$	3-9 $\sim I$
11	$\sim B$	$A / \equiv I$
12	$\sim A$	$A / \sim E$
13	A	$A / \equiv I$
14	$\sim B$	$A / \sim E$
15	A	13 R
16	$\sim A$	12 R
17	B	14-16 $\sim E$
18	B	$A / \equiv I$
19	$\sim A$	$A / \sim E$
20	B	18 R
21	$\sim B$	11 R
22	A	19-21 $\sim E$
23	A \equiv B	13-17, 18-22 $\equiv I$
24	$\sim (A \equiv B)$	1 R
25	A	12-24 $\sim E$
26	A $\equiv \sim B$	2-10, 11-25 $\equiv I$
27	A $\equiv \sim B$	$A / \equiv I$
28	A \equiv B	$A / \sim I$
29	B	$A / \sim I$
30	A	28, 29 $\equiv E$
31	$\sim B$	27, 30 $\equiv E$
32	B	29 R
33	$\sim B$	29-32 $\sim I$
34	$\sim B$	$A / \sim E$
35	A	27, 34 $\equiv E$
36	B	28, 35 $\equiv E$
37	$\sim B$	34 R
38	B	34-37 $\sim E$
39	$\sim (A \equiv B)$	28-38 $\sim I$
40	$\sim (A \equiv B) \equiv (A \equiv \sim B)$	1-26, 27-39 $\equiv I$

9. Equivalence

a. Derive: $\sim \sim A$

1	A	Assumption		
2	<table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">$\sim A$</td> <td></td> </tr> </table>	$\sim A$		A / \sim I
$\sim A$				
3	<table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">A</td> <td></td> </tr> </table>	A		1 R
A				
4	<table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">$\sim A$</td> <td></td> </tr> </table>	$\sim A$		2 R
$\sim A$				
5	$\sim \sim A$	2-4 \sim I		

Derive: A

1	$\sim \sim A$	Assumption		
2	<table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">$\sim A$</td> <td></td> </tr> </table>	$\sim A$		A / \sim E
$\sim A$				
3	<table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">$\sim A$</td> <td></td> </tr> </table>	$\sim A$		2 R
$\sim A$				
4	<table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">$\sim \sim A$</td> <td></td> </tr> </table>	$\sim \sim A$		1 R
$\sim \sim A$				
5	A	2-4 \sim E		

c. Derive: $A \vee A$

1	A	Assumption
2	$A \vee A$	A / \vee I

Derive: $A \vee A$

1	$A \vee A$	Assumption		
2	<table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">A</td> <td></td> </tr> </table>	A		A / \vee E
A				
3	<table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">A</td> <td></td> </tr> </table>	A		2 R
A				
4	A	1, 2-3, 2-3 \vee E		

e. Derive: $B \vee A$

1	$A \vee B$	Assumption		
2	<table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">A</td> <td></td> </tr> </table>	A		A / \vee E
A				
3	<table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">$B \vee A$</td> <td></td> </tr> </table>	$B \vee A$		2 \vee I
$B \vee A$				
4	<table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">B</td> <td></td> </tr> </table>	B		A / \vee E
B				
5	<table style="border-collapse: collapse;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">$B \vee A$</td> <td></td> </tr> </table>	$B \vee A$		4 \vee I
$B \vee A$				
6	$B \vee A$	1, 2-3, 4-5 \vee E		

Derive: $A \vee B$

1	$B \vee A$	Assumption
2	B	$A / \vee E$
3	$A \vee B$	$2 \vee I$
4	A	$A / \vee E$
5	$A \vee B$	$4 \vee I$
6	$A \vee B$	$1, 2-3, 4-5 \vee E$

g. Derive: $(A \vee B) \vee C$

1	$A \vee (B \vee C)$	Assumption
2	A	$A / \vee E$
3	$A \vee B$	$2 \vee I$
4	$(A \vee B) \vee C$	$3 \vee I$
5	$B \vee C$	$A / \vee E$
6	B	$A / \vee E$
7	$A \vee B$	$6 \vee I$
9	$(A \vee B) \vee C$	$7 \vee I$
10	C	$A / \vee I$
11	$(A \vee B) \vee C$	$10 \vee I$
12	$(A \vee B) \vee C$	$5, 6-9, 10-11 \vee I$
13	$(A \vee B) \vee C$	$1, 2-4, 5-12 \vee E$

Derive: $A \vee (B \vee C)$

1	$(A \vee B) \vee C$	Assumption
2	$A \vee B$	$A / \vee E$
3	A	$A / \vee E$
4	$A \vee (B \vee C)$	$3 \vee I$
5	B	$A / \vee E$
6	$B \vee C$	$5 \vee I$
7	$A \vee (B \vee C)$	$6 \vee I$
8	$A \vee (B \vee C)$	$2, 3-4, 5-7 \vee E$
9	C	$A / \vee E$
10	$B \vee C$	$9 \vee I$
11	$A \vee (B \vee C)$	$10 \vee I$
12	$A \vee (B \vee C)$	$1, 2-8, 9-11 \vee E$

i. Derive: $\sim B \supset \sim A$

1	A \supset B	Assumption
2	\sim B	A / \supset I
3	A	A / \sim I
4	B	1, 3 \supset E
5	\sim B	2 R
6	\sim A	3-5 \sim I
7	\sim B \supset \sim A	2-6 \supset I

Derive: $A \supset B$

1	\sim B \supset \sim A	Assumption
2	A	A / \supset I
3	\sim B	A / \sim E
4	A	2 R
5	\sim A	1, 3 \supset E
6	B	3-5 \sim E
7	A \supset B	2-6 \supset I

k. Derive: $A \equiv B$

1	(A & B) \vee (\sim A & \sim B)	Assumption
2	A & B	A / \vee E
3	A	A / \equiv I
4	B	2 &E
5	B	A / \equiv I
6	A	2 &E
7	A \equiv B	3-4, 5-6 \equiv I
8	\sim A & \sim B	A / \vee E
9	A	A / \equiv I
10	\sim B	A / \sim E
11	A	9 R
12	\sim A	8 &E
13	B	10-12 \sim E
14	B	A / \equiv I
15	\sim A	A / \sim E
16	B	14 R
17	\sim B	8 &E
18	A	15-17 \sim E
19	A \equiv B	9-13, 14-18 \equiv I
20	A \equiv B	1, 2-7, 8-19 \vee E

Derive: $(A \& B) \vee (\sim A \& \sim B)$

1	A \equiv B		Assumption
2	$\sim [(A \& B) \vee (\sim A \& \sim B)]$		A / \sim E
3	A		A / \sim I
4	B		1, 3 \equiv E
5	A & B		3, 4 &I
6	(A & B) \vee (\sim A & \sim B)		5 \vee I
7	$\sim [(A \& B) \vee (\sim A \& \sim B)]$		2 R
8	\sim A		3-7 \sim I
9	B		A / \sim I
10	A		1, 9 \equiv E
11	\sim A		8 R
12	\sim B		9-11 \sim I
13	\sim A & \sim B		8, 12 &I
14	(A & B) \vee (\sim A & \sim B)		13 \vee I
15	$\sim [(A \& B) \vee (\sim A \& \sim B)]$		2 R
16	(A & B) \vee (\sim A & \sim B)		2-15 \sim E

m. Derive: $(A \vee B) \& (A \vee C)$

1	A \vee (B & C)		Assumption
2	A		A / \vee E
3	A \vee B		2 \vee I
4	A \vee C		2 \vee I
5	(A \vee B) & (A \vee C)		3, 4 &I
6	B & C		A / \vee E
7	B		6 & E
8	A \vee B		7 \vee I
9	C		6 &E
10	A \vee C		9 \vee I
11	(A \vee B) & (A \vee C)		8, 10 &I
12	(A \vee B) & (A \vee C)		1, 2-5, 6-11 \vee E

Derive: $A \vee (B \& C)$

1		$(A \vee B) \& (A \vee C)$	Assumption
2		$A \vee B$	1 &E
3		A	A / \vee E
4		$A \vee (B \& C)$	3 \vee I
5		B	A / \vee E
6		$A \vee C$	1 &E
7		A	A / \vee E
8		$A \vee (B \& C)$	7 \vee I
9		C	A / \vee E
10		$B \& C$	5, 9 &I
11		$A \vee (B \& C)$	10 \vee I
12		$A \vee (B \& C)$	6, 7-8, 9-11 \vee E
13		$A \vee (B \& C)$	2, 3-4, 5-12 \vee E

10. Inconsistency

a. Derive: $B, \sim B$

1		$(A \supset B) \& (A \supset \sim B)$	Assumption
2		$(C \supset A) \& (\sim C \supset A)$	Assumption
3		$A \supset B$	1 &E
4		$A \supset \sim B$	1 &E
5		C	A / \sim I
6		$C \supset A$	2 &E
7		A	5, 6 \supset E
8		B	3, 7 \supset E
9		$\sim B$	4, 7 \supset E
10		$\sim C$	5-9 \sim I
11		$\sim C \supset A$	2 &E
12		A	10, 11 \supset E
13		B	3, 12 \supset E
14		$\sim B$	4, 12 \supset E

c. Derive: $A, \sim A$

1	$C \equiv \sim A$		Assumption
2	$C \equiv A$		Assumption
3	A		$A / \sim I$
4	C		2, 3 $\equiv E$
5	$\sim A$		1, 4 $\equiv E$
6	A		3 R
7	$\sim A$		3-6 $\sim I$
8	$\sim A$		$A / \sim E$
9	C		1, 8 $\equiv E$
10	A		2, 9 $\equiv E$
11	$\sim A$		8 R
12	A		8-11 $\sim E$

e. Derive: $A, \sim A$

1	$\sim [(A \vee B) \vee C]$		Assumption
2	$A \equiv \sim C$		Assumption
3	A		$A / \sim I$
4	$A \vee B$		3 $\vee I$
5	$(A \vee B) \vee C$		4 $\vee I$
6	$\sim [(A \vee B) \vee C]$		1 R
7	$\sim A$		3-6 $\sim I$
8	C		$A / \sim I$
9	$(A \vee B) \vee C$		8 $\vee I$
10	$\sim [(A \vee B) \vee C]$		1 R
11	$\sim C$		8-10 $\sim I$
12	A		2, 11 $\equiv E$

g. Derive: $B, \sim B$

1	A & (B \vee C)	Assumption
2	$(\sim C \vee H) \& (H \supset \sim H)$	Assumption
3	$\sim B$	Assumption
4	<u>B \vee C</u>	1 &E
5	<u>B</u>	A / \vee E
6	B	5 R
7	<u>C</u>	A / \vee E
8	$\sim C \vee H$	2 &E
9	<u>$\sim C$</u>	A / \vee E
10	<u>$\sim B$</u>	A / \sim E
11	C	7 R
12	$\sim C$	9 R
13	B	10-12 \sim E
14	<u>H</u>	A / \vee E
15	<u>$\sim B$</u>	A / \sim E
16	H $\supset \sim H$	2 &E
17	$\sim H$	14, 16 \supset E
18	H	14 R
19	B	15-18 \sim E
20	B	8, 9-13, 14-19 \vee E
21	B	4, 5-6, 7-20 \vee E
22	$\sim B$	3 R

11. Validity

a. Derive: M

1	S & F	Assumption
2	F \supset B	Assumption
3	<u>(B & \sim M) $\supset \sim$ S</u>	Assumption
4	<u>\sim M</u>	A / \sim E
5	F	1 &E
6	B	2, 5 \supset E
7	B & \sim M	4, 6 &I
8	\sim S	3, 7 \supset E
9	S	1 &E
10	M	4-9 \sim E

c. Derive: $\sim J$

1	(C \supset \sim R) & (R \supset L)		
2	C \equiv (C \vee L)		Assumption
3	J \supset R		Assumption
4	J		A / \sim I
5	R		3, 4 \supset E
6	R \supset L		1 &E
7	L		5, 6 \supset E
8	C \vee L		7 \vee I
9	C		2, 8 \equiv E
10	C \supset \sim R		1 &E
11	\sim R		9, 10 \supset E
12	\sim J		4-11 \sim I

e. Derive: $\sim M$

1	\sim (R \vee W)		
2	(R \equiv M) \vee [(M \vee G) \supset (W \equiv M)]		Assumption
3	M		A / \sim I
4	R \equiv M		A / \vee E
5	R		3, 4 \equiv E
6	R \vee W		5 \vee I
7	(M \vee G) \supset (W \equiv M)		A / \vee E
8	M \vee G		3 \vee I
9	W \equiv M		7, 8 \supset E
10	W		3, 9 \equiv E
11	R \vee W		10 \vee I
12	R \vee W		2, 4-6, 7-11 \vee E
13	\sim (R \vee W)		1 R
14	\sim M		3-13 \sim I

g. Derive: $H \supset J$

1	$(H \ \& \ T) \supset J$	Assumption
2	$(M \supset D) \ \& \ (M \vee D)$	Assumption
3	$\sim T \equiv (\sim D \ \& \ M)$	Assumption
4	H	A / \supset I
5	$\sim J$	A / \sim E
6	T	A / \sim I
7	H & T	4, 6 &I
8	J	1, 7 \supset E
9	$\sim J$	5 R
10	$\sim T$	6-9 \sim I
11	$\sim D \ \& \ M$	3, 10 \equiv E
12	M \supset D	2 &E
13	M	11 &E
14	D	12, 13 \supset E
15	$\sim D$	11 &E
16	J	5-15 \sim E
17	H \supset J	4-16 \supset I

i. Derive: $L \supset T$

1	$L \supset (C \vee T)$	Assumption
2	$(\sim L \vee B) \ \& \ (\sim B \vee \sim C)$	Assumption
3	L	$A / \supset I$
4	$C \vee T$	1, 3 $\supset E$
5	C	$A / \vee E$
6	$\sim B \vee \sim C$	2 $\&E$
7	$\sim B$	$A / \vee E$
8	$\sim L \vee B$	2 $\&E$
9	$\sim L$	$A / \vee E$
10	$\sim T$	$A / \sim E$
11	L	3 R
12	$\sim L$	9 R
13	T	10-12 $\sim E$
14	B	$A / \vee E$
15	$\sim T$	$A / \sim E$
16	B	14 R
17	$\sim B$	7 R
18	T	15-17 $\sim E$
19	T	8, 9-13, 14-18 $\vee E$
20	$\sim C$	$A / \vee E$
21	$\sim T$	$A / \sim E$
22	$\sim C$	20 R
23	C	5 R
24	T	21-23 $\sim E$
25	T	6, 7-19, 20-24 $\vee E$
26	T	$A / \vee E$
27	T	26 R
28	T	4, 5-25, 26-27 $\vee E$
29	$L \supset T$	3-28 $\supset I$

12. Inconsistency

a. 1	$(M \supset B) \ \& \ (B \supset P)$	Assumption
2	$M \ \& \ \sim P$	Assumption
3	M	2 $\&E$
4	$M \supset B$	1 $\&E$
5	B	3, 4 $\supset E$
6	$B \supset P$	1 $\&E$
7	P	5, 6 $\supset E$
8	$\sim P$	2 $\&E$

c.	1	B \supset I	Assumption
	2	(\sim B & \sim I) \supset C	Assumption
	3	\sim C & \sim I	Assumption
	4	B	A / \sim I
	5	I	1, 4 \supset E
	6	\sim I	3 &E
	7	\sim B	4-6 \sim I
	8	\sim I	3 &E
	9	\sim B & \sim I	7, 8 &I
	10	C	2, 9 \supset E
	11	\sim C	3 &E

e.	1	M \vee (F \supset T)	Assumption
	2	N \equiv \sim T	Assumption
	3	(F & N) & \sim M	Assumption
	4	M	A / \vee E
	5	M	4 R
	6	F \supset T	A / \vee E
	7	\sim M	A / \sim E
	8	F & N	3 &E
	9	F	8 &E
	10	T	6, 9 \supset E
	11	N	8 &E
	12	\sim T	2, 11 \equiv E
	13	M	7-12 \sim E
	14	M	1, 4-5, 6-13 \vee E
	15	\sim M	3 &E

13.a. We do not want this rule as a rule of *SD* because it is not truth-preserving. The truth of $\mathbf{P} \vee \mathbf{Q}$ does not entail the truth of \mathbf{P} .

c. We can show that Reiteration is dispensable by explaining how to derive \mathbf{P} whenever \mathbf{P} occurs on an earlier accessible line, without using Reiteration. Assume that \mathbf{P} occurs on an accessible line i and that we want to derive \mathbf{P} on a later line. We can do this as follows:

	i	P	...
	n	P & P	i, i &I
	n + 1	P	n &E

e. If \mathbf{P} is a theorem of SD then any argument of SL that has $\sim \mathbf{P}$ among its premises is valid in SD . We can construct a derivation of the conclusion, call it \mathbf{Q} , by taking the premises of the argument as our primary assumption. $\sim \mathbf{P}$ will be one of these assumptions. Next assume $\sim \mathbf{Q}$, derive both \mathbf{P} and $\sim \mathbf{P}$, and obtain \mathbf{Q} by Negation Elimination. We can obtain $\sim \mathbf{P}$ by Reiteration since it is one of the primary assumptions of the derivation. We can obtain \mathbf{P} because it is a theorem of SD and therefore can be derived from the empty set. If it can be derived from the empty set it can also be derived from the set consisting of the premises of the argument, by inserting the derivation of \mathbf{P} from the empty set within the scope of the assumption $\sim \mathbf{Q}$.

14. We here make use of a result established in Sections 6.3 and 6.4,

$\Gamma \vdash \mathbf{P}$ in SD if and only if $\Gamma \models \mathbf{P}$

a. Assume that a given argument is valid in SD . Then we know that its conclusion is derivable in SD from the set consisting of its premises. By the above result it follows that the conclusion of the argument is truth-functionally entailed by the set consisting of the premises of the argument. Therefore there is no truth-value assignment on which the members of the set, which are just the premises of the argument, are true and the conclusion of the argument false. So the argument is truth-functionally valid. Conversely, assume that the given argument is truth-functionally valid. So there is no truth-value assignment on which the premises of the argument are true and the conclusion false. From this it follows that the set consisting of the premises of the argument truth-functionally entails the conclusion of the argument. And by the above result it next follows that the conclusion of the argument is derivable from the set consisting of the premises of the argument, and from this it follows that the argument is valid in SD .

c. Assume that sentences \mathbf{P} and \mathbf{Q} of SL are equivalent in SD . Then each can be derived from the unit set of the other. By the above result it follows that the unit set of each truth-functionally entails the other. So there is no truth-value assignment on which \mathbf{P} is true and \mathbf{Q} false, and no truth-value assignment on which \mathbf{Q} is true and \mathbf{P} false. So \mathbf{P} and \mathbf{Q} are truth-functionally equivalent.

Now assume that \mathbf{P} and \mathbf{Q} are truth-functionally equivalent. Then there is no truth-value assignment on which \mathbf{P} is true and \mathbf{Q} is false, and no truth-value assignment on which \mathbf{Q} is true and \mathbf{P} is false. So \mathbf{P} truth-functionally entails \mathbf{Q} and \mathbf{Q} truth-functionally entails \mathbf{P} . By the above result it follows that \mathbf{Q} is derivable in SD from $\{\mathbf{P}\}$ and \mathbf{P} is derivable in SD from $\{\mathbf{Q}\}$.

Section 5.4E

1. Derivability

a. Derive: $\sim D$

1	$D \supset E$	Assumption
2	$E \supset (Z \ \& \ W)$	Assumption
3	$\sim Z \vee \sim W$	Assumption
4	$\sim (Z \ \& \ W)$	3 DeM
5	$\sim E$	2, 4 MT
6	$\sim D$	1, 5 MT

c. Derive: K

1	$(W \supset S) \ \& \ \sim M$	Assumption
2	$(\sim W \supset H) \vee M$	Assumption
3	$(\sim S \supset H) \supset K$	Assumption
4	$W \supset S$	1 &E
5	$\sim S \supset \sim W$	4 Trans
6	$\sim M$	1 &E
7	$\sim W \supset H$	2, 6 DS
8	$\sim S \supset H$	5, 7 HS
9	K	3, 8 \supset E

e. Derive: C

1	$(M \vee B) \vee (C \vee G)$	Assumption
2	$\sim B \ \& \ (\sim G \ \& \ \sim M)$	Assumption
3	$\sim B$	2 &E
4	$(B \vee M) \vee (C \vee G)$	1 Com
5	$B \vee [M \vee (C \vee G)]$	4 Assoc
6	$M \vee (C \vee G)$	3, 5 DS
7	$\sim G \ \& \ \sim M$	2 &E
8	$\sim G$	7 &E
9	$(M \vee C) \vee G$	6 Assoc
10	$M \vee C$	8, 9 DS
11	$\sim M$	7 &E
12	C	10, 11 DS

2. Validity

a. Derive: $Y \equiv Z$

1	~ Y \supset ~ Z		Assumption
2	~ Z \supset ~ X		Assumption
3	~ X \supset ~ Y		Assumption
4	Y		A / \equiv I
5	~ Z \supset ~ Y		2, 3 HS
6	Y \supset Z		5 Trans
7	Z		4, 6 \supset E
8	Z		A / \equiv I
9	Z \supset Y		1 Trans
10	Y		8, 9 \supset E
11	Y \equiv Z		4-7, 8-10 \equiv I

c. Derive: $I \supset \sim D$

1	(F & G) \vee (H & ~ I)		Assumption
2	I \supset ~ (F & D)		Assumption
3	I		A / \supset I
4	~ (F & D)		2, 3 \supset E
5	~ F \vee ~ D		4 DeM
6	~ ~ I		3 DN
7	~ H \vee ~ ~ I		6 \vee I
8	~ (H & ~ I)		7 DeM
9	F & G		1, 8 DS
10	F		9 &E
11	~ ~ F		10 DN
12	~ D		5, 11 DS
13	I \supset ~ D		3-12 \supset I

e. Derive: $I \vee H$

1	$F \supset (G \supset H)$		Assumption
2	$\sim I \supset (F \vee H)$		Assumption
3	$F \supset G$		Assumption
4	$\sim I$		A / $\supset I$
5	$F \vee H$		2, 4 $\supset E$
6	$\sim H$		A / $\sim E$
7	F		5, 6 DS
8	G		3, 7 $\supset E$
9	$G \supset H$		1, 7 $\supset E$
10	$\sim G$		6, 9 MT
11	H		6-10 $\sim E$
12	$\sim I \supset H$		4-11 $\supset I$
13	$\sim \sim I \vee H$		12 Impl
14	$I \vee H$		13 DN

g. Derive: $X \equiv Y$

1	$[(X \& Z) \& Y] \vee (\sim X \supset \sim Y)$		Assumption
2	$X \supset Z$		Assumption
3	$Z \supset Y$		Assumption
4	X		A / $\equiv I$
5	Z		2, 4 $\supset E$
6	Y		3, 5 $\supset E$
7	Y		A / $\equiv I$
8	$(X \& Z) \& Y$		A / $\vee E$
9	$X \& Z$		8 &E
10	X		9 &E
11	$\sim X \supset \sim Y$		A / $\vee E$
12	$Y \supset X$		11 Trans
13	X		7, 12 $\supset E$
14	X		1, 8-10, 11-13 $\vee E$
15	$X \equiv Y$		4-6, 7-14 $\equiv I$

3. Theorems

a. Derive: $A \vee \sim A$

1	$\sim (A \vee \sim A)$		A / $\sim E$
2	$\sim A \& \sim \sim A$		1 DeM
3	$\sim A$		2 &E
4	$\sim \sim A$		2 &E
5	$A \vee \sim A$		1-4 $\sim E$

c. Derive: $A \vee [(\sim A \vee B) \& (\sim A \vee C)]$

1	$\sim A$	$A / \supset I$
2	$\sim A \vee (B \& C)$	$1 \vee I$
3	$(\sim A \vee B) \& (\sim A \vee C)$	2 Dist
4	$\sim A \supset [(\sim A \vee B) \& (\sim A \vee C)]$	$1-3 \supset I$
5	$\sim \sim A \vee [(\sim A \vee B) \& (\sim A \vee C)]$	4 Impl
6	$A \vee [(\sim A \vee B) \& (\sim A \vee C)]$	5 DN

e. Derive: $[A \supset (B \& C)] \equiv [(\sim B \vee \sim C) \supset \sim A]$

1	$A \supset (B \& C)$	$A / \equiv I$
2	$\sim (B \& C) \supset \sim A$	1 Trans
3	$(\sim B \vee \sim C) \supset \sim A$	2 DeM
4	$(\sim B \vee \sim C) \supset \sim A$	$A / \equiv I$
5	$\sim (B \& C) \supset \sim A$	4 DeM
6	$A \supset (B \& C)$	5 Trans
7	$[A \supset (B \& C)] \equiv [(\sim B \vee \sim C) \supset \sim A]$	$1-3, 4-6 \equiv I$

g. Derive: $[A \supset (B \equiv C)] \equiv (A \supset [(\sim B \vee C) \& (\sim C \vee B)])$

1	$A \supset (B \equiv C)$	$A / \equiv I$
2	$A \supset [(B \supset C) \& (C \supset B)]$	1 Equiv
3	$A \supset [(\sim B \vee C) \& (C \supset B)]$	2 Impl
4	$A \supset [(\sim B \vee C) \& (\sim C \vee B)]$	3 Impl
5	$A \supset [(\sim B \vee C) \& (\sim C \vee B)]$	$A / \equiv I$
6	$A \supset [(B \supset C) \& (\sim C \vee B)]$	5 Impl
7	$A \supset [(B \supset C) \& (C \supset B)]$	6 Impl
8	$A \supset (B \equiv C)$	7 Equiv
9	$[A \supset (B \equiv C)] \equiv (A \supset [(\sim B \vee C) \& (\sim C \vee B)])$	$1-4, 5-8 \equiv I$

i. Derive: $[\sim A \supset (\sim B \supset C)] \supset [(A \vee B) \vee (\sim \sim B \vee C)]$

1	$\sim A \supset (\sim B \supset C)$	$A / \supset I$
2	$\sim \sim A \vee (\sim B \supset C)$	1 Impl
3	$\sim \sim A \vee (\sim \sim B \vee C)$	2 Impl
4	$A \vee (\sim \sim B \vee C)$	3 DN
5	$A \vee [(\sim \sim B \vee \sim \sim B) \vee C]$	4 Idem
6	$A \vee [\sim \sim B \vee (\sim \sim B \vee C)]$	5 Assoc
7	$(A \vee \sim \sim B) \vee (\sim \sim B \vee C)$	6 Assoc
8	$(A \vee B) \vee (\sim \sim B \vee C)$	7 DN
9	$[\sim A \supset (\sim B \supset C)] \supset [(A \vee B) \vee (\sim \sim B \vee C)]$	$1-8 \supset I$

4. Equivalence

a. Derive: $\sim (\sim A \ \& \ \sim B)$

1	$A \vee B$	Assumption
2	$\sim \sim A \vee B$	1 DN
3	$\sim \sim A \vee \sim \sim B$	2 DN
4	$\sim (\sim A \ \& \ \sim B)$	3 DeM

Derive: $A \vee B$

1	$\sim (\sim A \ \& \ \sim B)$	Assumption
2	$\sim \sim A \vee \sim \sim B$	1 DeM
3	$A \vee \sim \sim B$	2 DN
4	$A \vee B$	3 DN

c. Derive: $\sim (A \supset C) \supset \sim B$

1	$(A \ \& \ B) \supset C$	Assumption
2	$(B \ \& \ A) \supset C$	1 Com
3	$B \supset (A \supset C)$	2 Exp
4	$\sim (A \supset C) \supset \sim B$	3 Trans

Derive: $(A \ \& \ B) \supset C$

1	$\sim (A \supset C) \supset \sim B$	Assumption
2	$B \supset (A \supset C)$	1 Trans
3	$(B \ \& \ A) \supset C$	2 Exp
4	$(A \ \& \ B) \supset C$	3 Com

e. Derive: $A \vee (\sim B \equiv \sim C)$

1	$A \vee (B \equiv C)$	Assumption
2	$A \vee [(B \supset C) \ \& \ (C \supset B)]$	1 Equiv
3	$A \vee [(\sim C \supset \sim B) \ \& \ (C \supset B)]$	2 Trans
4	$A \vee [(\sim C \supset \sim B) \ \& \ (\sim B \supset \sim C)]$	3 Trans
5	$A \vee [(\sim B \supset \sim C) \ \& \ (\sim C \supset \sim B)]$	4 Com
6	$A \vee (\sim B \equiv \sim C)$	5 Equiv

Derive: $A \vee (B \equiv C)$

1	$A \vee (\sim B \equiv \sim C)$	Assumption
2	$A \vee [(\sim B \supset \sim C) \ \& \ (\sim C \supset \sim B)]$	1 Equiv
3	$A \vee [(C \supset B) \ \& \ (\sim C \supset \sim B)]$	2 Trans
4	$A \vee [(C \supset B) \ \& \ (B \supset C)]$	3 Trans
5	$A \vee [(B \supset C) \ \& \ (C \supset B)]$	4 Com
6	$A \vee (B \equiv C)$	5 Equiv

5. Inconsistency

a. Derive: $[(E \ \& \ F) \vee \sim \sim G] \supset M, \sim([(E \ \& \ F) \vee \sim \sim G] \supset M)$

1	$[(E \ \& \ F) \vee \sim \sim G] \supset M$		
2	$\sim [(G \vee E) \ \& \ (F \vee G)] \supset (M \ \& \ M)$		Assumption
3	$\sim [(G \vee E) \ \& \ (F \vee G)] \supset M$		2 Idem
4	$\sim [(G \vee E) \ \& \ (G \vee F)] \supset M$		3 Com
5	$\sim [G \vee (E \ \& \ F)] \supset M$		4 Dist
6	$\sim [(E \ \& \ F) \vee G] \supset M$		5 Com
7	$\sim [(E \ \& \ F) \vee \sim \sim G] \supset M$		6 DN
8	$[(E \ \& \ F) \vee \sim \sim G] \supset M$		1 R

c. Derive: $K \equiv \sim S, \sim (K \equiv \sim S)$

1	$M \ \& \ L$		Assumption
2	$[L \ \& \ (M \ \& \ \sim S)] \supset K$		Assumption
3	$\sim K \vee \sim S$		Assumption
4	$\sim (K \equiv \sim S)$		Assumption
5	$K \supset \sim S$		3 Impl
6	$[L \ \& \ M] \ \& \ \sim S \supset K$		2 Assoc
7	$(L \ \& \ M) \supset (\sim S \supset K)$		6 Exp
8	$L \ \& \ M$		1 Com
9	$\sim S \supset K$		7, 8 $\supset E$
10	$(K \supset \sim S) \ \& \ (\sim S \supset K)$		5, 9 $\ \& I$
11	$K \equiv \sim S$		10 Equiv
12	$\sim (K \equiv \sim S)$		4 R

e. Derive: $Z, \sim Z$

1	$\sim [W \ \& \ (Z \vee Y)]$		Assumption
2	$(Z \supset Y) \supset Z$		Assumption
3	$(Y \supset Z) \supset W$		Assumption
4	$\sim W \vee \sim (Z \vee Y)$		1 DeM
5	$\sim Z$		A / $\sim E$
6	$\sim (Z \supset Y)$		2, 5 MT
7	$\sim (\sim Z \vee Y)$		6 Impl
8	$\sim \sim Z \ \& \ \sim Y$		7 DeM
9	$\sim \sim Z$		8 $\ \& E$
10	$\sim Z$		5 R
11	Z		5-10 $\sim E$
12	$Z \vee Y$		11 $\vee I$
13	$\sim \sim (Z \vee Z)$		12 DN
14	$\sim W$		4, 13 DS
15	$\sim (Y \supset Z)$		3, 14 MT
16	$\sim (\sim Y \vee Z)$		15 Impl
17	$\sim \sim Y \ \& \ \sim Z$		16 DeM
18	$\sim Z$		17 $\ \& E$

6. Validity

a. Derive: $\sim B$

1		$(R \supset C) \vee (B \supset C)$	Assumption
2		$\sim (E \ \& \ A) \supset \sim (R \supset C)$	Assumption
3		$\sim E \ \& \ \sim C$	Assumption
4		$\sim E$	3 &E
5		$\sim E \vee \sim A$	4 \vee I
6		$\sim (E \ \& \ A)$	5 DeM
7		$\sim (R \supset C)$	2, 6 \supset E
8		$B \supset C$	1, 7 DS
9		$\sim C$	3 &E
10		$\sim B$	8, 9 MT

c. Derive: $\sim W \supset \sim A$

1		$A \supset [W \vee \sim (C \vee R)]$	Assumption
2		$\sim R \supset C$	Assumption
3		$\sim W$	A / \supset I
4		A	A / \sim I
5		$W \vee \sim (C \vee R)$	1, 4 \supset E
6		$\sim (C \vee R)$	3, 5 DS
7		$\sim \sim R \vee C$	2 Impl
8		$R \vee C$	7 DN
9		$C \vee R$	8 Com
10		$\sim A$	4-9 \sim I
11		$\sim W \supset \sim A$	3-10 \supset I

e. Derive: $J \supset \sim (E \vee \sim M)$

1		$\sim (J \ \& \ \sim H)$	Assumption
2		$\sim H \vee M$	Assumption
3		$E \supset \sim M$	Assumption
4		J	A / \supset I
5		$\sim J \vee \sim \sim H$	1 DeM
6		$\sim \sim J$	4 DN
7		$\sim \sim H$	5, 6 DS
8		M	2, 7 DS
9		$\sim \sim M$	8 DN
10		$\sim E$	3, 9 MT
11		$\sim E \ \& \ \sim \sim M$	9, 10 &I
12		$\sim (E \vee \sim M)$	11 DeM
13		$J \supset \sim (E \vee \sim M)$	4-12 \supset I

g. Derive: $\sim A \supset [H \supset (F \& B)]$

1	$(H \& \sim S) \supset A$		Assumption
2	$\sim B \supset \sim S$		Assumption
3	$\sim S \vee C$		Assumption
4	$C \supset F$		Assumption
5	$\sim A$		$A / \supset I$
6	H		$A / \supset I$
7	$H \supset (\sim S \supset A)$		1 Exp
8	$\sim S \supset A$		6, 7 $\supset E$
9	$\sim \sim S$		5, 8 MT
10	C		3, 9 DS
11	F		4, 10 $\supset E$
12	$\sim \sim B$		2, 9 MT
13	B		12 DN
14	$F \& B$		11, 13 $\&I$
15	$H \supset (F \& B)$		6–14 $\supset I$
16	$\sim A \supset [H \supset (F \& B)]$		5–15 $\supset I$

7. Inconsistency

a. Derive: $C, \sim C$

1	$B \vee \sim C$		Assumption
2	$(L \supset \sim G) \supset C$		Assumption
3	$(G \equiv \sim B) \& (\sim L \supset \sim B)$		Assumption
4	$\sim L$		Assumption
5	$\sim L \vee \sim G$		4 $\vee I$
6	$L \supset \sim G$		5 Impl
7	C		2, 6 $\supset E$
8	$\sim L \supset \sim B$		3 $\&E$
9	$\sim B$		4, 8 $\supset E$
10	$\sim C$		1, 9 DS

8.a. The rules of replacement are two-way rules. If we can derive **Q** from **{P}** by using only these rules, then we can also derive **P** from **{Q}** using the same rules but in reverse order.

c. Suppose that before a current line **n** of a derivation, an accessible line **i** contains a sentence of the form **P \supset Q**. The sentence **P \supset (P & Q)** can be derived by using the following routine:

i	P \supset Q		
n	P		Assumption
n + 1	Q		i, n $\supset E$
n + 2	P & Q		n, n + 1 $\&I$
n + 3	P \supset (P & Q)		n – n + 2 $\supset I$