
CHAPTER SEVEN

Section 7.1E

- 1.a. 'The President' is a singular term, 'Democrat' is not.
 x is a Democrat
- c. 'Sarah' and 'Smith College' are the singular terms.
 x attends Smith College
 Sarah attends x
 x attends y
- e. The singular terms are 'Charles' and 'Rita'.
 w and Rita are siblings.
 Charles and w are siblings.
 w and z are siblings.

Section 7.2E

- 1.a. Formula and sentence
 $Ba \& Hc$ truth-functional
 Ba atomic
 Hc atomic
- c. Formula and sentence
 $\sim(\forall y)Fya$ truth-functional
 $(\forall y)Fya$ quantified
 Fya atomic
- e. Not a formula: ' $(\exists a)$ ' is not a quantifier as ' a ' is a constant, not a variable.
- g. Formula and sentence
 $(\forall x)(\forall y) \sim Hxy$ quantified
 $(\forall y) \sim Hxy$ quantified
 $\ominus Hxy$ truth-functional
 Hxy atomic
- i. Formula and sentence
 $(\forall y) \sim Fy \oplus \sim(\exists w)Fw$ truth-functional
 $(\forall y) \sim Fy$ quantified
 $\ominus(\exists w)Fw$ truth-functional
 $\ominus Fy$ truth-functional
 Fy atomic
 $(\exists w)Fw$ quantified
 Fw atomic

- k. Formula and sentence
- | | |
|-------------------------------|------------------|
| $(\exists z)(Fz \& \sim Baz)$ | quantified |
| $Fz \& \sim Baz$ | truth-functional |
| Fz | atomic |
| $\neg Baz$ | truth-functional |
| Baz | atomic |
- m. Formula and sentence
- | | |
|---|------------------|
| $(\exists x)Fx \vee \sim (\exists x)Fx$ | truth-functional |
| $(\exists x)Fx$ | quantified |
| Fx | atomic |
| $\neg (\exists x)Fx$ | truth-functional |
| $(\exists x)Fx$ | quantified |
| Fx | atomic |
- o. Not a formula. There is no 'y' variable for ' $(\exists y)$ ' to bind.
- q. Formula and sentence
- | | |
|----------------------------|------------------|
| $Fa \supset (\exists x)Fx$ | truth-functional |
| Fa | atomic |
| $(\exists x)Fx$ | quantified |
| Fx | atomic |
- s. Formula but not a sentence. The first 'w' is free.
- | | |
|---------------------------------------|------------------|
| $\sim Fw \supset \neg (\exists w)Gww$ | truth-functional |
| $\neg Fw$ | truth-functional |
| Fw | atomic |
| $\neg (\exists w)Gww$ | truth-functional |
| $(\exists w)Gww$ | quantified |
| Gww | atomic |
- 2.a. $(\forall x)(Fx \supset Ga)$ quantified
- c. $\neg (\forall x)(Fx \supset Ga)$ truth-functional
- e. $\neg (\exists x)Hx$ truth-functional
- g. $(\forall x)(Fx \equiv (\exists w)Gw)$ quantified
- i. $(\exists w)(Pw \supset (\forall y)(Hy \equiv \sim Kyw))$ quantified
- k. $\neg [(\exists w)(Jw \vee Nw) \vee (\exists w)(Mw \vee Lw)]$ truth-functional
- m. $(\forall z)Gza \supset (\exists z)Fz$ truth-functional
- o. $(\exists z)\sim Hza$ quantified
- q. $(\forall x)\sim Fx \equiv (\forall z)\sim Hza$ truth-functional
- 3.a. Maa & Fa
- c. $\sim (Ca \equiv \sim Ca)$
- e. $(Fa \& \sim Gb) \supset (Bab \vee Bba)$
- g. $\sim (\exists z)Naz \equiv (\forall w)(Mww \& Naw)$

- i. $Fab \equiv Gba$
- k. $\sim (\exists y)(Hay \ \& \ Hya)$
- m. $(\forall y)[(Hay \ \& \ Hya) \supset (\exists z)Gza]$

- 4.a. $(\forall y)Ray \supset Byy$ No. Missing parentheses—‘ $(Ray \supset Byy)$ ’
- c. $(\forall y)(Rwy \supset Byy)$ No. ‘w’ in ‘Rwy’ needs to be replaced by a constant.
- e. $(\forall y)(Ryy \supset Byy)$ No. The first ‘y’ in ‘Ryy’ needs to be replaced by a constant
- g. $(Ray \supset Byy)$ No. Universal quantifier is missing.
- i. $Rab \supset Bbb$ No. Not close: almost everything is wrong.

Section 7.3

- 1.a. $(Gb \ \& \ Gc) \ \& \ (Gd \ \vee \ Ge)$
- c. $(Ad \ \& \ Le) \supset Med$
- e. $Gd \supset [(Gb \ \& \ Gc) \ \& \ (Gd \ \& \ Ge)]$
- 2.a. $(\exists x)Ox \ \& \ (\exists x)Ex$
- c. $\sim (\exists y)Lya$
- e. $Lbc \ \& \ (\exists x)Lcx$
- g. $\sim (\forall x)Px \ \& \ \sim (\forall x)Ex$
- i. $(\forall y)Ey \equiv (\forall y) \sim Oy$
- k. $(\exists x)Lxd$
- 3.a. $(\forall w)(Gw \supset Jw)$
- c. $(\exists y)Gy \supset (\forall x)Gx$
- e. $(\exists x)Ax \supset (Ac \ \& \ Ad)$
- g. $(\forall z)(Gz \vee \sim Gz)$
- i. $(\forall x)[Lx \supset (\forall y)(\sim Ly \supset Mxy)]$
- 4.a. $(\exists x)(\forall y)(Py \supset Lxy)$
- c. $(\forall x)(\forall y)[(Px \ \& \ Py) \supset \sim Txy]$
- e. $(\forall x)(\forall y)[(Px \ \& \ Lay) \supset \sim Txy]$
- g. $(\forall x)(\forall y)((Dxy \ \& \ Lay) \supset [(Ex \ \& \ Ey) \vee (Ox \ \& \ Oy)])$

Section 7.4

1. a. This English sentence may well be true --people want to do all sorts of odd and even impossible things. But the proposed translation into *PL* is obviously false, for it says, in part, that there is at least one vampire when in fact there are no such creatures as vampires. A better symbolization would be

$\forall j$

where ‘ $\forall x$ ’ symbolizes ‘x wants to catch a vampire’ rather than ‘x is a vampire’.

c. The left conjunct is an appropriate symbolization of ‘Sue believes there are vampires’, but the right conjunct is not an appropriate symbolization of ‘Sue doesn’t want to catch a vampire’, for two reasons. First, the right conjunct is true simply because there are no vampire. Hence it does not tell us anything about Sue, unlike the sentence it is supposed to symbolize. (If we replace ‘s’ with a constant designating someone else, anyone else, the result is a true sentence, again because there are no vampires.) Secondly, ‘Sue believes there are vampires but doesn’t want to catch one’ clearly indicates that Sue does not want to be in possession of a vampire. But the proposed symbolization of the right conjunct does not say this. Rather, it tells us only that there is no specific vampire that Sue wants to catch. That is, it is compatible with Sue wanting to catch a vampire, any vampire, which she doesn’t. An adequate symbolization of ‘Sue doesn’t want to catch a vampire’ is ‘ $\sim Ax$ ’ where ‘ Ax ’ symbolizes ‘x wants to catch a vampire’.

e. This is an appropriate symbolization. The use of the existential quantifier is appropriate because the sentence being symbolized makes it clear that there is a particular moose that Sue wants to see and Jeremy wants to ride.

2. If Helen has been dealing with a sales clerk who has disappeared into the back room, and Helen is now waiting for the return of that clerk, then ‘ $(\exists z)(Sz \ \& \ Whz)$ ’ is a correct symbolization of ‘Helen is waiting for a sales clerk’. But if Helen has not yet been helped by a sales clerk and is waiting for one, any one, of the clerks, to help her, then the proposed symbolization is not appropriate, because it says there is a particular clerk Helen is waiting for and she is not. In this latter case a better symbolization would be ‘Wh’, where ‘h’ designates Helen and ‘Wx’ symbolizes ‘x is waiting for a sales clerk’.

3. a. $(\exists y)[Ry \ \& \ (Cy \ \& \ Ly)]$

c. $\sim (\forall w)[(Rw \ \& \ Lw) \supset Cw]$

e. $\sim (\forall x)(\forall y)[(Rx \ \& \ Syx) \supset Ry]$

g. $\sim (\forall x)(\forall y)[(Rx \ \& \ (Dyx \ \vee \ Syx)) \supset Ry]$

i. $(\forall z)[(Rz \ \& \ (\exists w)(Swz \ \vee \ Dwz)) \supset Lz]$

k. $Sr \ \vee \ (\exists y)(Ry \ \& \ Dry)$

m. $(Rr \ \& \ (\forall z)[(Dzr \ \vee \ Szr) \supset Rz]) \ \vee \ (Rj \ \& \ (\forall z)[(Dzj \ \vee \ Szj) \supset Rz])$

4. a. $(\forall x)[Ax \supset (\exists y)(Fy \ \& \ Exy)] \ \& \ (\forall x)[Fx \supset (\exists y)(Ay \ \& \ Exy)]$

c. $\sim (\exists y)(Fy \ \& \ Eyp)$

e. $\sim (\exists y)(Fy \ \& \ Eyp) \ \& \ (\exists y)(Cy \ \& \ Eyp)$

g. $\sim (\exists w)(Aw \ \& \ Uw) \ \& \ (\exists w)(Aw \ \& \ Fw)$

i. $(\exists w)[(Aw \ \& \ \sim Fw) \ \& \ (\forall y)[(Fy \ \& \ Ay) \supset Ewy]]$

k. $(\exists z)[Fz \ \& \ (\forall y)(Ay \ \supset \ Dzy)] \ \& \ (\exists z)[Az \ \& \ (\forall y)(Fy \ \supset \ Dzy)]$

m. $(\forall x)[(\forall y)Dxy \supset (Px \ \vee \ (Ax \ \vee \ Ox))]$

5. a. An even integer times any integer is even.

c. If the sum of a pair of integers is even, then either both integers are even or both are odd.

e. There is no prime that is larger than every prime.

g. There are no primes such that their product is prime.

i. There is a prime such that it times every positive integer is even.

k. The product of a pair of integers is odd if and only if both members of the pair are odd.

- m. If a pair of integers are both odd, then their product is odd and their sum is even.
- o. The sum of an odd integer and an even integer is odd, and their product is even.
- q. There is an integer that is larger than one and less than three that is prime and even.

Section 7.5

- 1. a. $(\forall x)[(Wx \ \& \ \sim x = d) \supset Sx]$
- c. $(\forall x)[(Wx \ \& \ \sim x = d) \supset [Sx \vee (\exists y)[Sy \ \& \ (Dxy \vee Sxy)]]]$
- e. $Sdj \ \& \ (\forall x)(Sxj \supset x = d)$

- g. $(\exists x)[(Sxr \ \& \ Sxj) \ \& \ (\forall y)[(Syr \ \vee \ Syj) \ \supset \ y = x]]$
 i. $(\exists x)(\exists y)[((Dxr \ \& \ Dyr) \ \& \ (Sx \ \& \ Sy)) \ \& \ \sim x = y]$
 k. $(\exists x)[(Sxj \ \& \ Sx) \ \& \ (\forall y)(Syj \ \supset \ y = x)] \ \& \ (\exists x)(\exists y)(([Sx \ \& \ Sy] \ \& \ (Dxj \ \& \ Dyj)) \ \& \ \sim x = y) \ \& \ (\forall z)[Dzj \ \supset \ (z = x \ \vee \ z = y)]]$
- 2.a. Every positive integer is less than some positive integer [or] There is no largest positive integer.
 c. There is a positive integer than which no integer is less.
 e. 2 is even and prime, and it is the only positive integer that is both even and prime.
 g. The product of any pair of odd positive integers is itself odd.
 i. If either of a pair of positive integers is even, their product is even.
 k. There is exactly one prime that is greater than 5 and less than 9.
- 3.a. Symmetric only
 $(\forall x)(\forall y)(Nxy \ \supset \ Nyx)$
 c. Neither reflexive, nor symmetric, nor transitive
 e. Symmetric and transitive
 $(\forall x)(\forall y)(Rxy \ \supset \ Ryx)$
 $(\forall x)(\forall y)(\forall z)[(Rxy \ \& \ Ryz) \ \supset \ Rxz]$
- g. Reflexive and transitive (in UD: Physical Objects)
 $(\forall x)Txx$
 $(\forall x)(\forall y)(\forall z)[(Txy \ \& \ Tyz) \ \supset \ Txz]$
- i. Symmetric and reflexive (in UD: People)
 $(\forall x)(\forall y)(Exy \ \supset \ Eyx)$
 $(\forall x)Exx$
- k. Symmetric, transitive, and reflexive (in UD: Physical Objects)
 $(\forall x)(\forall y)(Wxy \ \supset \ Wyx)$
 $(\forall x)(\forall y)(\forall z)[(Wxy \ \& \ Wyz) \ \supset \ Wxz]$
 $(\forall x)Wxx$
- m. Transitive only
 $(\forall x)(\forall y)(\forall z)[(Axy \ \& \ Ayz) \ \supset \ Axz]$
- o. Symmetric, transitive, and reflexive (in UD: People)
 $(\forall x)(\forall y)(Lxy \ \supset \ Lyx)$
 $(\forall x)(\forall y)(\forall z)[(Lxy \ \& \ Lyz) \ \supset \ Lxz]$
 $(\forall x)Lxx$
- 4.a. Sjc
 c. $Sjc \ \& \ (\forall x)[(Sxc \ \& \ \sim x = j) \ \supset \ Ojx]$
 e. $(\exists x)[(Dxd \ \& \ (\forall y)[(Dyd \ \& \ \sim y = x) \ \supset \ Oxy]) \ \& \ Px]$
 g. $Dcd \ \& \ (\forall x)[(Dxd \ \& \ \sim x = c) \ \supset \ Ocx]$
 i. $(\exists x)[(Sxh \ \& \ (\forall y)[(Syh \ \& \ \sim y = x) \ \supset \ Txy]) \ \& \ Mcx]$
 k. $(\exists x)[(Bx \ \& \ (\forall y)(By \ \supset \ y = x)) \ \& \ (\exists w)((Mx \ \& \ (\forall z)(Mz \ \supset \ z = w)) \ \& \ x = w)]$
 m. $(\exists x)[(Mxc \ \& \ Bxj) \ \& \ (\forall w)(Bwj \ \supset \ x = w)]$

- 5.a. $\sim (\exists y)a = f(y)$
 c. $(\exists x)(Px \ \& \ Ex)$
 e. $(\forall x)(\exists y)y = f(x)$
 g. $(\forall y)(Oy \supset Ef(y))$
 i. $(\forall x)(\forall y)[Ot(x,y) \supset Et(f(x), f(y))]$
 k. $(\forall x)(\forall y)[Os(x,y) \supset [(Ox \ \& \ Ey) \vee (Oy \ \& \ Ex)]]$
 m. $(\forall x)(\forall y)[(Px \ \& \ Py) \supset \sim Pt(x,y)]$
 o. $(\forall z)[(Ez \supset Eq(z)) \ \& \ (Oz \supset Oq(z))]$
 q. $(\forall x)[Ox \supset Ef(q(x))]$
 s. $(\forall x)[(Px \ \& \ \sim x = b) \supset Os(b,x)]$
 u. $(\exists x)(\exists y)[(Px \ \& \ Py) \ \& \ t(x,y) = f(s(x,y))]$