Present Bias, Preference Heterogeneity, and Wealth Inequality over the Life-cycle

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Abstract

This paper studies present bias in a life-cycle model with heterogeneous preferences. Previous research outlining the role of behavioral biases in the macroeconomy largely falls into one of three categories: (1) partial equilibrium, (2) infinite horizon, and (3) homogenous preferences. In partial equilibrium models, present bias implies a high degree of wealth inequality and low aggregate wealth relative to a society of time-consistent discounters. I find general equilibrium price adjustments mitigate the impact of present bias on both wealth inequality and aggregate wealth. However, present bias affects the timing of wealth accumulation and leads to a dramatic increase in wealth inequality beginning at retirement. Heterogeneous exponential and present biased discounting improve the fit of the model economy by producing a higher concentration of wealth among the top 1% and a higher Gini coefficient, compared to a model in which agents are only heterogeneous in their exponential discount factor. This occurs as time-consistent individuals benefit from the aggregate mis-optimization of their present biased peers, an effect absent in homogenous preference models or models in which prices do not adjust in response to aggregate savings.

Keywords: Present Bias, Wealth Inequality, Retirement Wealth, Life-cycle Model, Preference Heterogeneity

1 Introduction

According to a national survey commissioned by Experian in 2016, 54% of Americans believe they will never fully pay off their debt, 46% of Americans have less savings today than they expected to have five years ago, and 71% of Americans aren't saving enough for retirement. Although some of these shortcomings can be explained by unexpected labor and capital earnings shocks or information frictions in the marketplace, a growing literature characterizing the widespread nature of present bias may provide an alternative viewpoint to help reconcile the observed gap between intentions and actions.

Present bias[1] refers to a preference anomaly wherein individuals discount tradeoffs between all future periods and the current period at a higher rate than they discount tradeoffs between any two periods in the future. First outlined by Phelps and Pollak (1968) and Strotz (1956) and later reintroduced to the economics literature by Laibson (1997), present bias offers a convenient theoretical framework for explaining savings phenomena like those outlined in the Experian survey. Even in an environment in which individuals have perfect information regarding future earnings, present bias creates a time inconsistency in preference that leads individuals to abandon their consumption savings plan in favor of re-optimizing (and over consuming relative to their previous plan) each period. I aim to elicit the avenues through which this consistent re-optimization impacts aggregate wealth and the timing of wealth accumulation over the life-cycle.

There is a robust empirical literature outlining the degree to which individuals are present biased in both laboratory settings and over real world tradeoffs that lends further credence to the examination of these types of non-standard preferences in a carefully calibrated general equilibrium framework[2]. I imbed both present biased and exponential discounters into a quantitative life-cycle model with uninsurable idiosyncratic risk in order to characterize the marginal contribution of each agent type towards generating wealth dispersion in the model economy. A consistent feature of this class of models is the gross under-prediction

[1] Also referred to as quasi-hyperbolic, quasi-geometric, or hyperbolic discounting.

[2] See Della Vigna (2009) for a survey of the literature characterizing behavioral biases in economics.

of wealth inequality in the model economy relative to U.S. data. Therefore, I am careful to characterize the contribution of each preference type to the overall dispersion of wealth and the evolution of wealth inequality over the life-cycle. I find the impact of present bias is largest when household planning horizons are short and for households with low wealth[3]. As older households have both shorter planning horizons and lower wealth (on average) than working age households, special attention is paid to the role of preference heterogeneity and present bias on the accumulation of assets of older households.

Three key results stand out from my analysis. First, if present biased agents are imbedded in a partial equilibrium model, the impact of time inconsistency will be drastically overstated. In a model economy examined in general equilibrium (where prices adjust fully to the actions of agents) in which all agents are present biased, present bias does very little to augment the aggregate wealth distribution relative to a model in which all agents behave rationally. This occurs as the inclusion of present biased agents drives up the market clearing interest rate and changes the savings incentives of agents who would otherwise be tempted to over-consume and under-save. This result may be of particular interest as a number of previous studies examining the macroeconomic consequences of present biased optimization imbed agents in a partial equilibrium framework.

Second, although the aggregate wealth distribution is not particularly sensitive to the inclusion of present biased agents[4], I find present bias increases wealth inequality as cohorts age and changes the timing of wealth accumulation in the model economy relative to a baseline model in which all agents behave rationally. Models that abstract from the richness of a life-cycle and instead place agents in an infinite horizon context will understate the role of present bias in the macroeconomy. This occurs as models with infinitely lived agents miss the interaction of shortened planning horizons and low average wealth inherent to older agents that amplifies the negative effects of present bias.

Third, the inclusion of discount rate and present bias heterogeneity improves the fit of

[3] See Section 3.6 for further exposition on this topic.
[4] When all agents are assumed to be present biased.

the model economy to U.S. data. Although a number of authors have shown heterogeneous exponential discount factors lead to increased wealth dispersion in a life-cycle model, to the best of my knowledge this paper marks the first attempt to characterize the contribution of heterogeneous present bias to aggregate economic outcomes in a life-cycle framework. When some agents are present biased and others are time-consistent, the time-consistent agents benefit from the aggregate mis-optimization of their present biased peers. Thus, present bias heterogeneity leads to increased wealth dispersion and an increased concentration of wealth in the hands of the richest 1% relative to a model in which agents are only heterogeneous in the exponential discount factor. The remainder of the paper is structured in the following way: Section 2 reviews the literature, Section 3 sets up the model, Section 4 describes the calibration approach, Section 5 highlights my results, and Section 6 concludes.

2 Review of the Literature

2-A. Research on Wealth Inequality

There is a vast empirical literature outlining the extent of wealth and income inequality in the United States. The primary takeaways from this literature are: wealth is much more highly concentrated than earnings[5], higher income households save a higher percentage of their wealth[6], and households with identical lifetime earnings retire with significantly different values of wealth[7]. Aiyagari (1994) and Huggett (1996) mark two of the first attempts to generate model economies for which the equilibrium wealth distribution matches the observed distribution of wealth in US data. Each author explores a model of idiosyncratic income risk in the presence of incomplete markets, with Aiyagari focusing on infinitely lived agents and Huggett focusing on finitely lived agents in a quantitate life-cycle framework. Each author finds that after endowing agents with a realistic degree of income risk and solving for optimal household, firm, and government decisions, the resulting equilibrium

[5]See Piketty and Zucman (2014)
[6]See Dynan et al. (2004)
[7]See Hurst et al. (1998)

wealth distribution falls well short of recreating the concentration of wealth observed in the data. In particular, the richest 1%, 5% and 10% of households hold far too little wealth in equilibrium relative to the analogous wealth holdings of these percentiles in the data. This result stems from the model's inability to induce high savings for wealthy agents, who prefer to disinvest and prioritize consumption in model economies. Over the past 20 years, a literature aimed at remedying this shortcoming has emerged.

Heer (2001), DeNardi (2004) and DeNardi and Yang (2014) explore the role of intentional and unintentional bequests in cultivating wealth inequality across generations. A heterogeneous agent overlapping generations model is augmented to accommodate lifetime uncertainty and the role of different bequest motives on equilibrium wealth accumulation is described. These authors find that modeling bequests, whether intentional or unintentional, does very little to augment the equilibrium distribution of wealth in a model economy. However, DeNardi and Yang find the introduction of non-linear bequest motives (only the richest households leave intentional bequests) improves the concentration of wealth in their model economy. Even in the presence of this assumption the proportion of wealth held by the richest agents still falls well short of that observed in the data.

Quadrini (2000), Cagetti and DeNardi (2006), and Benhabib, Bisin, and Zhu (2011) all consider models in which agents receive heterogeneous returns to capital income. Quadrini and Cagetti and DeNardi imbed entrepreneurs and workers in an overlapping generations framework and find the inclusion of entrepreneurs (who receive higher rates of return on their investments) considerably improves the fit of their models to U.S. data. Benhabib, Bisin, and Zhu find the inclusion of capital risk leads to an equilibrium wealth distribution that is much closer to the U.S. distribution of wealth than models with income risk alone. Although these authors are able to generate realistic wealth distributions in their model economies, they do so by making a significant departure from the benchmark models in the literature in which agents face a common interest rate and are only subject to risk in their labor endowments.

A separate strand of the literature more in line with the original works of Aiyagari and Huggett explores the role of preference heterogeneity in generating equilibrium wealth dispersion. Krusell and Smith (1998) find the inclusion of a small degree of heterogeneity in discount rates leads to a much more realistic wealth distribution in the model economy. However, this result hinges on a modeling environment in which agents are infinitely lived and receive frequent, transitory shocks to their labor earnings. If earnings are instead represented by a persistent process (as U.S. data indicates), then the Krusell Smith economy no longer generates a realistic wealth distribution. This results as agents no longer save high proportions of positive earnings shocks to protect against lower earnings in the future. Rather, an increase in earnings today leads to an increase in expected income tomorrow which induces agents to consume out of their new income at the expense of accumulating savings.

Hendricks (2007) augments a model closely resembling that of Huggett (1996) to accommodate heterogeneous discount factors. He exploits the fact that preference heterogeneity affects the distribution of wealth within a cohort as individuals age. Following this insight, the distribution of discount rates is targeted in equilibrium to match a set of age specific Gini coefficients. Hendricks finds a large degree of preference heterogeneity induces a modest increase in wealth inequality. In his model, the equilibrium wealth distribution is far less sensitive to preference heterogeneity than in the Krusell Smith model, exactly because agents face realistic earnings risk.

2-B. Present Bias and Preference Heterogeneity: Empirical Evidence

Following the re-introduction of present bias to economics by David Laibson (1997), a strong literature has emerged outlining the impact of present bias on consumer behavior. Meier and Sprenger (2009) find present biased individuals are more likely to have credit card debt and have significantly higher credit card debt than their non present biased peers. Recent work by Brown and Previtero (2017) indicates that present bias plays a large role in the determination of financial behaviors of households. They find that present biased individuals are less likely to participate in a supplemental savings plan and, conditional on participation, they contribute less and are more likely to take a lump sum (vs an annuity) than individuals who display less procrastination.

Huffman et al. (2017) explore the role of discount rate heterogeneity and present bias among older households. Although they are unable to distinguish between present bias and heterogeneity in exponential discount rates, they find that individuals displaying less patience have lower retirement wealth and less planing for end of life care than more patient households. As in Brown and Previtero, Schreiber and Weber (2016) find individuals who answer time discounting questions inconsistently have a stronger tendency to choose lump sum payments over fair annuities. Further, they show the likelihood of an impatient household choosing a lump sum is increasing in age. My results offer a specific channel through which this result can be rationalized. In a general equilibrium framework, present biased households behave near rationally when young and are more tempted by their biases as they reach retirement. The relationship between my work and the above literature is discussed further in Section 6.

2-C. Present Bias in Macroeconomics

İmrohoroğlu et al. (2003) consider the role of social security in an economy populated by overlapping generations of time-inconsistent optimizers. They find that unfunded social security lowers the capital stock, output and consumption for time-consistent and time-inconsistent individuals. However, time-inconsistent individuals may have higher welfare under a system providing unfunded social security, depending on the degree of their time inconsistency.

Angeletos et al. (2001) integrate hyperbolic discounting into a standard model of life-cycle behavior and find the inclusion of hyperbolic discounting can help rationalize observations in wealth holdings, debt accumulation and consumption paths in response to predictable income changes. However, their analysis is performed in a partial equilibrium setting in which prices are fixed and do not respond to aggregate behavior. I extend their model environment to a setting in which prices adjust to the savings decisions of households who are heterogeneous both in terms of the present biased discount factor and the exponential discount factor.

My work most closely resembles that of Harris and Laibson (2001), Krusell et al. (2002) and Maliar and Maliar (2006). Harris and Laibson construct a partial equilibrium model in which present biased agents are subject to transitory labor income shocks. Krusell et al. focus on a deterministic general equilibrium model with present biased optimizers. However, agents are homogenous in their degree of present bias and in their exponential discount factor. Mailar and Maliar introduce persistent labor earnings shocks and preference heterogeneity to a neoclassical growth model. However, they model infinitely lived agents and are unable to come remotely close to the wealth distribution of the US economy. I extend the work of Maliar and Maliar by imbedding heterogenous present biased agents into a life-cycle model (as in Harris and Laibson and Angeletos et al.) in which agents are subject to persistent labor income shocks and preferences, earnings ability, and wealth are all partially inherited. Further, I consider heterogeneity in both the present bias discount factor (as in Maliar and Maliar) and in the exponential discount factor (as in Hendricks). This exercise is particularly valuable, as my results indicate present bias and discount rate heterogeneity have drastically different implications for the accumulation of wealth over the life-cycle. This result hinges on the interaction of preferences and age, and is therefore lost in infinite horizon settings.

3 The Model

The modeling environment is a stylized version of the stochastic incomplete markets life-cycle model which is commonly used to examine the distribution of wealth. I build off of the work of Hugget (1996) and Hendricks (2007) while integrating insights from İmrohoroğlu et al. (2003) and Harris and Laibson (2001) with regard to modeling present bias. The economy is comprised of a continuum of agents of unit mass, a single representative firm, and a government. I restrict my attention to examining the economy in steady state with competitive markets.

3-A. Households

An agent is born at model age 1, works for the first R periods of life, and dies with certainty after N>R periods. The probability of an agent surviving to age t conditional on surviving to age $t-1$ is given by s_t. When an agent dies, they are replaced by a child of age 1 who inherits the after tax value of their parents' wealth and imperfectly inherits their parents preferences and labor endowments. The population grows at a constant rate n with stable demographic patterns so that age $t \leq$N agents make up a constant portion of the population.

Agents inelastically supply $l = h(t)e$ units of labor to the market each period from birth to retirement, where $h(t)$ is a deterministic age-earnings profile and e is a labor endowment shock. An agent's initial labor endowments (e_1) is inherited stochastically from their parents and earnings evolve over the life-cycle according to Markov transition matrix, P_e. A new agent inherits their parents age a_{IG} labor endowment with probability ρ_{IG}. With probability $1 - \rho_{IG}$ an agent draws an initial labor endowment from the distribution governing the economy wide distribution of initial labor shocks. An agent's draw of e_1 depends on their parent's labor endowment in period a_{IG} and not their parent's labor endowment in their terminal period[8]. Upon retirement, all agents receive a social security transfer, τ_R, and consume out of their savings and annual social security allocation.

Agents share identical preferences over consumption and leisure with the exception of their discount factors, β and δ. These heterogeneous preference parameters are drawn at birth from a finite set of discrete values J and remain constant over an agent's life. Preferences are stochastically inherited with probability ρ_j. With probability 1-ρ_j preferences are drawn randomly from the stable distribution over agent types. Upon drawing preference parameters, households maximize the expected discounted sum of lifetime utility in the

[8]This assumption implies parents stochastically pass on their human capital to their children during the their working lifetime, and not when old.

following way:

$$U = \max \left\{ u(c_1) + \delta_j \, \mathrm{E}\left[\sum_{t=1}^{N} \beta_j^t \left(\prod_{i=1}^{t} s_i \right) u(c_{t+1}) \right] \right\} \tag{1}$$

where c_t represents age t consumption, s is the probability of surviving one additional year conditional on being alive today, δ_j is the present biased discount factor of a type j agent and β_j is the exponential discount factor of a type j agent[9]. All bequests are assumed to be unintentional, as previous research finds intentional bequests play a small role in generating realistic wealth dispersion in the absence of nonlinear bequest motives[10].

3-B. Firms

Output is produced using capital (K) and labor (L) by a representative firm with production given by $Y = F(K, L) = AK^{\alpha}L^{1-\alpha}$. The representative firm seeks to maximize profit in a competitive market, $F(K, L) - q_K K - q_L L$, where q_K and q_L represent the rental rates for capital and labor, respectively. In each period, capital depreciates at a constant rate δ_k.

3-C. Government

The government levies taxes on labor income (τ_w) and bequests (τ_B) and provides state dependent lump sum transfers ($\tau(x)$). Transfers can be broken down into two categories; social security transfers to retired households (τ_R if $t > R$) as well as unconditional lump sum transfers ($\tilde{\tau}$) to all agents. The government does not tax capital income. Therefore, the relevant household prices are given by $w = (1 - \tau_w)q_L$ and $r = q_K - \delta$. Aggregate transfer payments are given by $T = \int_x \Lambda(x)\tau(x)dx$ where x denotes a potential state and $\Lambda(x)$ denotes the density of households over states. All tax revenue beyond that needed to fund aggregate transfers is assumed to be discarded so that the government balances its budget in each period.

3-D. Dynamic Programming Problem

[9]I have reversed the meaning of the β and δ parameters from Laibson's exposition of quasi-hyperbolic discounting so that the discount factor β retains its standard interpretation found throughout the macroeconomic literature.

[10]For further exposition on this topic, please see section 2-A in which the results of DeNardi (2004) and DeNardi and Yang (2014) are discussed.

An agent of type j has a state vector x given by $x = (k, e, t, j)$ where k is wealth, e is the current period's labor endowment shock, and t is agent age. The agent's optimization problem can be written as a dynamic program where the Bellman equation is given by:

$$V(x) = \max_{(c,\ k')} \ u(c) + \delta_j \beta_j s' \mathrm{E}\big[\tilde{V}(x'|x)\big] \tag{2}$$

subject to

$$c + k' \leq (1+r)k + wl + \tilde{\tau} + \tau_R, \tag{3}$$

$$k' \geq \underline{k},\ \ c \geq 0,\ \ k' > 0 \text{ if } t = N,\ \ V(x) = \tilde{V}(x) = 0 \text{ if } t = N+1. \tag{4}$$

Common transfers $\tilde{\tau}$ and retirement transfers τ_R are independent of earnings history and depend only on agent age.[11] If δ_j=1, this is the well known value function of an exponential discounter. When $\delta_j < 1$, I must structure the beliefs of agents with regard to their future selves.

The two primary cases studied in the behavioral literature are naive and sophisticated present bias. A sophisticated agent will behave in a present biased way today, but will seek commitment devices to keep their future self from violating their planned consumption, savings profile. I proceed by assuming that agents behave in a naive manor. This assumption is convenient for several reasons. First, there is not a strong consensus in the behavioral literature regarding the true nature of agent sophistication, but recent work from Laibson (2015) concludes "a demand for commitment is a special case rather than the general case". Second, my model does not offer agents any vehicle for commitment. Therefore, knowledge regarding one's own biases is not exploitable, as agents do not have savings vehicles that can effectively constrain their decisions even if they are wary of their future self's inability

[11] I proceed with this assumption as this approach is shared by both Huggett (1996) and Hendricks (2007). Whenever possible, I aim to reduce the distinctions between my model and these previous models to limit the avenues through which my results may differ from their work.

to follow through with a plan of action. A naive agent solves their dynamic programming problem under the belief that, although they have a history of present bias and behave in a present biased way today, they will act in a time-consistent manner in the future. This assumption can be represented by defining the continuation payoff $\tilde{V}(x)$ as:

$$\tilde{V}(x) = \max_{(c,\ k')}\ u(c) + \beta_j s' \mathrm{E}\big[\tilde{V}(x'|x)\big] \tag{5}$$

subject to constraints (3) and (4) outlined above.

The continuation payoff of a present biased agent, which informs an individuals' consumption-savings decisions over all future periods, is identical to the value function of an exponential discounter. The only distinction between an agent with $\delta_j < 1$ and an agent with $\delta_j = 1$ is the additional discounting of future utility made every period. It is exactly this additional discount factor that leads present biased agents to behave inconsistently.

3-E. Equilibrium

A stationary, competitive equilibrium consists of aggregate quantities $(K,\ L,\ C,\ T,\ B)$, prices $(w,\ r,\ q_L,\ q_K)$, transfers $(\tau(x))$, current and continuation value functions ($V(x)$ and $\tilde{V}(x)$), policy functions ($c(x)$ and $k(x)$) and a distribution over agent types $(\Lambda(x))$ such that:

- The policy functions ($c(x)$ and $k(x)$) along with the value function $V(x)$ and the continuation value function $\tilde{V}(x)$ solve the agent's optimization problem.
- Firms maximize profits.
- The government balances its budget.
- The distribution of households over states, $\Lambda(x)$, is stationary.
- Prices are given by $w = (1 - \tau_w)q_L$ and $r = q_K - \delta$.
- Markets Clear:
  (i) $K = \int \Lambda(x)k(x)dx$
  (ii) $L = \int \Lambda(x)l(x)dx$

(iii) $F(K, L) = C + \delta K$ where $C = \int_x \Lambda(x) c(x) dx$

3-F. Present Bias: The Euler Equation and Discounting the Future

When agents are not present biased, $\delta = 1$, the discount factor is equal to β in every period. However, if individual's do behave in a present biased manor, $\delta < 1$, the discount factor is no longer constant. Rather, the discount factor becomes an endogenous variable that depends on an agent's current state. I denote the effective discount factor as $\beta_{x'}$ to highlight the new dependence of discounting on the state next period, x'. Following from the optimization problem set up in the previous section, the present biased Euler equation can be written as:

$$u'(c_t) \geq \beta_{x'}(1 + r) s_{t+1} E_t(u'(c_{t+1})) \tag{6}$$

The effective discount factor is equal to[12]:

$$\beta_{x'} \equiv \beta_{x'}(k_{t+1}, e_{t+1}, t+1) = \beta \left[1 - \frac{1 - \delta}{1 + r} \frac{E_t[u'(c_{t+1}) c_k(k_{t+1}, e_{t+1}, t+1)]}{E_t[u'(c_{t+1})]} \right] \tag{7}$$

Where $c_k(k_{t+1}, e_{t+1}, t+1)$ is the derivative of the optimal consumption function with respect to capital selected in period t. When $\delta = 1$, $\beta_{x'} = \beta$ in each period. However, when $\delta < 1$, the effective discount factor is a function of the state in period $t + 1$[13]. This generates a direct relationship between the effective discount factor and an agent's wealth, k. As shown in Maliar and Maliar (2006), if the consumption function is strictly concave, then the effective discount factor of present biased agents will be strictly increasing in wealth. Thus, if two agents have the same degree of present bias but different levels of wealth, the richer of the two agents will behave more patiently than the poorer agent.

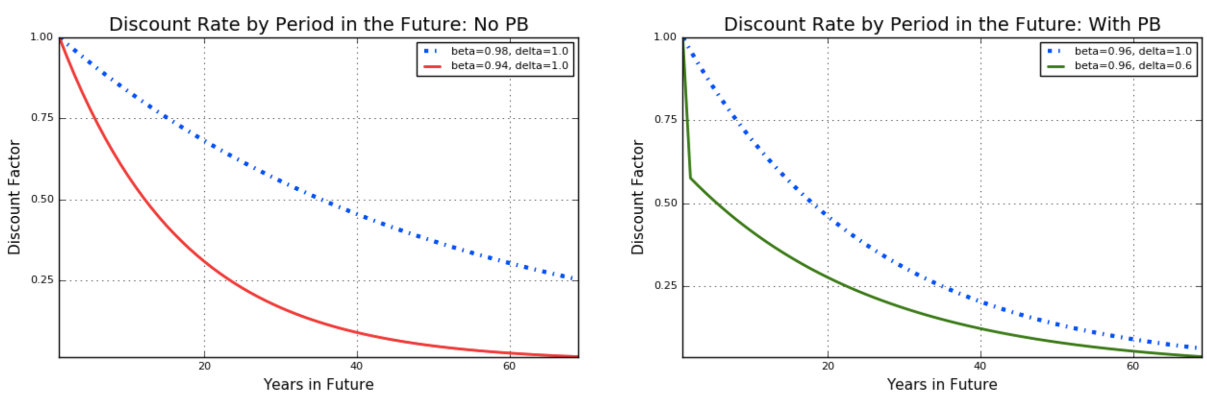
This result is not particularly surprising as diminishing marginal returns to consumption

[12] For further exposition of this derivation, see Appendix B.

[13] For a full exposition regarding the effective discount factor under quasi-hyperbolic discounting, see Harris and Laibson (2001).

is a typical feature of an agent's utility function. A present biased individual ends each day with a consumption savings plan laid out for their future self only to wake up the next morning and violate this plan. However, individuals who are very wealthy have less of an incentive to deviate from their planned consumption profile as their marginal utility gains are much smaller than those of a similarly biased individual with lower wealth (as wealthy agent's are at a flatter point on their utility curve with respect to consumption). It is this exact relationship between effective discounting and wealth that leads to increased dispersion in savings between rich and poor households. Thus, two present biased agents with identical lifetime earnings will display different savings behavior based on the amount of wealth they have accumulated.

The second avenue through which present bias impacts the accumulation of capital is the horizon over which decisions are being made. Consider Figure 1 which shows the effective discount rate applied to utility n periods in the future. Panel (a) shows the discount rate for



(a) Homogenous δ, heterogenous β

(b) Heterogenous δ, homogenous β

Figure 1: Discounting of Future Utility

exponential discounters ($\delta = 1$) with $\beta = 0.98$ and 0.94, respectively. The impact of heterogeneity in the exponential discount factor leads to small discrepancies in discounting over a short planning horizon but large discrepancies in discounting over tradeoffs further into the future. This heterogeneity leads to the creation of savers and spenders in the model, as agents all face the same interest rate but have different preferences regarding tradeoffs between now and any future period based on their discount factor. This intuition is confirmed in section 5, where I find the inclusion of heterogeneity in the exponential discount factor leads to a level shift in wealth inequality at each age in the model economy relative to a baseline model populated with homogenous exponential discounters.

Panel (b) shows the discount rate for present biased discounters with the same exponential discount factor ($\beta = 0.96$) and $\delta = 1.0$ and 0.6, respectively. Unlike heterogeneity in the exponential discount factor, which leads to large differences across household discounting as horizons move further into the future, present bias creates the largest disagreements regarding time discounting over short planning horizons. Further, as individuals apply their present biased discount factor to every future period, this leads agents to continually re-evaluate their desired tradeoff between current and future utility as they get closer to an event. This results in consistently higher discounting of a future event relative to an exponential discounter as the planning horizon between now and said event is shortened. It is this exact feature of present bias that leads agents in my model to decumulate wealth more quickly in old age when they are present biased relative to exponential discounters.

To recap, if an individual is present biased they behave in a more biased way the less wealth they have. Thus, in an economy comprised of present biased individuals with identical preferences, there will still be a degree of effective preference heterogeneity if individuals have accumulated different amounts of wealth. This will serve to amplify wealth inequality, as it will generate savers and spenders relative to the prevailing market interest rate, albeit to a lesser extend than heterogenous exponential discount factors. Further, when individuals are present biased, the gap between their discounting of the future and the discounting of an exponential discounter is largest when planning horizons are short. Therefore, present bias will have it's greatest impact on agent behavior when: (a) wealth holdings are low and (b) planning horizons are short. This characterization of the impact of present bias on agent behavior offers support for the findings of Schreiber and Weber (2016). Younger agents are less tempted by their present bias than old agents, thus older households are more likely to behave irrationally and choose a lump sum over a fair annuity.

4 Calibrating Model Paramaters

4-A. Demographics

Households are born at the age of 22 and live at most 69 periods (age 90). Households retire and begin receiving social security transfers at a model age of 43 (age 65). The probability of survival from one period to the next (1 year) is given by the mortality rates listed in the Social Security Life Tables [14].

4-B. Labor endowments

Agent's labor endowments consist of a deterministic age efficiency profile, $h(a)$, and a stochastic labor productivity shock, e. The age efficiency profile is modeled after Huggett 1996, using 1990 PUMS data. The transition matrix for labor endowment shocks, P_e, is the Markov transition matrix associated with the Markov approximation of the following autoregressive process:

$$\ln(e_t) = \rho \ln(e_{t-1}) + \epsilon_t \tag{8}$$

where e_{t-1} is the labor shock experienced in the previous period and $\epsilon_t \sim N(0, \sigma_\epsilon^2)\ \forall\ t$. I've selected values of ρ and σ_ϵ^2 that are consistent with the persistence of annual earnings and the variance of log earnings for individuals over their working lifetime. This provides a grid of potential labor shocks and a Markov transition matrix describing the probability of moving from shock e_i to any other shock e_j, $j \in [1, 7]$.

The initial productivity shock of young agents, e_1, is equal to their parent's shock at age a_{IG} with probability 0.41 and is drawn from a normal distribution governing initial labor endowments with probability 0.59[15]. Initial labor endowments are distributed $N \sim (\bar{e}_1, \sigma_1^2)$.

[14] A weighted average of Male and Female survival probabilities is used for model calibration. All values are taken from the 2015 Social Security Life Tables

[15] I follow Hendricks (2007) in setting $\rho_{IG} = 0.41$ and setting $a_{IG} = 39$.

Although individuals in the model are not replaced by children until their death, I choose a value of $a_{IG} < a_D$. That is, agents inherit their parent's labor endowment in the middle of their parent's working life. This assumption is both reasonable, as human capital is likely transmitted between generations when parents are middle aged, and computationally convenient, as the observed persistence of intergenerational earnings cannot be matched if endowments are transferred late in life.

4-C. Preferences

Preferences over consumption utility are given by $u(c) = c^{(1-\sigma)}/(1-\sigma)$. The curvature parameter of the household utility function is set to $\sigma = 1.5$. This is the value chosen in Hendricks (2007a), Huggett (1996), and DeNardi and Yang (2014), among others.

In terms of the calibration of present bias, Laibson, Repetto and Tobacman (2007) argue for a degree of present bias equal to 0.7, while Paserman (2008) finds an average degree of present bias of 0.65. However, Paserman finds the degree to which individuals are present biased varies significantly with income. He finds low income households display a degree of present bias as low as 0.40 whereas high income households are significantly less time inconsistent (0.89). Meier and Sprenger (2015) estimate the value of the present bias discount factor using a series of survey questions and find average degree of present bias to be between 0.69 and 0.82 depending upon their model specification. They also find a significant degree of present bias heterogeneity, with poorer women displaying a higher degree of present bias than wealthier women. Tanaka et al. (2010) perform a similar survey based experiment on 181 rural Vietnamese households. They find an average degree of present bias of 0.644. However, unlike Meier and Sprenger or Paserman, they do not find any correlation between household characteristics and the degree of present bias. Due to the disparate findings outlined above in the empirical literature, I calibrate present bias as 0.7 in each experiment performed in Section 5.[16]

[16]The results reported in Section 5, while quantitatively sensitive to the selection of δ do not change qualitatively when alternate parameterizations are considered.

Table 1: Model Parameters

Demographics	
$a_D = 69$	Maximum lifetime (Physical age of 90)
$a_R = 43$	Retirement age (Physical age of 64)
P_s	Matches Mortality rates of couples found in Social Security Administration Period Life Tables 2015
Labor endowments	
$\eta_e = 7$	Size of labor endowment grid
$\rho = 0.96$	Persistence of labor endowments
$\sigma_e = 0.212$	Standard deviation of transitory shocks
$\rho_{IG} = 0.41$	Intergenerational persistence of labor endowments
$\sigma_{e_1} = 0.616$	Standard deviation of age 1 endowment shock
$a_{IG} = 19$	Age of intergenerational transmission (physical age 40) taken from Hendricks (2007)
Preferences	
$\sigma = 1.5$	Consistently used throughout the literature
$\rho_j = 0.5$	Intergenerational preference transmission is set to 0.5
Technology	
$\alpha = 0.36$	Capital income share
$\delta_k = 0.076$	Jointly set with A to normalize $q_L = 1$ when $r = 0.04$
$A = 0.89$	Jointly set with δ_k to normalize $q_L = 1$ when $r = 0.04$
Government	
$\tau_w = 0.40$	Tax rate on labor income, Trostel(1993), Hendricks (2007a)
$\tau_R = 0.4 * (AvgEarnings)$	Retirement transfer set to 40% of average household earnings

The parameters listed above are common to all model specifications. In the results section I distinguish experiments by the proportion of agents endowed with each β_j and δ_j.

4-D. Technology

The production function in the model economy is of the Cobb-Douglas form. $F(K, L) = AK^{\alpha}L^{(1-\alpha)}$. The capital share, α, is set equal to 0.36. A and δ are chosen so that $q_L = 1$ when the interest rate, r, equals 0.04 and the capital to output ratio, $\frac{K}{Y}$, equals 3.1.

4-E. Government

Wages are taxed at a rate of 0.40, following Huggett (1996)[17]. Bequests are not taxed in the baseline case ($\tau_B = 0$) following a convention in much of the quantitate life-cycle literature. Preliminary analysis indicates my results are not sensitive to assumptions regarding

[17]Note, the results presented in the next section are robust to changes in the tax rate.

bequest taxation.

5 Results

In this section, I draw distinctions between several experiments regarding the distribution of societal preferences over β and δ. As there is no consensus in the literature regarding the true distribution of household preferences, I remain agnostic with respect to which experiment corresponds to the "correct" model[18]. Instead, I analyze the wealth distribution corresponding to each distinct model economy and comment on the areas in which said distributions match wealth targets in SCF and PSID wealth data, which are commonly used as benchmarks in this literature. To impose discipline across modeling experiments, the aggregate capital to output ratio is set so that $\frac{K}{Y}= 3.1$ in each model economy and the interest rate is set so that $r = 4\%$. Each experiment (and the subsequent distribution of wealth associated with the experiment) is disciplined by the selection of a common discount factor β_c applied to every household in the economy so that equilibrium r and $\frac{K}{Y}$ match the targets outlined above. In models without exponential discount factor heterogeneity, β_c is simply the exponential discount factor used by each household when making tradeoffs between utility in any two future periods[19]. The term "Avg. β" refers to the average exponential discount factor across households in the model economy.

It is important to note that rather than iterating over β_c to match the targeted interest rate (and capital to output ratio), one could just as easily iterate over the interest rate for a given value of β_c to find an equilibrium. I avoid this approach for several reasons. First, changing the interest rate necessarily changes the capital to output ratio across modeling experiments and a fixed capital stock makes for more consistent graphical comparisons across models. Second, by constraining the capital to output ratio to be constant across models I am fixing an observable statistic in an economy (r) and varying an unobservable variable

[18]See Frederick, Loewenstein and O'Donogue (2002) for a discussion of the empirical estimates of β dispersion. Their results indicate that low end estimates of β can be close to 0 and high end estimates can be greater than 1.

[19]More detailed information regarding the solution algorithm can be found in Appendix B.

(β_c). Third, changing interest rates changes the price agents receive for renting their capital holdings and creates savers and spenders in a model economy with heterogeneous preferences. Changing the value of β_c does *the exact same thing* in the model economy.

Although prices are fixed across experiments, an agent's response to prices is *not* fixed. Thus, a change in β_c can be interpreted as a change in the *relative price* of capital when discussing distinctions across models in the following section. When agents have heterogenous preferences, a high value of β_c and a fixed interest rate of r=4% will generate individuals who view this interest rate as too high (relative to the value of β that would lead to consumption smoothing in their consumption Euler equation) and therefore become savers, and individuals who view this interest rate as too low and therefore become spenders. This mechanism for generating disparate savings decisions is equivalent to comparing model economies with a fixed value of β_c in which prices (r) respond to the distribution of households over types.

5-A. Outline of Experiments

The experiments outlined in the following section are characterized in Table 2:

Table 2: Outline of Experiments

	Proportion of Households by Type						
		β		δ			
Experiment	(0.98	0.94	0.90)	(1.0	0.7)	β_c	Avg. β
Base	1.0	-	-	1.0	-	0.983	0.963
BasePB	1.0	-	-	-	1.0	1.017	0.999
BaseHet	0.5	0.5	-	1.0	-	0.997	0.957
		β		δ			
Experiment	(0.98	0.96	0.90)	(1.0	0.7)	β_c	Avg. β
Full	.44	.12	.44	1.0	-	1.003	0.945
FullPB	.44	.12	.44	-	1.0	1.027	0.968
*FullPBHet$_\lambda$	.44	.12	.44	λ	$1-\lambda$	-	-

where the "Proportion of Households by Type" represents the stable distribution of households over each potential discount factor in the model economy.

The experiment labeled "Base" corresponds to a baseline model in which all agents are exponential discounters ($\delta = 1.0$) and have the same $\beta = 0.963$ ($0.98 \times \beta_c = 0.98 \times 0.983$) in equilibrium. "BasePB" refers to a model in which all agents are present biased ($\delta = 0.7$) and have an exponential discount factor equal to 0.999 in equilibrium. "BaseHet" refers to a model in which all agents are exponential discounters and half of all agents have an equilibrium discount factor of 0.977 (0.98×0.997) and the other half have an equilibrium discount factor of 0.937 (0.94×0.997).

All experiments labeled "Full" refer to a richer model in which agents are heterogeneous in their exponential discount factor β. Calibration for preference heterogeneity is modeled after Hendricks (2007) for a model with partial intergenerational transmission of preference and ability and full inheritance of wealth via accidental bequests. Hendricks backs out the distribution of households over preference types by matching age specific Gini coefficients in the model economy with their corresponding moments in PSID data. Although Hendricks allows for five potential discount factors, his approach for preference calibration results in meaningful weight being placed on just three potential β values. Therefore, I allow for just three values of β_j with weights close to those implied by Hendrick's calibration as opposed to selecting the degree of heterogeneity at random.

"Full" refers to a model in which agents are all exponential discounters ($\delta = 1.0$) with 44% of households endowed with $\beta = 0.98 \times 1.0032$, 12% of households endowed with $\beta = 0.96 \times 1.003$, and 44% of households endowed with $\beta = 0.90 \times 1.003$ where 1.003 is equilibrium β_c for the "Full" model. "FullPB" refers to a model with the same breakdown of exponential discount factors in the model economy and $\delta = 0.7$ for all agents. That is, every agent in the "FullPB" model economy is present biased. Finally, the "FullPBHet" model refers to a set of calibrations in which households preferences are distributed over β as described in "Full", but a proportion λ of agents endowed with each β are not present biased ($\delta = 1.0$) and $(1 - \lambda)$ are present biased ($\delta = 0.7$). I consider values of $\lambda = 1$, 0.75, 0.5, 0.25, and 0 where $\lambda = 1$ corresponds to the "Full" model and $\lambda = 0$ corresponds to the "FullPB" model. Results for this calibration are discussed in Section 5-C.

5-B. Results- Building Intuition

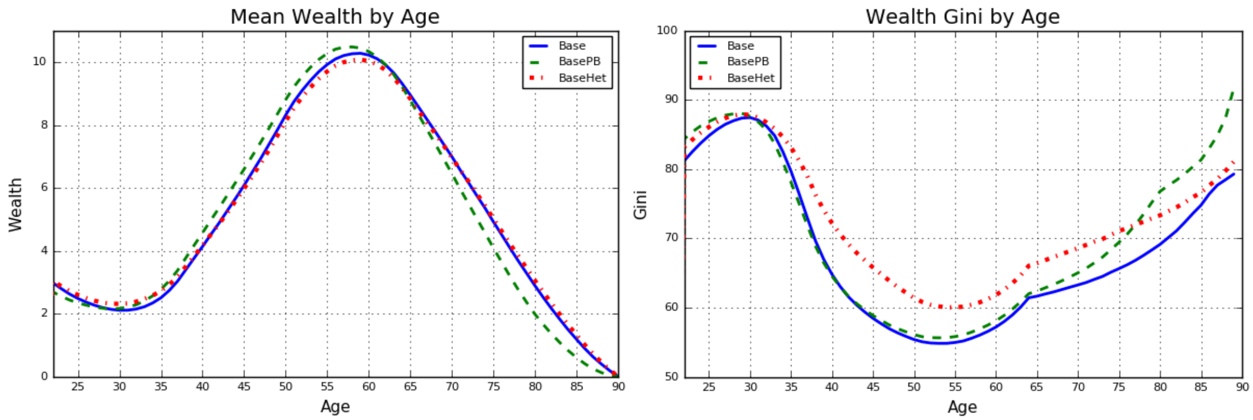
In this section I present results from the "Base" experiments. This exercise is useful as it provides a means of understanding the margins on which present bias and discount rate heterogeneity differentially impact the accumulation of wealth in model economies. Table 3 outlines the Gini coefficient and selected elements of the Lorenz Curve for both US data (SCF and PSID data) and for the "Base" model economies outlined above.

Table 3: Wealth Distribution in the U.S. and "Hugg" Model Economies

	Gini	99-100	95-99	90-95	80-90	40-80	0-40
PSID (2003)	0.76	25.3	21.8	14.0	16.3	21.8	0.9
SCF (1998)	0.80	34.7	23.1	11.3	12.7	17.2	1.0
Base	0.70	12.0	21.8	16.8	21.8	25.9	1.6
BasePB	0.72	11.8	22.4	17.4	22.3	25.3	0.7
BaseHet	0.74	13.1	24.0	18.3	20.2	23.6	0.8

As shown in Huggett (1996), a model economy with homogenous exponential discounters generates a Gini coefficient that is lower than that found in the data. Further, the fraction of wealth held by the 99^{th} percentile of households is just 12%, less than half of the wealth share of the top 1% of households reported in the PSID. Relative to the baseline model, adding present bias increases the Gini coefficient in the model economy to better match the data. However, this improved fit in the Gini coefficient is generated by a distribution of households in which the poorest 40% of households are too poor and the richest 1% of households are too poor relative to the data *and* the baseline model of time-consistent discounters. As shown in Krussell-Smith (1998) and Hendricks (2007a), the inclusion of heterogeneity in the exponential discount factor leads to a marked improvement in the Gini coefficient (0.74 compared to 0.70 in the baseline model) and the model's ability to match the wealth holdings of the top 1% of earners (13.1% compared with 12%).

To better understand the distinct role of present bias relative to that of preference hetero-



(a) Mean Wealth by Age

(b) Gini Coefficient by Age

Figure 2: Evolution of Wealth over the Life-cycle

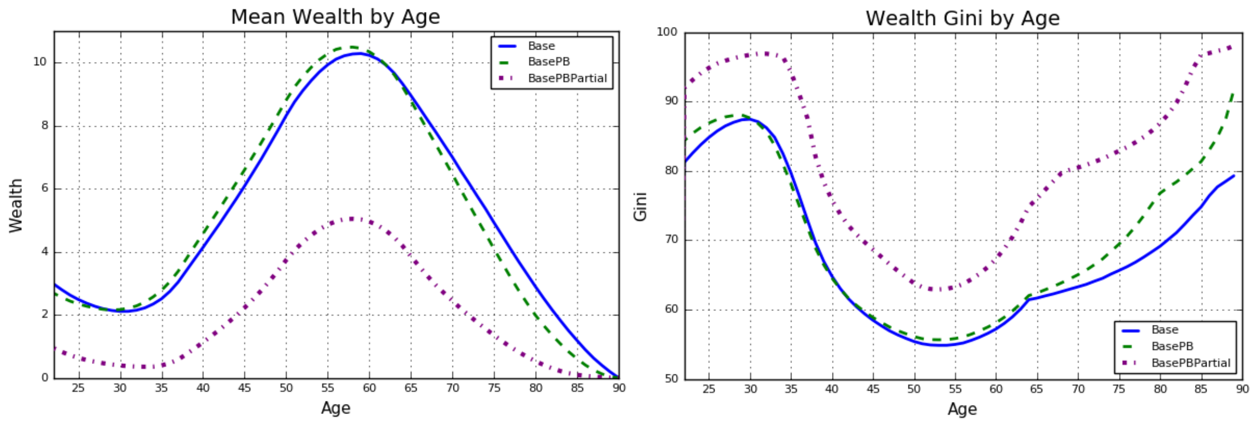
geneity in generating equilibrium wealth dispersion, consider the graphs in Figure 2 representing the average wealth accumulated in the model economy by age and the Gini coefficient in the model economy by age across different assumptions regarding economy wide preferences.

Average wealth in the "Base" model with homogenous exponential discounters is nearly indistinguishable from the average wealth accumulated in the "BaseHet" model with heterogenous exponential discounters. However, in the "BasePB" model economy, in which all agents are homogenous in their exponential discount factor and present biased, mean wealth peaks at a slightly earlier age and a slightly higher value than in the non present biased economies. Further, wealth is depleted at a faster rate when all agents are present biased following peak earnings, particularly after households retire (age 65). As each model is constrained to produce the same capital to output ratio and interest rate for the given distribution of households over types, total capital across model economies is also constrained to be identical. In spite of this fact, the inclusion of present biased discounters has a significant impact on the timing of wealth accumulation.

Panel (b) sheds further light on the distinct role played by present bias relative to preference heterogeneity in generating wealth inequality over the life-cycle. Heterogeneity in discount rates leads to a level shift in the wealth Gini at every age over the life-cycle compared to a model with homogenous discounters. Present bias, on the other hand, does very little to impact wealth inequality over the life-cycle *until households near retirement*. The Wealth Gini by Age in a present biased economy is nearly identical to that in an economy populated by homogenous exponential discounters until agents reach age 55. At this age, we see average wealth holdings rapidly decrease in the present biased model economy and the Gini coefficient by age begins steadily increasing. At retirement, the "BaseHet" economy and the homogenous exponential discounter economy see a slight uptick in wealth inequality, but wealth inequality in the present biased economy increases at a much faster rate to a much higher level than either of the exponential economies. This fact follows conveniently from the insights gained in Section 3-F. Present bias is the most costly to consumers when planning horizons are short and when agents have low wealth. We see wealth inequality drastically increase upon retirement as a wedge is driven between households with sufficient wealth to behave near rationally and households with low wealth who's biases are exacerbated by the short planning horizon over their remaining lifetime.

Early in the life-cycle, an economy comprised of present biased optimizers looks very similar to one comprised of homogenous discounters, as general equilibrium effects impose a higher β_c in the present biased society than the "Base" society. Thus, a high relative value of savings overwhelms present biased agents' desire to over consume while young, as they are extremely patient relative to the market interest rate in the long run compared with agents in the "Base" model who have a much lower β_c. Models that do not place present biased agents in a general equilibrium framework will surely miss this fact. To this point, consider a version of the "BasePB" model that is *not* in general equilibrium, called "BasePB$_{partial}$". Every agent is endowed with $\beta_c = 0.963$ as in the "Base" economy and the interest rate is still set to $r = 4\%$.

The resulting economy has a Gini coefficient of 0.83 and the top 1% of households hold over 19% of all wealth. These are both significant improvements over the baseline model's ability to match inequality in the data, and may lead one to believe that homogenous present



(a) Mean Wealth by Age (b) Gini Coefficient by Age

Figure 3: Wealth Evolution: General vs Partial Equilibrium

bias plays a tremendous role in generating wealth inequality. However, this increase in wealth dispersion is generated by a much poorer society that has far greater wealth inequality at every age when compared with the the “Base” and “BasePB” societies. Figure 3 shows the mean wealth by age and Gini coefficient by age for the “Base” and “BasePB” models relative to the “BasePB$_{partial}$” economy. The above graphs shows a model analyzing the role of present bias outside of general equilibrium will drastically misrepresent the role played by present bias, particularly with regard to any measure that depends on aggregate wealth or the interest rate. By allowing the relative price of capital (the combination of β_c and r) to vary across modeling assumptions, model economies do not display increased wealth dispersion over an agent’s working lifetime when all individuals are assumed to be present biased.

In general equilibrium models, the largest deviation between present bias and non-present biased societies comes late in life as households with low wealth are far more tempted by their shortened planning horizon than high wealth present biased households. As there is already some degree of wealth inequality at retirement across all model economies due to heterogeneity in labor earnings, inequality is amplified in a present biased society when the shortened planning horizon of old age is interacted with lower average wealth and increased wealth dispersion. Just as partial equilibrium models will overstate the importance of present bias in generating wealth inequality, infinite horizon models will understate the role of present bias *even in general equilibrium* as the largest impact of present bias on household outcomes occurs as households reach retirement. Thus, models missing the richness associated with life-cycle dynamics will understate the role of present bias.

5-C. Adding Present Bias to a Model with Preference Heterogeneity

Having established the role of present bias and preference heterogeneity in generating wealth inequality in the simple economy comprised of homogenous exponential discounters, I now turn to a set of experiments in which agents display a large degree of exponential discount rate dispersion to better understand the margins on which present bias (and present bias heterogeneity) can improve the fit of my model to data. Results from the "Full" experiments (outlined in Table 1) are presented in Table 4. Recall, each "Full" experiment is calibrated so that 44% of the population has $\beta = 0.98$, 12% of the population has $\beta = 0.96$, and 44% of the population has $\beta = 0.90$, where each β is multiplied by the relevant β_c specific to each calibration[20]. In each experiment, I vary the percentage of the population with present biased preferences, where a proportion λ of the agents in the model are not present biased ($\delta = 1.0$) and $(1 - \lambda)$ of the agents are present biased ($\delta = 0.7$). For each experiment, the proportion of individuals in the model economy endowed with each β is unaffected by the percentage of individuals that are present biased. For example, when $\lambda = 0.5$, 44% of individuals have $\beta = 0.98$, 12% have $\beta = 0.96$, and 44% have $\beta = 0.90$ but half of the individuals endowed with each β are endowed with $\delta = 1.0$ and half are endowed with $\delta = 0.7$.

The results from my calibration of the "Base" model as well as the wealth moments in PSID and SCF data are reported in Table 4 for ease of comparison. As shown in Hendricks (2007), the "Full" model in which there is a large degree of discount rate heterogeneity and no individuals are present biased offers a significant improvement in fit relative to the baseline "Base" model. The Gini coefficient on wealth is 0.77 (compared with 0.70 in the

[20]This follows from the calibration of the distribution of preferences over household types in Hendricks (2007) for a model with accidental bequests.

Table 4: Wealth Distribution in the U.S. and "Hend" Model Economies

	λ	β_c	Gini	99-100	95-99	90-95	80-90	40-80	0-40
PSID (2003)			0.76	25.3	21.8	14.0	16.3	21.8	0.9
SCF (1998)			0.80	34.7	23.1	11.3	12.7	17.2	1.0
Base	1	0.983	0.70	12.0	21.8	16.8	21.8	25.9	1.6
Full	1	1.003	0.77	13.8	26.3	19.6	20.6	19.7	0.1
FullPBHet$_{.75}$	3/4	1.008	0.78	14.0	27.2	19.6	20.7	18.5	0.0
FullPBHet$_{.5}$	1/2	1.013	0.79	14.2	27.9	19.6	20.8	17.5	0.0
FullPBHet$_{.25}$	1/4	1.020	0.80	14.4	27.9	19.9	20.7	17.1	0.0
FullPB	0	1.027	0.79	13.6	27.5	20.2	21.0	17.6	0.0

"Base" model and 0.76-0.80 in the data) and the percentage of wealth held by 99^{th} percentile of households is 13.9 (compared to 12.0 in the "Base" calibration). Augmenting the baseline "Full" model so that every individual is present biased ("FullPB") results in a slightly higher Gini coefficient of 0.79. As β_c in the "FullPB" model is larger than β_c in any of the other model calibrations considered, agents in this experiment have an incredibly high incentive to save. Thus, the percentage of wealth held by the top 1% of households is actually reduced by 0.2 relative to a model with no present bias. Again, we see the powerful role played by general equilibrium as the negative impact of agent's biases are somewhat mitigated by the increased relative prices of capital.

As I vary the proportion of individuals who are present biased from 0 to 1, there is a steady increase in both the Gini coefficient **and** the percentage of wealth held by the 99^{th} percentile of earners. The model in which 75% of households are present biased ("FullPB$_{.25}$") results in a wealth share for the top 1% of earners of 14.4, which is 20% higher than the equivalent wealth share in the "Base" calibration and 4.5% higher than the wealth share in the "Full" calibration in which no agents are present biased. The Gini coefficient in this experiment is equal to 0.80, which matches the Gini reported in SCF data exactly. Thus, a model in which there is a high degree of discount rate heterogeneity and heterogeneity in whether households are present biased generates an equilibrium wealth distribution that better matches both the Gini coefficient and the wealth share of the top 1% of households.

The rationale behind this finding is fairly straightforward. As shown in section 5-B, if we fail to imbed present biased agents in a general equilibrium model in which prices respond to their actions, the resulting model economy will overstate the role of present bias in generating wealth inequality. When agents are heterogenous with regard to their present bias, present biased agents are essentially imbedded in a partial equilibrium model, albeit one in which the price (or in this case, the value of β_c) is closer to their desired price than it would be if every agent in the economy was not present biased. In the following section, I show the present biased and non present biased sub-economies when $\lambda = 0.75$, 0.50, and 0.25. As the percentage of agents in the model economy who are present biased increases, the gap in average wealth between present biased and non-present biased individuals widens. This occurs as non present biased individuals exploit an interest rate that is quite high relative to their discount factor exactly because the behavior of present biased individuals has bid this value up in equilibrium. By examining mean wealth and Gini coefficients by age for both present biased and non present biased agents, I am able to show that the increased wealth dispersion reported in Table 4 is the result of *across* group inequality. *Within group* inequality is reduced as the percentage of individuals in the economy who are present biased is increased.

5-D. The Present Biased Sub-Economy

In this section I outline distinctions between two agent types in the "FullPBHet" model experiments: non present biased (non PB) and present biased (PB) individuals. Recall, each "FullPBHet" calibration includes a proportion λ of non PB agents and a proportion $(1 - \lambda)$ of PB agents. In each of these calibrations, I define the economic behavior of non PB agents as the *exponential sub-economy* and the economic behavior of present biased agents as the *present biased sub-economy.* In Table 4, I report the equilibrium value of β_c for each experiment that normalizes the interest rate and capital-to-output ratio across models. The "Full" model in which all agents are non PB results in a value of $\beta_c = 1.003$

Table 5: Wealth Inequality for PB vs non PB Agents

	Gini	Top 1%	Top 5%	Top 20%
PSID (2003)	0.76	25.3	47.1	77.4
SCF (1998)	0.80	34.7	47.8	81.8
Full	0.77	13.8	40.1	80.3
FullPBHet ($\lambda = .75$)	0.78	14.0	41.2	81.5
-Non PB Agents	0.76	12.9	38.8	79.1
-PB Agents	0.83	17.8	48.1	88.7
FullPBHet ($\lambda = 0.50$)	0.79	14.2	42.1	82.5
-Non PB Agents	0.75	11.9	37.3	77.8
-PB Agents	0.82	16.6	46.0	86.4
FullPBHet ($\lambda = 0.25$)	0.80	14.4	42.3	82.9
-Non PB Agents	0.74	10.7	36.0	76.7
-PB Agents	0.81	15.3	43.6	84.6
FullPB	0.79	13.6	41.1	82.3

and the "FullPB" model in which all agents are PB results in a value of $\beta_c = 1.027$. As the proportion of individuals endowed with $\delta = 0.7$ increases, the value of equilibrium β_c increases as well. Thus, when some agents are present biased, non PB agents have access to an effective interest rate ($\beta_c \times r$) that is *higher* than what they face in a model comprised entirely of exponential discounters, and PB agents face an effective interest rate that is *lower* than what they face in a model economy comprised entirely of present biased agents. It is this discrepancy that places both agents types in a partial equilibrium sub-economy in which prices have not fully adjusted to the individual preferences of each respective type.

Table 5 highlights the Gini coefficient and the wealth holdings of the top 1%, 5% and 20% of households in each "Full" model economy as well as in SCF and PSID data. For each "FullPBHet" calibration, these same statistics are evaluated for exponential and present biased sub-economies. By evaluating the sub-economy associated with each agent type, I am able to provide insight into the avenue through which increased wealth dispersion is arising in the model economy. As shown in Table 4, as the percentage of agents who are present biased increases, overall inequality increases. The Gini coefficient and wealth holdings of the top 1%, 5%, and 20% of households increase to levels much closer to that in the data in response to the increased proportion of agents displaying time inconsistent preferences. However, wealth inequality in each sub-economy is highest *for both PB and non PB agents* when the percentage of agents endowed with present biased preferences is low!

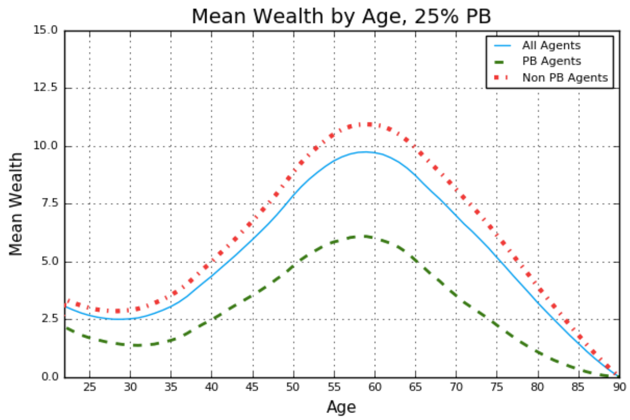
When $\lambda = 0.75$, the present biased sub-economy (25% of the total population) has a Gini coefficient on wealth of 0.83 and the wealth ownership of the top 1% of PB households is 17.8% of all wealth in the present biased sub-economy. The exponential sub-economy has a Gini coefficient of 0.76 and wealth ownership of the top 1% of households is 12.9%. As λ increases, the Gini coefficient in each sub-economy decreases and the wealth holdings of the top 1%, 5% and 20% of households in each sub-economy decrease as well. However, in spite of the reduction in inequality within each sub-economy, overall inequality is increased as the proportion of present biased agents increases.

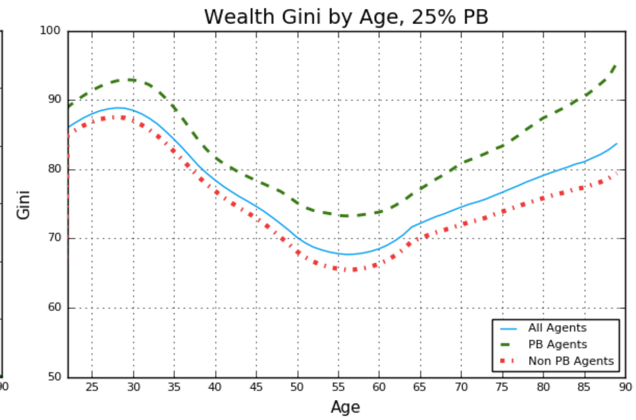
Again, we return to the importance of general equilibrium effects when agents are endowed with time inconsistent preferences. As more present biased individuals are added to the model economy, the common discount factor β_c is bid up by the impatient behavior of these individuals. As demonstrated by the distinction between the "FullPB" and "Full" model economies, if all agents are present biased wealth inequality is *reduced* relative to a world in which all agents are exponential discounters. Thus, the higher the proportion of present biased agents in the economy, the greater the savings incentive for both present biased and non present biased agents. As all agents face an increased savings motive when more time inconsistent discounters are in the economy, within type inequality is reduced because both present biased and non present biased agents accumulate more wealth in the face of relatively inexpensive capital. Despite this fact, inequality in the economy as a whole increases because wealth dispersion *across* agent types increases with the percentage of present biased agents in the model economy.

Figure 4 provides further insight into the evolution of wealth inequality in each "FullPB-Het" calibration. The dashed line represents the mean wealth by age of present biased agents and the wealth Gini by age across present biased agents for $\lambda = 0.75$, 0.50, and 0.25. The dash-dot line represents these same statistics for the non present biased agents in each experiment, and the solid line represents the mean wealth by age and wealth Gini by age for all agents in the model economy. Comparing panels (a), (c), and (e), it is apparent that as the proportion of individuals in the model economy endowed with present bias increases, the gap in mean wealth holdings between PB and non PB agents increases. Although a higher β_c leads to an increase in the mean wealth holding by age for both the present biased and exponential sub-economies, as $(1-\lambda)$ increases, agents in the exponential sub-economy take advantage of the increased effective interest rate they face when coexisting in an economy with a large number of present biased individuals and amass much higher average wealth than their present biased peers. Thus, as the percentage of individuals in the economy endowed with present biased preferences increases, inequality between time-inconsistent and time-consistent agents increases.

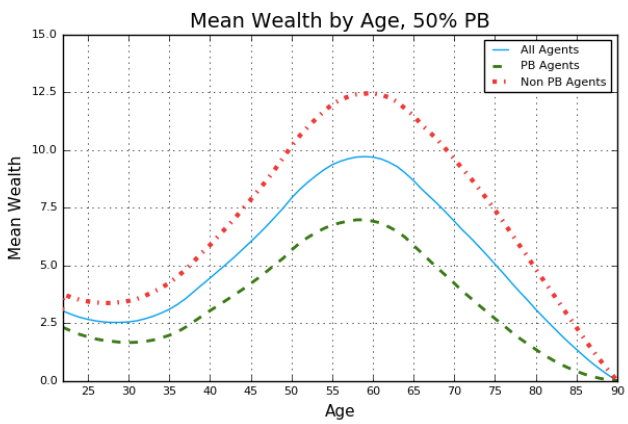
As cross group inequality increases in response to a higher proportion of present biased agents in the model economy, within group inequality is somewhat reduced in both the exponential and present biased sub-economies. Consider the Gini coefficient on wealth by age displayed in panels (b), (d), and (f) of Figure 4. As more present biased agents are added to the model economy, the Gini coefficient at every age is lowered (very slightly) for both present biased and exponential agents. As shown in Table 5, this decrease in within group inequality results from an increased savings incentive for all agents as an increase in $(1-\lambda)$ drives up β_c. When all agents face an increased savings incentive, they accumulate more wealth earlier in their lifetime and inequality is driven down by an economy comprised of net savers. Further, the gap in wealth Gini by age between the present biased and exponential sub economies is largely unaffected by the percentage of individuals endowed with PB preferences. That is, the distance between the dashed line (PB agents) and the dashdot line (non PB agents) is nearly identical for each calibration. This gap is smallest when agents are young and begins widening at a much faster rate when agents near retirement and the shortened planning horizon of older households interacts with the natural inclination

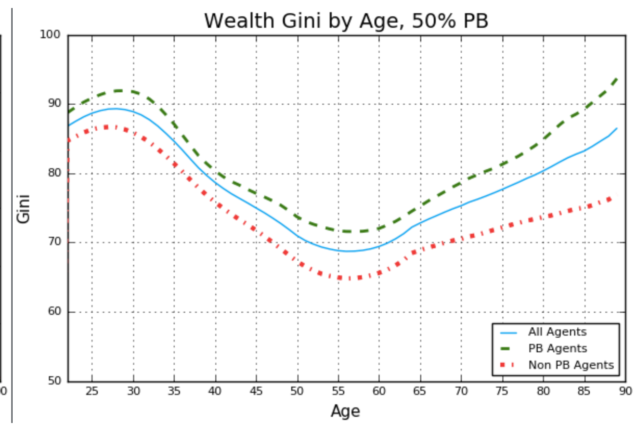
(a) (b)



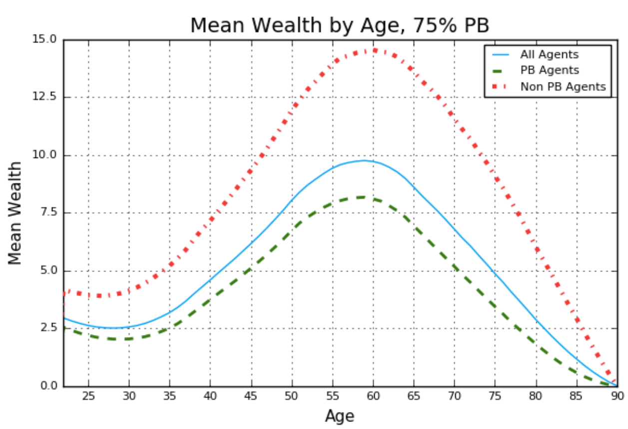


(c) (d)





(e) (f)



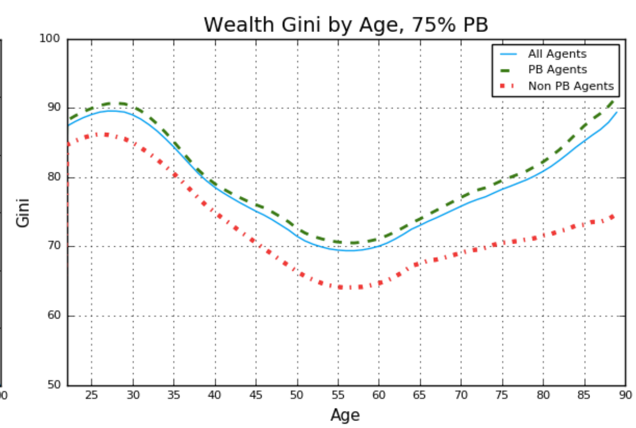


Figure 4: Evolution of Wealth over the life-cycle

to under-save inherent to present biased optimizers (as shown in Section 5-B).

A final note of interest from Figure 4 pertains to the exact shape of the wealth Gini by age for all agents in the model economy (the solid line in all three experiments). In panels (b), (d), and (f) this line reaches its minimum between age 55 and 60, but the rate at which inequality evolves following its trough depends on the percentage of agents in the model economy who are present biased. In panel (b), when only 25% of agents are present biased, the wealth Gini by age increases at a much slower rate following age 60 to a lower overall level than it does in panel (d) (50% of agents PB) or panel (f) (75% of agents PB). Thus, my model offers a convenient means of backing out the percentage of households who display present biased preferences via the examination of the evolution of wealth inequality as agents age. Exponential discounting will result in a low degree of wealth dispersion following age 60, while present bias implies a rapid increase in wealth dispersion after age 60. By measuring the actual dispersion of wealth amongst older households, one could infer the percentage of households endowed with each preference type.

This result has the potential to resolve an issue discussed in Hendricks (2007). Recall Hendricks' insight that heterogeneous exponential discounting leads to dispersion in wealth as households age. Thus, he calibrated the distribution of households over exponential discount factors to minimize the distance between model Gini coefficients on wealth by age and moments in U.S. data. Although it may appear that this approach could be used to replicate the wealth distribution perfectly, Hendricks notes "since preference heterogeneity increases inequality among young and old households, it is not possible to match inequality among the old without overstating inequality among the young". As shown throughout Section 5, present bias offers an avenue through which old age wealth inequality can be increased without overstating inequality among younger households. A future endeavor aimed at calibrating the distribution of households over exponential and present biased discount factors may offer a resolution to this issue proposed by Hendricks. A calibration approach of this type could, at the very least, shed light on the percentage of households endowed with present biased preferences.

6 Conclusion

I imbed present biased agents into an overlapping generations, quantitative life-cycle general equilibrium framework in which agents face uninsurable idiosyncratic income shocks. If all agents are assumed to be present biased, the distribution of wealth is largely unaffected relative to a baseline economy in which all agents are exponential discounters. However, stark differences arise between a present biased society and a society of homogenous, exponential discounters when the accumulation of wealth over the life-cycle is analyzed. A present biased society is characterized by an earlier peak in mean wealth by age and by increased dispersion in wealth as household reach retirement. I then consider a model economy populated by agents who are heterogenous across both their exponential and present biased discount factor. I find the inclusion of some present biased households improves the fit of the model economy to the data. The increase in wealth dispersion resulting from an increase in the percentage of households that are present biased arises as time-consistent discounters amass higher wealth relative to their present biased peers due to the general equilibrium impact of present biased optimization on the effective interest rate.

The work outlined above highlights several avenues through which present bias may help to rationalize gaps in the literature pertaining to the savings decisions of retired households. As noted in Schreiber and Weber (2016) and Huffman et al. (2017), some puzzling behaviors associated with retired households appear to be correlated with present biased preferences. Schreiber and Weber highlight the fact that older households characterized as impatient are more likely to choose a lump sum over a fair annuity relative to patient older households and younger (age 30-50) impatient households. This result fits nicely with the general equilibrium implications of present bias; older households are more tempted by their behavioral biases than younger households due to their shortened planning horizon and lower average wealth. Huffman et al. find there is a large degree of heterogeneity in time discounting of older households, but due to data restrictions they are unable to distinguish between heterogeneity in the exponential discount factor and heterogeneity in the present biased discount factor. Thus, they conclude that as less patient households have significantly lower net wealth in retirement, this is "probably indicating that the least patient save less and therefore arrive at old age with fewer assets." My results imply that this may not be the entire story. If agents are present biased, then wealth dispersion in retirement will be a function of both pre-retirement savings decisions **and** post-retirement savings decisions. Thus, further work eliciting the proportion of individuals displaying present biased preferences could help to distinguish the role played by high discounting due to a low draw of β versus a draw of $\delta < 1$.

Although I have primarily focused on the interaction of present bias and old age, it is important to note that the key interaction of shortened planning horizons and low wealth that leads older populations to suffer most from their biases is not an age dependent phenomenon. In the modeling environment utilized throughout this paper, the only finite horizon for an agent is the entire life-cycle. However, as post-secondary educational investment occurs early in life, a young agent is effectively looking at a short finite horizon over which they must decide how much schooling to attain. Previous work by Nighswander (2017) imbeds present biased agents in a simple three period framework in which time in the first period of life is split between supplying low skill labor and acquiring human capital via costly education. I find that present biased individuals acquire less education leading to lower lifetime earnings and lower retirement consumption than time-consistent discounters. I intend to extend this simple model of educational choice to a life-cycle framework in which agents must decide how much education to acquire while young before entering the labor market. This endeavor may help to amplify lifetime inequality in the model economy to levels closer to US data.

Appendix A. Computational Algorithm

Outline of Equilibrium Solution Algorithm:

1. Propose a candidate interest rate, r, and common discount factor, β_c.For a given β_c:
2. Solve the household problem (find $c(x)$ and $k(x)$) given prices r and w that solve the firm optimization problem for the capital stock implied by the interest rate r.
3. Using the optimized capital decision rule, $k(x)$, compute individual savings decisions for the stable distribution of households.
4. Compute aggregate capital, K_1, the capital stock in the model economy given preferences β_c and the interest rate r. Compute the implied capital to output ratio, K_1/Y_1.
5. If K_1/Y_1 is sufficiently close to the target capital output ratio of 3.10, stop. If not, propose a new β_c and repeat steps (a)-(d) until convergence is achieved.

Appendix B. The Effective Discount Rate

If the Bellman Equation outlined by equations (2)-(5) has an interior solution, such a solution satisfies the present biased Euler Equation:

$$u'(c_t) \geq \beta s_{t+1} E_t\{u'(c_{t+1})[1 + r - (1 - \delta)c_k(k_{t+1}, e_{t+1}, t + 1)]\} \tag{9}$$

where u' is the derivative of the utility function and c_k represents the the derivative of the optimal consumption function w.r.t assets. Equation (9) can be re-written in the following way:

$$
\begin{aligned}
u'(c_t) &\geq \beta s_{t+1} E_t\{u'(c_{t+1})[1 + r - (1-\delta)c_k(k_{t+1}, e_{t+1}, t+1)]\} \\
\Leftrightarrow u'(c_t) &\geq \beta s_{t+1} E_t\{u'(c_{t+1})\}(1+r) - \beta s_{t+1} E_t\{u'(c_{t+1})(1-\delta)c_k(k_{t+1}, e_{t+1}, t+1)\} \\
\Leftrightarrow u'(c_t) &\geq \beta(1+r) s_{t+1} E_t\{u'(c_{t+1})\}\left[1 - \frac{1-\delta}{1+r} E_t\{c_k(k_{t+1}, e_{t+1}, t+1)\}\right] \\
\Leftrightarrow u'(c_t) &\geq \beta(1+r) s_{t+1} E_t\{u'(c_{t+1})\}\left[1 - \frac{1-\delta}{1+r} \frac{E_t\{u'(c_{t+1})c_k(k_{t+1}, e_{t+1}, t+1)\}}{E_t\{u'(c_{t+1}\}}\right] \\
\Leftrightarrow u'(c_t) &\geq \beta\left[1 - \frac{1-\delta}{1+r} \frac{E_t\{u'(c_{t+1})c_k(k_{t+1}, e_{t+1}, t+1)\}}{E_t\{u'(c_{t+1}\}}\right](1+r) s_{t+1} E_t\{u'(c_{t+1})\}
\end{aligned}
$$

let $\beta_{x'} = \beta\left[1 - \frac{1-\delta}{1+r} \frac{E_t\{u'(c_{t+1})c_k(k_{t+1}, e_{t+1}, t+1)\}}{E_t\{u'(c_{t+1}\}}\right]$, then the present biased Euler Equation can be written as:

$$
u'(c_t) \geq \beta_{x'}(1+r) s_{t+1} E_t\{u'(c_{t+1})\}
$$

which is exactly equation (6) in the text.

References

Aiyagari, R. (1994). Uninsured Idiosyncratic Risk and Aggregate Savings. *Quarterly Journal of Economics 109*, 239-354.

Angeletos, G.M., Laibson, D., Repetto, A., Tobacman, J., and S. Weinberg (2001). The Hyperbolic Consumption Model: Calibration, Simulation, and Empirical Observation. *Journal of Economic Perspectives, vol. 15 no. 3*, 47-68.

Benhabib, J., Bisin, A., and S. Zhu (2011). The Distribution of Wealth and Fiscal Policy in Economics with Finitely Lived Agents. *Econometrics 79(1),* 123-157.

Brown, J.R., and A. Previtero (2017). Saving for Retirement, Annuities and Procrastination. *Working Paper.*

Cagetti, C. and M. DeNardi (2006). Entrepreneurship, Frictions, and Wealth. *Journal of Political Economy, 114,* 835-870.

DellaVigna, S. (2009). Psychology and Economics: Evidence from the Field. *Journal of Economic Literature, Vol. 47 (2)* 315-372.

DeNardi, M. (2004). Wealth Inequality and Intergenerational Links. *Review of Economic Studies 71(3)*, 743-768.

DeNardi, M. and F. Yang (2014). Bequests and Heterogeneity in Retirement Wealth. *European Economic Review 72*, 182-196.

Dynan, K.E., Skinner, J., and S.P. Zeldes (2004). Do the Rich Save More? *Journal of Political Economy, 112,* 397-444.

Frederick, S., Loewenstein, G., and T. O'Donoghue (2002). Time Discounting and Time Preference: A Critical Review. *Journal of Economic Literature, Vol. 40 (2).* 351-401.

Harris, C. and D. Laibson (2001). Dynamic Choices of Hyperbolic Consumers. *Econometrica, 69(4),* 935-957.

Heer, B. (2001). Wealth Distribution and Optimal Inheritance Taxation in Life-Cycle Economies. *Scandinavian Journal of Economics, 103,* 445-465.

Hendricks, L. (2007). How Important is Discount Rate Heterogeneity for Wealth Inequality? *Journal of Economic Dynamics and Control, 31(9),* 3042-3068.

Huffman, D., Maurer, R., and O. Mitchell (2017). Time Discounting and Economic Decision-Making in the Older Population. *The Journal of the Economics of Ageing.*

Huggett, M. (1996). Wealth Distribution in Life-Cycle Economies. *Journal of Monetary Economics, 38*, 469-494.

Hurst, E. and K. Charles (2003). The Correlation of Wealth Across Generations. *Journal of Political Economy, 111(6),* 1155-1182.

İmrohoroğlu, A., İmrohoroğlu, S, and D.H. Joines. (2003). Time-inconsistent preferences and social security. *Quarterly Journal of Economics*, 745-784.

Krusell, P. and A.A. Smith (1998). Income and Wealth Heterogeneity in the Macroeconomy. *Journal of Political Economy 106,* 868-896.

Krusell, P., Kuruscu, B., and A.A. Smith (2002). Equilibrium Welfare and Government Policy with Quasi-geometric Discounting. *Journal of Economic Theory, 105,* 42-72.

Laibson, D. (1997). Golden eggs and hyperbolic discounting. *The Quarterly Journal of Economics- Volume 112, Issue 2*, 443-477.

Laigson, D. (2015). Why don'e present-biased agents make commitments? *American Economic Review Papers and Proceedings; 105(5)*, 267-272.

Laibson, D. Repetto, A., and J. Tobacman (2007). Estimating Discount Rate Functions with Consumption Choices Over the Lifecycle. *Technical Report, National Bureau of Economic Research.*

Maliar, L. and S. Maliar (2006). The Neoclassical Growth Model with Heterogeneous Quasi-Geometric Consumers. *Journal of Money, Credit, and Banking 38(3),* 635-654.

Meier, S. and C.D. Sprenger (2015). Temporal Stability of Time Preferences. *The Review of Economics and Statistics, 97(2),* 273-286.

Nighswander, T. (2017). Human Capital Formation and Quasi-Hyperbolic Discounting. *Working Paper.*

Paserman, M.D. (2008). Job Search and Hyperbolic Discounting: Structural Estimation and Policy Evaluation. *The Economic Journal, 118,* 1418-1452.

Phelps, E.S., and R.A. Pollak (1968). On Second-Best National Saving and Game Equilibrium Growth. *Review of Economic Studies, XXXV*, 185-199.

Piketty, T. and G. Zucman (2014). Capital is Back: Wealth-Income Rations in Rich Countries 1700-2010. *Quarterly Journal of Economics,* 1255-1310.

Quadrini, V. (2000). Entrepreneurship, Savings and Social Mobility. *Review of Economic Dynamics, 3* 1-40.

Schreiber, P. and M. Weber (2016). Time Inconsistent Preferences and the Annuitization Decision. *Journal of Economic Behavior & Organization, volume 129*, 37-55.

Strotz, R. H. (1956). Myopia and Inconsistency in Dynamic Utility Maximization. *Review of Economic Studies, XXIII*, 165-180.

Tanaka, T. Camerer, C.F., and Q. Nguyen (2010). Risk and Time Preference: Linking Experimental and Household Survey Data from Vietnam. *The American Economic Review, Volume 11 No. 1*, 557-571.