Innis Lecture: Environmental crises: past, present, and future

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Abstract. Environmental crises are distinguished by rapid and largely unexpected changes in environmental quality that are difficult if not impossible to reverse. Examples would be major extinctions and significant degradations of an ecosystem. I argue there are three preconditions for crisis: failures in governance, an ecological system exhibiting a tipping point, and an economy/environment interaction with positive feedbacks. I develop a simple model to illustrate how a crisis may arise, and draw on our knowledge of past and present crises to highlight the mechanisms involved. I then speculate as to whether climate change is indeed a crisis in the making.

Grandes crises environnementales: passé, présent et futur. Les crises environnementales sont caractérisées par des changements rapides et largement non-anticipés dans la qualité de l'environnement qui sont difficiles sinon impossibles à renverser. Des exemples pourraient être des extinctions d'espèces et des dégradations significatives d'un écosystème. L'auteur suggère qu'il faut trois conditions pour qu'il y ait crise : faillites de gouvernance, un système écologique au point de bascule, et une interface environnement/économie riche de rétroactions positives. L'auteur développe un modèle simple pour illustrer comment une crise peut émerger et s'appuie sur la connaissance des crises passées et présentes pour souligner les mécanismes en jeu. On spécule en fin de texte sur la question à savoir si le changement climatique est une crise en train de prendre.

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1. Introduction

The current financial crisis has given many economists pause. Do we have a good understanding of the likelihood of crises, their cause, or their cure; or is the economics of crises so different from the normal economics we teach and write about, that we can offer very little guidance to policymakers today? While I am not a macroeconomist, even a layman's read of the business press suggests that something unique has transpired over the last two years, something largely unexpected by most economists, and something that has proven difficult and very costly to reverse.

With the financial crisis in the news every day, it is hard to think about anything else. And as the crisis dragged on, I started to think about the likelihood of a similar environmental crisis. Even though I have studied some very destructive environmental events, I have always viewed the probability of any future crisis as very close to zero. This view was more an article of faith than a reasoned position, and I found myself questioning it. Since environmental economics does not have a standard theory of crises to guide my thoughts or help me organize data and history in a coherent manner, perhaps a future crisis was possible.

This paper describes my attempt to develop a theory of environmental crises as a means of answering this question. A recent exchange between William Nordhaus and Martin Weitzman over catastrophic climate change has brought 'environmental crises' to the fore of environmental economics, so perhaps the time is ripe for a simple model that helps us understand the mechanics of crises more fully.[1] Macroeconomics has, after all, well-known models of banking and foreign exchange crises, but at present there is no similar work in environmental economics. My goal is to provide a first step by identifying a set of preconditions for a crisis. By doing so I hope to provide an answer to my own question, but more important to spur others to investigate further.

My method is deliberately simple. First, I provide a definition of an environmental crisis that is both narrow enough to be useful for theory, and broad enough to include several real-world examples. Second, I revisit the classic Gordon-Schaefer model of resource use (hereafter the GS model) to understand why crises are so difficult to generate in this framework.[2] The GS model is the simplest dynamic model we use to think about many of the most pressing resource issues of the day – overfishing, biodiversity loss, and global warming. The model, however, admits little room for crisis. Fortunately, it does provide us with clues as to why it is crisis resistant and, by making three simple changes, I construct a far less resistant variety I refer to as the 'Crisis Model'.

1 I am refering to the recent fat tailed versus thin tailed exchange between William Nordhaus and Martin Weitzman. See Weitzman (2009a), a comment by Nordhaus (2009) and two replies by Weitzman (2009b,c).

2 For the original contributions see Gordon (1954) and Schaefer (1957). For a thorough treatment see Clark (1990).

The changes I introduce allow for imperfect regulation in resource industries, a tipping point in natural growth, and a positive feedback effect arising from the interaction of economic activity and the environment. These are the preconditions for crisis. I then examine the mechanics of crises using a combination of phase-plane techniques and comparative steady state analysis. I provide analytical results and highlight these results by simulating the model's distribution of outputs given a distribution of input shocks. These simulations are neither calibrations nor tests of the model; they are just useful ways to illustrate the possibilities the model presents. Throughout I draw on well-known environmental crises in the very distant and not so distant past in the hope of convincing the reader that the mechanisms I identify are more than just theoretical constructs. Finally, using the theory developed, I ask whether a future crisis is possible. While there are perhaps many candidates for future crises, I consider climate change.

To investigate the role played by each of the preconditions, I examine them in isolation. I eliminate one or more of the preconditions and focus on the remaining force. I adopt assumptions to ensure that in the absence of feedbacks or tipping points, the model would be well behaved; that is, it generates a stable, interior, steady state. With this method I am able to identify the novel features introduced by each precondition and to show how they combine to deliver a theory of environmental crises.

I first revisit the Brander and Taylor (1998) analysis of Easter Island, but now introduce governance that limits overharvesting and a tipping point that generates a truly catastrophic outcome. I show that the tipping point divides the state space into two basins of attraction: one basin leads to an interior steady state; the other to a catastrophic outcome with extinction of both the resource and population. More effective governance moves the economy's interior steady state away from the catastrophe, and shrinks its basin of attraction. While more effective governance is in some sense stabilizing, it can be all for naught. When the interior steady state occurs at a resource stock that is below a critical threshold (related to, but far above, the tipping point), then the steady state is unstable, and all trajectories (save one) lead to the catastrophic outcome. Tipping points therefore introduce dramatic changes in the system's dynamics.

While a tipping point plus poor governance can generate a crisis, these two forces alone are unlikely to be the whole story. In the Easter Island case in particular, the catastrophic outcome is guaranteed only if the tipping point is approximately 30% of carrying capacity. While this is possible, it is in some sense unsatisfactory to develop a theory where crises are always and everywhere a function of large tipping points. Consequently, I proceed to investigate the role of feedback effects.

To sharpen the focus on feedback effects I eliminate the tipping point entirely and rule out any government intervention. Under these assumptions, positive feedbacks arise quite naturally in a variety of circumstances that we often discuss in environmental economics. I offer examples of positive feedbacks drawn from the 19th-century elimination of the American bison and the passenger pigeon, and link my discussion to recent work on catastrophic climate change. I show how positive feedbacks magnify shocks to the system, but are, in an important sense, self-limiting. As a result, I am forced to conclude that positive feedbacks cannot be the whole story either.

The final section puts the three preconditions together. I demonstrate how a relatively small tipping point, weak governance, and positive feedbacks can produce a crisis. The mechanism I propose is simple. A shock hits the system (the precipitating event), and it is allowed to propagate because of weak governance. It is magnified by positive feedbacks, and by doing so it crosses a boundary related to the tipping point. The dynamics of the system change irrevocably, environmental quality is driven to its lowest possible level, and we have a crisis. I provide an analytical result showing how positive feedback effects and tipping points are complements in crisis creation. The tipping point can be very small because of this complementarity (e.g., in my simulation the tipping point is only 4% of the carrying capacity); and positive feedbacks can be self-limiting because they are not required to carry the day. Together these two forces provide a more nuanced theory of crises creation than either could alone.

To illustrate these results I simulate the model by drawing 100,000 endowment shocks from a simple symmetric distribution and generate the resulting histogram (or sample probability density function) of steady state outcomes for the environment. The resulting probability density over outcomes is symmetric when feedback effects are absent, but its mass is shifted towards extreme outcomes as feedbacks grow in strength. Once I introduce a tipping point, the probability mass bifurcates and the resulting probability density is neither fat tailed nor thin tailed. The density is instead discontinuous, with the majority of the mass centred on ‘normal interior outcomes, with the remaining probability mass concentrated on just one point – the catastrophic outcome.

This depiction of a crisis has many attractive features. The crisis is a truly unique event: it is surrounded by a sea of zero probability with no neighbouring outcomes ever realized. The crisis is extremely difficult to reverse. This is because the local dynamics at the crisis point are decidedly unfriendly, since we are past the tipping point. And finally, the crisis is difficult to learn about and difficult to learn from. Crises are difficult to learn about because they are by definition infrequent, low-probability events. They are difficult to learn from because much of the useful variation in the forcing variable, which we would typically use to link cause with effect, is compressed into one outcome – the crisis.

With at least a rudimentary theory of environmental crises in hand, I turn to assess the likelihood of a future climate crisis. I ask whether the preconditions for a crisis are met in this case, and argue that the run-up in carbon dioxide (CO_2) concentrations since the industrial revolution could be the precipitating event. Since the preconditions are met and a precipitating event is in place, I am forced to conclude that a future climate crisis is indeed possible. It is not a certainty, nor even likely, but the current state of affairs with regard to climate change does satisfy the requirements for crisis.[3]

1.1. Environmental crisis: a definition

I define an environmental crisis as a dramatic, unexpected, and irreversible worsening of the environment leading to significant welfare losses. This definition includes and precludes several things. First, the change has to be dramatic and rapid in its pace. Therefore, the slow reduction in species numbers worldwide or the gradual reduction in a fish stock does not constitute an environmental crisis under this definition. The environmental change has to be 'unexpected,' and by this I mean it is a low probability event. Dramatic changes and an element of unexpectedness distinguish crises from what I would refer to as 'resource tragedies.' Resource tragedies are situations in which resource overuse has been long-standing, the only remaining uncertainty being exactly when the train comes off the tracks. These situations are also worthy of study, but they are not true crises.[4] An element of irreversibility is also important. If resources or nature are quick healing, then it is difficult to see how any change in the environment should be of much concern, but if recovery took a century or more, things would be quite different. Finally, the change in the environment must produce a significant welfare loss; therefore, the scale of the damage cannot be small.

If a crisis is ever to emerge, I have to either limit the ability of agents to forecast the future, or reduce the government's ability to enforce first-best outcomes. In what follows, I will assume governments are less than perfect, while agents are trapped in a system where self-interested actions produce aggregate welfare losses.

1.2. The world according to Gordon-Schaefer

The Gordon-Schaefer model comprises one definition plus three assumptions. The definition is written as a differential equation linking changes in the resource stock to the difference between natural growth and harvesting; the assumptions are on the technology for harvesting, the biology of natural growth, and an objective function that values the resource by the discounted sum of rents it can provide. A typical representation would be

$$
\begin{aligned}
dS/dt &= R(S) - H \\
H &= \alpha L_h S \\
R(S) &= r S(1 - S/K) \\
W &= \int_0^\infty [pH - w L_h] e^{-\delta\tau} d\tau, \qquad (1)
\end{aligned}
$$

3 What should we do? Emission reductions plus investment in geoengineering options seem to be the obvious answer. Emission reductions will move us back from any potential tipping point in the climate system; geoengineering needs to be explored because we may need a means to lower temperatures rapidly in the unlikely event that a crisis occurs. For some interesting recent work in the economics of geoengineering see Moreno-Cruz (2009). A useful introduction is Keith (2000).

4 For example, the collapse of the cod fishery did create a political crisis, but most resource economists fully expected the cod fishery to collapse; it was just a question as to when.

where $R(S)$ is natural growth taken to be logistic, r is the intrinsic rate of resource growth, and K is the carrying capacity of the environment. H is harvesting, L_h is labour employed in harvesting, p and w are prices for the harvest and labour input, respectively, while α and δ are parameters reflecting the state of technology and the strength of time preference.

The narrow interpretation of S is that it represents a commercial species subject to harvest, such as fish or wildlife, but the model is commonly employed to examine the economics of deforestation, air pollution, water quantity and quality, soil erosion, antibiotic resistance, and issues related to long-run growth. The only limit to the model's applicability is that the 'resource' in question be renewable, while 'harvesting' is difficult to control because it is diffuse and hard to monitor.

To make the model operational we add an assumption on the success or failure of government policy in regulating the harvest (typically policy is either first best or entirely absent); an assumption on how the opportunity costs of labour in the harvesting sector is determined (the wage w is typically taken as given); and some method of price determination (often the price p is fixed or a partial equilibrium demand function is specified).

Under standard conditions, this model cannot produce a crisis. If policy is absent, open access produces rent dissipation, but rent dissipation alone does not constitute a crisis. Alternatively, if policy is present and perfect, a most rapid approach path is sometimes optimal, but this rapid depletion path produces no welfare losses. Under even tighter conditions, extinction of the resource can be optimal, but this too does not constitute a crisis under my definition. Similarly, shocks to prices, wages, technologies, and other parameters never produce a crisis; and this is, in fact, why the model and its many variants have proven to be so useful.[5]

2. Three steps to a crisis

2.1. Governance

The first step in generating a crisis is to limit the omniscience of government. While the first best is a useful theoretical construct, in industries that draw on common pool resources that are hard to define and control, governments face severe monitoring problems. Therefore, the first-best policy derived under the assumption of perfect information and costless enforcement may bear little resemblance to those policies that could be implemented in practice. Accordingly, I assume the best governments can do is scale back harvesting effort.

Define the strength of governance, G, as the extent to which policymakers can constrain harvesting below the unconstrained open-access outcome. Let L_h^O denote the labour allocation in the H sector under open access and let L_h^F denote the first best allocation of labour; then G is defined by

$$L_h = G L_h^O \quad \textit{where} \quad G \in \left[L_h^F / L_h^O, 1\right]. \tag{2}$$

5 If we add uncertainty to resource growth then further possibilities present themselves some of which are 'crisis-like.' See, for example Pindyck (1984).

When G is at its minimum, it implements the first best with $L_h = L_h^F$; when it is at its maximum, it implements open access with $L_h = L_h^O$.[6]

While capturing governance in this simple way is ad hoc, in recent work Brian Copeland and I develop a theory of resource management where the implementation of the first best is constrained by the government's inability to monitor harvests perfectly. In Copeland and Taylor (2009) we combine a simple moral hazard model with a general equilibrium version of the GS model I developed earlier with James Brander (see Brander and Taylor 1997). The resulting theory shows how the constrained first-best outcome lies somewhere between the unconstrained first best (without the monitoring problem) and the open access result, and links the extent of policy failure to country characteristics such as enforcement power, the extent of overcapacity in the resource sector, and what we refer to as the incentive to extinguish the resource.

What I am assuming here is more than a shortcut to the Copeland-Taylor analysis, as it makes two further restrictions. First, I take G as constant both in and out of steady state, whereas the Copeland and Taylor (2009) analysis is concerned almost exclusively with steady states. Second, I vary G freely rather than link it to primitives of the economic system.

2.2. Positive feedbacks

The second step in generating a crisis is to introduce a positive feedback effect. The GS model exhibits a strong negative feedback from stock reduction to harvesting productivity and this negative feedback insulates the economy from extreme outcomes.

To generate a positive feedback, I introduce an explicit general equilibrium structure where harvesting H competes with another sector M for labour. Activity in the M sector will not help or harm the environment, but its productivity may be sensitive to changes in the environment. Specifically, I write the production functions in the two sectors as follows:

$$
\begin{aligned}
H &= h(L_h, S) \\
M &= m(L_m, S), \qquad (3)
\end{aligned}
$$

where h and m are homogeneous of degree one in labour and increasing in the measure of the environment's health, S. M will be the numeraire, and I choose units so that one unit of labour produces one unit of M (when S is at its carrying capacity).

These assumptions ensure the model is 'Ricardian' with a linear production possibility frontier at every point in time. A positive feedback is created when productivity changes brought about by a worsening of the environment feed back

6 Open access refers to a situation where withdrawals from the resource are free. Zero pollution regulation implies open access to the atmosphere; no harvesting limits implies open access to wildlife, to forest stocks, or to an acquifer.

through labour or product markets to generate additional activity lowering S. For example, the direct effect of a reduction in S is to lower harvesting for a given labour allocation; but if relative prices or productivity in the M sector change, then the resulting general equilibrium adjustment may shift labour into harvesting. If the general equilibrium reallocation of labour is towards (away from) the H sector, then under my definition there is a positive (negative) feedback effect.

For example, suppose each unit of H production emitted pollution, and this pollution lowered S. Suppose, further, that M is an agricultural industry whose productivity is greatly affected by S, whereas productivity in the polluting industry H is unaffected by S. Then a small expansion of the harming industry H could lead, by lowering S, to a further expansion because the general equilibrium adjustment will shift labour towards the polluting industry. This negative cross-industry externality exhibits a positive feedback effect under my definition.[7]

To be precise, in the Crisis Model the extent of feedbacks, F, is given by

$$F \equiv [\sigma - 1][\epsilon_m - \epsilon_h], \tag{4}$$

where ϵ_m and ϵ_h are the partial elasticities of m and h to a change in S (and are assumed to be constant), and σ is the representative consumer's elasticity of substitution between the two goods. By definition both partial elasticities are non-negative, but their difference can be positive, negative, or zero.[8]

The standard GS model assumes $\epsilon_h = 1$ and has only one sector, so $\epsilon_m = 0$. It often takes product prices as given, which is equivalent to letting $\sigma \to \infty$. As a result, under my definition the GS model exhibits a very strong negative feedback.

2.3. Tipping point

The final step in generating a crisis is to alter the natural growth assumptions of the GS model. The standard GS model adopts the specific logistic form $R(S) = rS(1 - S/K)$. More generally, we could assume $R(S)$ is strictly concave, with $R(0) = R(K) = 0$ and $R'(S) > 0$ for S low and $R'(S) < 0$ for S high. Either assumption amounts to much the same thing – the natural growth of the environment is modelled as purely compensatory; that is, the percentage rate of growth of the environment, $R(S)/S$, is higher the lower is the resource stock.[9] Under the logistic growth assumption of the GS model, $R(S)/S$ is linear, as shown in the top right panel of figure 1. This characteristic of natural growth implies, for example, that the percentage growth rate of a species, the cleansing rate of the atmosphere, or the resistance of humans to bacteria, responds more aggressively the harder it is hit by harvesting, by pollution, or by infection. This feature

7 See section 4 for more examples and for the precise requirements for this case to exhibit positive feedbacks.
8 See the appendix for a further explanation and derivation of the Feedback effect.
9 See Clark (1990) for a discussion of various classes of growth functions (compensatory, depensatory, critically depensatory, etc.)

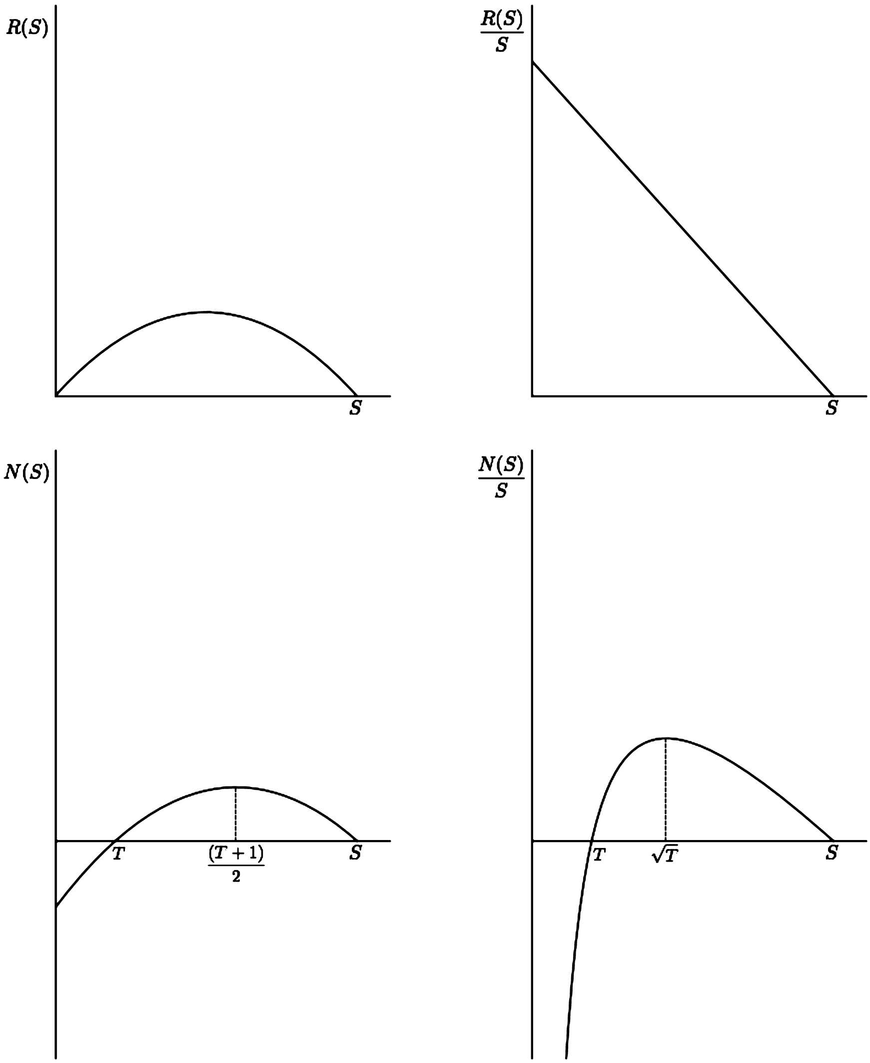

FIGURE 1

provides a nice sort of buffering power to nature and by itself makes crisis-like outcomes hard to generate. A related feature is the absence of any point of no return, or tipping point, where the dynamics of natural growth change for the worst. This is evident in the top left panel of figure 1, where $R(S)$ is everywhere (at least weakly) positive.

My last change to the GS model removes these properties while simultaneously addressing one other concern. Natural scientists are often alarmed when economists capture the environment in one variable S. Their concern is typically not the heroic aggregation we undertake to do so – after all they undertake equally heroic aggregations themselves – but rather that a one-variable depiction of nature rules out many important and interesting physical or ecological system interactions. For example, a biologist may be offended by our treatment of one harvestable species in isolation, without a consideration of how its competitors react when selective harvesting weakens their competition; a climatologist may be alarmed when we ignore additional physical processes such as sea ice melt or permafrost thaw in our one-variable analysis of climate change; and ecologists surely find it disconcerting that our assumptions imply an entire ecosystem can be grown in a coffee cup.

The simplest way to address some of these concerns is to enrich the model by adding another physical process. Specifically, I assume this additional process, $P(S)$ lowers natural growth available for harvesting, cleansing, and so on. $P(S)$ has the following properties:

$$P(S) \geq 0,\ P'(S) < 0,\ P(0) > 0,\ P(K) = 0. \tag{5}$$

Net (natural) growth is then $N(S) = R(S) - P(S)$.

The interpretation of $P(S)$ varies with the application. For example, if S represents a harvestable species, then $P'(S) < 0$ implies that competitive pressure from other species is most intense when the population of the harvested species is low. This additional pressure $P(S)$ results in lower growth. If S measures the density of trees within a given area, then $P(S)$ could represent the impact of soil erosion. Soil erosion intensifies as the forest is thinned, and erosion lowers forest growth. And if S is a measure of the climate's health, then sea ice melt (through the albedo effect) or permafrost thaw (because of degassification) would become larger contributors to climate change as temperatures rose and the climate's health worsened.

If I make the further assumption that $P(S)$ is linear, then net natural growth can be written as a slight variant of the standard logistic growth function:[10]

$$N(S) = r(S - T)(1 - S/K), \tag{6}$$

where T is a measure of the tipping point in units of the stock.

This new growth function has three properties that are shown in the bottom two panels in figure 1. In the bottom left panel it is apparent that natural growth is negative if the stock falls below the tipping point T. Once this barrier is breached, even zero harvesting cannot restore the stock. Also noteworthy is that the rate

10 Let $P(S)$ take the linear form $P(S) = c[K - S]$, where $c = rT/K$; then we obtain the simple form in the text. The critical assumptions are that $P(0) > 0$ and $P'(S) < 0$. To be fair, not all excluded physical processes will, once added, work in this way. For example, in the case of climate change, when warming increases evaporation and cloud cover, this increases the world's albedo, which decreases warming.

of net growth at $S = 0$ is strictly negative. Since a negative stock is not possible, these dynamics will imply a sudden stop to stock depletion as the $S = 0$ barrier is crossed. This has the flavour of a car hitting a brick wall at $S = 0$ and decelerating to zero instantaneously. This feature seems suitable – even desirable – in a paper focusing on catastrophic outcomes.

The percentage rate of growth, $N(S)/S$ is graphed in the bottom right panel. As shown, nature can no longer continually compensate for stock reduction. Nature at first compensates with faster percentage growth as it is diminished, but eventually delivers lower percentage growth as it is pushed too far (below $\sqrt{T}$). Growth per unit time (percentage or absolute) is negative, of course, for any stock $S < T$. To make things a bit cleaner, I have chosen units such that $K = 1$. Under this choice, net natural growth, $N(S)$, takes its maximum at $(T + 1)/2$, as shown. The percentage rate of (net) growth, $N(S)/S$, takes its maximum at $\sqrt{TK} = \sqrt{T} > T$.

With these assumptions in hand, the Crisis Model is almost complete. To close the model I assume a representative consumer with tastes given by a symmetric CES utility function. That is,

$$u = \left[H^{\frac{\sigma-1}{\sigma}} + M^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \text{ with } \sigma \geqslant 0. \tag{7}$$

Identical tastes seems like an innocuous assumption when all agents have the same income. Homothetic tastes rule out income changes working to offset or accelerate feedback effects as the economy grows or shrinks.

The complete model is described by the production functions in (3), the growth function in (6), the full employment condition for labour, $L_m + L_h = L$, the differential equation linking growth, harvesting, and stock accumulation $dS/dt = N(S) - H$, and the preferences given in (7). Since the resource stock is held in common and agents are atomistic, they face no intertemporal problem at all, and with no other store of value in the economy, period-by-period optimization determines decisions. These harvesting decisions, however, are constrained by government policy that tries to manage the resource intertemporally.[11]

3. Revisiting the past

I start by revisiting a past environmental crisis to examine the roles played by governance and tipping points. To sharpen my focus, I eliminate any role for feedback effects by setting $\sigma = 1$. To precipitate a crisis, I rely on natural variation in state variables caused by economic growth and adopt the simplest model of

11 In Copeland and Taylor (2009) agents face an intertemporal optimization problem, where they choose to cheat on government sanctioned allocations and risk punishment or obey by harvesting within limits. The government in turn maximizes the sum of current and future living generations, subject to the requirement that any harvesting policy has to be incentive compatible. All of these complications are swept under the rug here.

growth possible – Malthusian population growth. I take as my illustrative example the rise and fall of the Easter Island civilization.

The record of Easter Island offers us a compelling case of a once prosperous and advanced civilization that crashed dramatically because of environmental degradation. The island is littered with architectural vestiges of a vibrant, wealthy, but ultimately failed society. There are over 800 giant statues (*Moai*), with an average weight of 12.5 tons and a height of 4 metres; several hundred ceremonial platforms (*Ahu*) created with hundreds of tons of smooth sea stone, rubble, and volcanic tuff; and numerous communal houses (*Hare Paenga*) constructed of individual stones weighing as much as 10 tons.

Although much of Easter Island's history was known for hundreds of years, important pieces of the puzzle were not uncovered until the 1980s, when palynologist John Flenley reconstructed the environmental history of Easter Island from pollen spore counts taken from lake sediment (see Bahn and Flenley 1992, chap. 4). This remarkable reconstruction showed Easter Island once contained a large and productive forest, which had disappeared by the time of European discovery. On Easter Sunday 1722 the Dutch captain Jacob Rogeveen was the first to confront the two key mysteries of Easter Island. How did the poorly nourished islanders he encountered build such monuments without the materials necessary for rope or levers or sleds? How did this meagre barren island provide the economic surplus needed to support what must have been a rich and vibrant society?

In some sense, Flenley's discovery explained the mysteries. The lush forest would have provided the materials for rope, levers, rollers and sleds necessary to construct and move the giant statues from their quarry to locations around the island. It would also have provided the materials needed to sustain a productive fishery and the tools for agriculture. The forest also provided ecosystem services such as water retention and purification, while limiting soil erosion and providing a wind break. The forest provided all these materials and services, at least until the islanders destroyed it.

However, this 'explanation' is more of an ex post description of events than a true explanation. The real task of explanation is twofold. The first task is to explain how and why the crash occurred on Easter Island, but the second task is to explain why this dramatic development pattern is not observed on other islands settled by Polynesians. The Polynesians colonized hundreds of islands in the Pacific stretching from Hawaii in the north, New Zealand in the south, and Easter Island in the east. This Polynesian triangle represents the single largest area of colonization by any stone age culture, and the colonizers brought with them their common technology, language, customs, and religion. Easter is but one observation in a sea of data; therefore, an Easter Island specific explanation is really not much of an explanation at all.

Brander and Taylor's (1998) explanation was built on a general equilibrium version of the GS model, amended to allow for population growth in a Malthusian fashion. The answer we provided was that Easter Island's environment was unique in Polynesia: it had a lower average temperature, it was an outlier in terms of low rainfall, and the predominant palm tree that grew on Easter was (most closely related to) the incredibly slow growing *Jubea Chilensis* (the Chilean wine palm). All of these environmental factors suggest that what was unique about Easter was that its resource base – something we termed the forest/soil complex – grew slowly relative to the rest of the islands in Polynesia. A surprising feature of the Brander-Taylor model was that a low resource growth rate implied a dramatic boom and bust path from colonization to the steady state – which we argued fit the prehistory of Easter Island, whereas an otherwise identical island with a faster growth rate exhibited a far less dramatic development path – which we argued was typical for much of the rest of Polynesia. This two-pronged explanation – with only small additional details – is the one presented in Jared Diamond's 2005 book, *Collapse*.[12]

Critics of the original paper focused on three potential weaknesses. The first was that open-access conditions were assumed throughout ($G = 1$), when in fact Polynesians had some rules governing resource use. A second was posed as a question: who cut down the last tree? Wouldn't the islanders have seen the end in sight and then made a mid-course correction to save themselves? Finally, where is the crash? The model predicts cyclical adjustment to a strictly interior steady state. For some readers the development path described by the model was not dramatic enough, and the coincidence of Rogeveen's discovering the island after just one boom and bust cycle was just too fortuitous. These are, I think, all valid questions, which can be addressed with a slightly expanded version of the Crisis Model.

To start, I allow for some degree of control over resource use, $G < 1$ and introduce a tipping point, $T > 0$. To set feedback effects to zero, I set $\sigma = 1$ and follow the original analysis by assuming agents tastes are Cobb-Douglas: $u = H^{\beta} M^{1-\beta}$. To introduce a means of economic growth I again follow the original by assuming population growth is Malthusian:

$$\dot{L} = L[b + \phi[H/L]], \tag{8}$$

where $b < 0$ is the death rate, $\phi > 0$ is a fertility parameter, and H/L is the per capita consumption of the harvest good. If we combine (8) with the Crisis Model, we obtain a simple generalization that allows for economic growth and fluctuations.

It proves useful to start by revisiting the original Brander and Taylor analysis by setting $G = 1$ and $T = 0$. When we combine our earlier assumptions, the dynamics of the resource sector become:[13]

12 A close read of Brander and Taylor (1998) will show we argued that the collapse suffered by the Mayans, the Anazazi, the Assyrians, and that on the remaining mystery islands of Polynesia also fit our theory. Three of these four cases are also discussed in Diamond (2005).

13 See the appendix for a derivation of the equilibrium harvest function H.

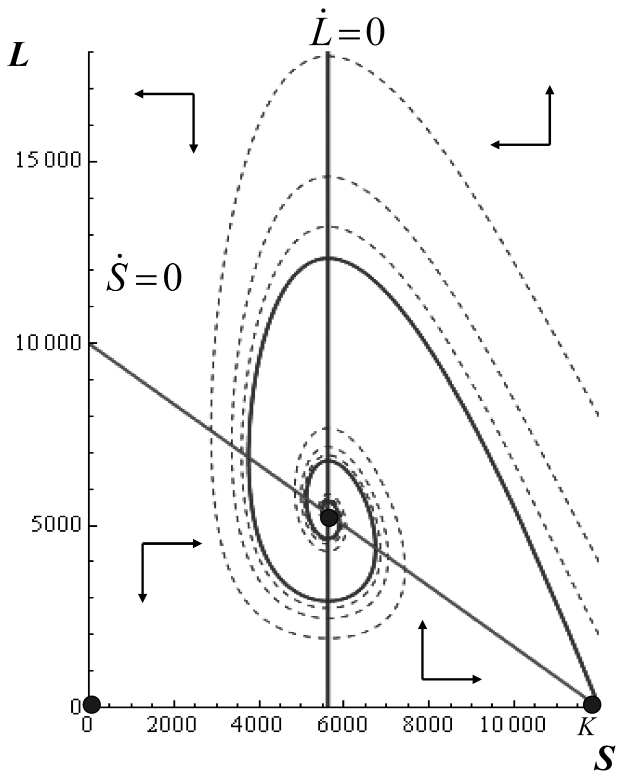

FIGURE 2

$$\dot{S} = rS(1 - S) - H \tag{9}$$

$$H = \alpha\beta LS. \tag{10}$$

Under a simple parameter restriction,[14] which I will adopt throughout, the system described by (8), (9), and (10) has one interior stable steady state shown by the intersection of the two isoclines in figure 2.

The negatively sloped isocline gives those combinations of population size and resource stock that are sustainable. Higher population levels imply lower resource stocks along this sustainability isocline. Points above the isocline imply

14 The restriction ensures an interior steady state in the absence of any tipping point. It is $b + \alpha\beta\phi > 0$.

negative growth in the resource stock, as too much harvesting occurs; points below imply positive growth. The vertical isocline indicates there is one and only one level of per capita consumption of the harvest good that is consistent with zero population growth. This fertility isocline is vertical because a Malthusian system does not admit a demographic transition. Higher resource stocks imply higher per capita consumption and positive population growth; lower resource stocks imply negative population growth. Only one interior point is both sustainable and consistent with zero population growth. This point is the interior intersection of the fertility and sustainability isoclines. There are also two other steady states. There is a steady state at $\{S=0, L=0\}$, which is a saddlepoint; and there is a steady state at $\{S=K, L=0\}$, which is also a saddle point. Brander and Taylor (1998) show that, starting from any strictly interior point, the system always converges to the unique interior steady state and hence it is globally stable.

When we draw in the arrows of motion for the system, it is straightforward to see that there are two possible transition paths to the interior steady state from an initial position with a pristine environment and some small founding population. Along one path, the resource stock consistently falls and the population level consistently rises. This transition path is not very dramatic and does not fit the prehistory of Easter Island. The other possible transition path exhibits cyclical adjustment to the steady state. Several such trajectories are shown in figure 2. The population level grows very quickly and overshoots its long-run sustainable level by a large margin; as a result, the resource stock is then driven down well below its eventual steady state. This in turn drives population growth negative, and so on. This type of trajectory fits the overshoot and collapse story of Easter Island fairly well. The key difference between these two outcomes is that the overshoot and collapse scenario arises in situations where humans grow quickly relative to the environment. When this occurs, the human population overshoots its long-run value and this calls for a subsequent collapse in the resource and population. When the growth of humans is slower, the environment and human population adjust in tandem towards the steady state.

Consider what happens to this analysis when we introduce a tipping point and some degree of control over resource use as in the Crisis Model. This new setup is shown in the three panels of figure 3. To maximize the comparability with figure 2, I have used the same parameter values; hence, the carrying capacity now appears as 12,000 in the figure rather than $K=1$ as in figure 1. I have set the tipping point at 20% of the carrying capacity and vary governance from none at all, $G=1$, through $G=.9$, to $G=.8$ (harvesting is 80% of the open-access level).

With governance in place, $H^G = G\alpha\beta LS$ and less harvesting is conducted because $G \leq 1$. From (8) it is apparent that zero population growth requires $H^G/L = G\alpha\beta S = -b/\phi$. Since $-b/\phi > 0$ is a constant, a reduction in G implies an increase in the steady state resource stock S. The fertility isocline shifts to the right with improved management, but improved governance does nothing to raise

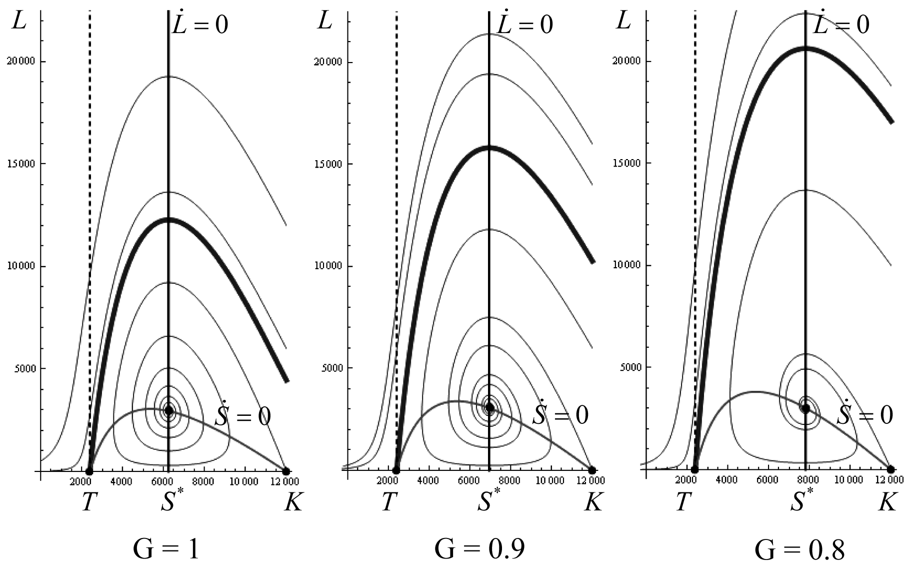

FIGURE 3

per capita consumption of the harvest good in the long run. This demonstrates that the key externality here is population growth.

The impact of the tipping point is reflected in the shape of the sustainability isocline. In figure 2 this isocline was linear and negatively sloped throughout, but the new sustainability isocline is positively sloped from its starting point at $S = T$ to its maximum at $S = \sqrt{TK}$ and negatively sloped thereafter. The negatively sloped segment corresponds to set of resource stocks where growth is compensatory: $N(S)/S$ rises when the stock is diminished. The positively sloped portion is the new possibility introduced by the tipping point. This is, of course, the region where percentage growth falls with stock reduction.

The Crisis Model version of Easter Island also exhibits three possible steady states. There is, as before, a unique interior steady state shown by the intersection of the two isoclines. However, this intersection must occur at a stock greater than T as shown, and this requires a further parameter restriction.[15] There are also two other steady states that lie on the horizontal axis at $\{S = T, L = 0\}$ and $\{S = K, L = 0\}$. It is easily verified that when the interior steady state satisfies $S^* > \sqrt{TK}$ (as shown in the panels) it is locally stable. The other two steady states are saddlepoints.[16]

There are three types of trajectories shown in the panels. First, there is a unique trajectory ending at the saddlepoint $\{S = T, L = 0\}$. We use the arrows of motion as our guide; this trajectory must approach the steady state from above and to

15 The restriction ensures an interior steady state exists to the right of the tipping point. It is $b + \alpha\beta\phi\, T < 0$.

16 See the proof of proposition 1 in the appendix for details concerning the dynamics of the system.

the right. Working backwards shows the trajectory must first be positively sloped and then turn downwards after it crosses the fertility isocline. This trajectory is drawn in bold, since it divides the state space into two basins of attraction. Such a trajectory is called a separatrix.

Second are the set of trajectories passing through points above this separatrix. These trajectories must stay above the separatrix.[17] Starting from a pristine environment and a positive population size, these trajectories take the economy above the sustainability isocline, turn back towards the horizontal axis after they cross the fertility isocline, and then escape to crash – at non-zero speed – into the vertical axis where $S = 0$. These trajectories exhibit a very dramatic overshoot and collapse in the population and the total destruction of the stock. The collapse is also compressed in calendar time, since the approach to the vertical axis is not exponentially decaying in time.

Third are the trajectories that lie below the separatrix. These trajectories approach the interior steady state either monotonically or in dampened oscillations just as in the original Brander and Taylor analysis; again, a slow growth rate works towards dramatic oscillation, as shown in the three panels.

From this graphical analysis alone we can draw several conclusions. Improved governance lowers harvesting and produces higher and healthier resource stocks and this moves the interior steady state away from the tipping point at T. This is to be expected. Less obvious is that the separatrix moves up with improved governance and by doing so shrinks the basin of attraction for the catastrophic outcome. The logic for this result is simply that with tighter harvesting controls in place, per capita consumption is less, and this constrains population growth. The set of initial conditions leading to irreversible overshooting is then smaller.

This result also implies that if an economy is on a path above its current separatrix and therefore is heading to the catastrophic outcome, a sufficiently strong mid-course correction can place it below the system's new separatrix. Mid-course corrections are possible only if the resource has not fallen below the tipping point T, and they may need to be draconian because any such correction must occur when the population is well above its sustainable level.

Finally, the three panels beg a question – in all panels the interior steady state is shown at $S^* > \sqrt{TK}$. But what if the tipping point was larger, so the interior steady state occurred in the region where nature was pushed too far? It turns out that the dynamics in this case are very different.

PROPOSITION 1. *Assume $S^* < \sqrt{TK}$; then the interior steady state is unstable while the steady states at $\{S = T, L = 0\}$ and $\{S = K, L = 0\}$ remain saddle points. Only one trajectory enters the saddle point $\{S = T, L = 0\}$ from a point interior $\{S > 0, L > 0\}$. All other trajectories escape to crash into the vertical axis with $S = 0$ in finite time. Any initial position with $\{S = K, L > 0\}$ produces a single boom to end in a crash.*

17 The system is autonomous and continuously differentiable at all interior points; therefore, no trajectories can ever cross.

Proof. See the appendix.

Proposition 1 is surprising. All trajectories – save one – now lead to the catastrophic outcome. The key to this result is that the interior steady state is now unstable, because it occurs at a point where natural growth no longer compensates for stock reduction with more rapid growth.[18] By adding a tipping point I have changed the system from one with a unique and globally stable interior steady state (Brander and Taylor 1998) into one without any stable interior steady state at all. It demonstrates that the point of no return is given not at the tipping point T, but instead at a point much earlier, where natural growth starts to weaken. This is somewhat unsettling. We may know, for example, that if we deplete a harvestable species to $1/4$ of its maximum population, then recovery is impossible ($T = 1/4$); what is surprising is that if we push the stock to less than or equal to $1/2$ of its carrying capacity ($S = \sqrt{TK}$, $T = (1/4)K$), then virtually all trajectories crash. Since the point where the system's dynamics change dramatically is far from the physical tipping point, it is much easier to see how past societies may have met their end even though they were relatively sophisticated. The ground around them gave way even though they were a long way from the edge.

What do these results tell us about Easter Island? Is this explanation superior to that of Brander and Taylor (1998)? Proposition 1 does address some of the unanswered questions. The answer to who cut down the last tree on Easter Island is simply that no one needed to cut it down – nature itself could have dealt the final blow if the Islanders drove the stock below T but didn't realize the implications of their actions. The single boom and bust is now a robust feature of the dynamics, and the crash is truly dramatic. However, these results require a restriction on parameters to generate $S^* < \sqrt{T}$. If we stick to the original parameters employed by Brander and Taylor (1998), then it is possible to show that the tipping point T must be about 30% of the island's carrying capacity for proposition 1 to hold. If T is smaller than this, then we are back to the original analysis, although it is altered by the appearance of the separatrix. Thus, it seems that tipping points alone may not be enough to generate environmental crises. Certainly, large tipping points will be enough, but smaller, less obvious ones may not. Therefore, it may be time to consider a complementary mechanism that can amplify shocks to the economy, that is, positive feedback effects.

4. Examining the present

In this section I examine the potential for positive feedback effects to create crises. To sharpen my focus I eliminate any role for variation in either tipping points or governance by setting $T = 0$ and $G = 1$. I return to our CES preferences with $\sigma \neq 1$ and the $K = 1$ normalization. To ensure a focus on novel elements introduced

18 As the steady state moves from the right to left, it passes from local stability to instability through a steady state at exactly $\sqrt{TK}$, which exhibits limit cycles.

by feedbacks, I again restrict parameters so that a strictly interior steady state exists in the absence of positive feedbacks.[19] With CES preferences, demand for good H, in aggregate, can be written as

$$H = \left[\frac{I}{p_h^{1-\sigma} + p_m^{1-\sigma}} \right] \left[\frac{1}{p_h^{\sigma}} \right].$$

When $G = 1$, income equals wage income wL, and prices are again tied directly to productivities. Substituting for prices with their marginal costs and choosing good m as the numeraire, I obtain a new harvesting function under these conditions:

$$H = \frac{\alpha L S^{\epsilon_h}}{q(S)} \quad \text{where} \quad q(S) \equiv 1 + uS^F \geqq 1, \; u = \alpha^{1-\sigma}. \tag{11}$$

The harvesting function in the Easter Island case was linear in the stock. Once feedback effects are in play, the set of possibilities becomes much wider. To see how wide, totally differentiate (11) and rearrange to obtain

$$\frac{dH}{dS} = \frac{H}{S}\left[\epsilon_h - b_m(S)F\right], \quad \text{where}\, b_m(S) = [q(S) - 1]/q(S), \tag{12}$$

where b_m is the budget share spent on good M and $F = [\sigma - 1][\epsilon_m - \epsilon_h]$ is the measure of feedback effects. In order to facilitate comparison with the GS model I set $\epsilon_h = 1$ and assume $F > 0$ in order to focus on positive feedbacks.

With feedback effects present, a reduction in the resource stock creates two effects. First, the productivity of labour in harvesting falls and this lowers harvesting. This is captured by the term ϵ_h in (12) and reflects the negative feedback built into the GS model. Second, there is a reallocation of labour across sectors and, when $F > 0$, this raises harvesting. For example, suppose relative prices were fixed and S falls. Then, if the productivity of labour falls relatively more in the M sector, $[\epsilon_m - \epsilon_h] > 0$ and labour moves into the H sector in response. This is a positive feedback because it tends to raise harvesting in the face of a declining stock.

If prices are allowed to change, more possibilities emerge. When S falls and $[\epsilon_m - \epsilon_h] > 0$, the relative price of good M must rise and, under elastic demand $\sigma > 1$, this shifts expenditure towards the consumption of the harvest good. This again creates a positive feedback because both $[\sigma - 1]$ and $[\epsilon_m - \epsilon_h]$ are positive. In fact, the fixed price situation is just a special case of $[\sigma - 1] > 0$ where σ is infinite.

19 In the previous section we had $H = \alpha\beta LS$ with $(G = 1)$, and coupling this with our growth function when $T = 0$ necessarily produces a single interior stable steady state for any β, if $r > \alpha L$. I will impose this condition throughout.

Alternatively, when S falls and $[\epsilon_m - \epsilon_h] < 0$, relative prices move in the opposite direction. But if demand is now inelastic, $\sigma < 1$, then labour is again shifted towards the H sector and we have another positive feedback.

There are several examples of positive feedbacks that fit this formalization. The first arises when one industry's pollution negatively affects the productivity of another. This cross-industry negative externality was first discussed by Baumol and Bradford (1972) and has led to a large literature on the impact of non-convexities. Assume H is a harming industrial activity that produces pollution one for one with output and M is an agricultural good whose productivity is negatively affected by pollution. Then, with no controls on pollution, $G = 1$, and with fixed prices, we generate a positive feedback because $\epsilon_h = 0$ (the environment is irrelevant to productivity in the industrial sector) and $\epsilon_m > 0$.

A second example arises from the combination of a within-industry negative externality and inelastic demand. Suppose $\epsilon_m = 0$, but $\epsilon_h > 0$. H is the harvest of an agricultural crop that requires water as an input in fixed proportions. Let S be an inverse measure of an aquifer's salinity, which is made higher by excessive withdrawals for irrigation. Then, if the demand for this agricultural crop is inelastic, a positive feedback exists. A shock that raises the salinity of the aquifer lowers the productivity of irrigation and H production. This drives up the relative price of the crop and works to raise the demand for irrigation.[20]

Further afield, positive feedbacks exist in many situations where local increasing returns can generate large multipliers. There are two well-known extinctions or near extinctions in the 19th century, where hunting activity may have been hastened by positive feedbacks. The first occurred during the 1870s when vast herds of the North American bison were slaughtered across the Great Plains of the U.S. Estimates of their population circa 1870 were 10–15 million, but by the mid-1880s only 100 were thought to be left wild in the Great Plains states. In recent work I have argued that the slaughter on the plains was initiated by the introduction of a tanning innovation allowing the bison hides to be turned into thick industrial leather.[21] The innovation was created in the U.K. and Germany and international markets played a key role in the kill-off. Although my explanation does not require positive feedbacks, but instead focuses on the role of the tanning innovation, this shock may have been magnified by positive feedbacks.

One potential feedback was the creation of a specialized transport industry moving the hides from hunters in the field to towns and railroad stations for transport. When the market was relatively small, each buffalo hunter left the killing fields in order to transport hides for sale in nearby towns. But once the market was large, enterprising freighters bought hides directly from the hunters

20 Positive feedbacks appear to have played a large role in the destruction of the Aral Sea in Central Asia. Both excessive irrigation and incompatible industries feature prominently in this disaster. See, for example, Ellis (1990), Wines (2002), and Micklin (2007).

21 See Taylor (2007) for details. Hornaday (1889) is still a fantastic read and reference. Gilbert, Remiger, and Cunningham (2003) contains interesting personal information concerning hunters and freighters.

in the field and transported them in large wagon trains pulled by up to 50 oxen. Since this new industry could be exploited only at higher hunter levels, and it helped propagate the shock that brought about the bison's demise, it worked much like the positive feedbacks captured in the Crisis Model.

A similar mechanism was at work in the case of the passenger pigeon.[22] The passenger pigeon was at one time the most bountiful bird in North America. If estimates are to be believed, several billion of these birds nested in the large forests of the U.S. and Canada. Starting in the mid-1800s, commercial hunting of the bird increased and hit a peak in the late 1880s when hundreds of millions of pigeons were transported by the railroad and sold in east coast markets. Hunting passenger pigeons was quite simple. Once a nesting site was found, a variety of methods were employed to trap, snare, and shoot the birds. The critical input was information about nesting sites, since they contained literally millions of birds. While individual trappers conducted their own search and had little incentive to share information about nesting sites, shipping agents did so. When pigeons started to be shipped from a new location, the railroad informed its own trappers as to the location.[23]

The shipping companies internalized what was an external economy arising from search and, by doing so, lowered unit hunting costs. The shock in the passenger pigeon case was the advent of railroads, which made shipping birds to eastern markets possible; the positive feedback magnifying the shock was the internalization of a search externality. By the late 1890s commercial hunting of the passenger pigeon was all but finished; the last known wild pigeon was shot in 1907, and the sole remaining bird in captivity died in 1914.

Given the examples I provided above, it is not surprising that positive feedbacks often produce multiple (and sometimes unstable) equilibria. To make some headway I start by proving an existence result and then move to examine whether a shock to the economy could precipitate a crisis.

PROPOSITION 2. *Assume $T = 0$ and $F > 0$; then a stable interior steady state exists.*

Proof. See the appendix.

Proposition 2 tells us that positive feedbacks cannot eliminate all equilibria from a previously well-behaved system. Recall that my method is to impose parameter restrictions on the model consistent with the existence of equilibria in

22 Discussion of the extinction can be found in Schorger (1955) and Farrow (1995).

23 'In this very large country there would seem to be every chance of losing a body of birds and not finding out where they are. But a very good system has been established for keeping track of them, which is specially looked after by the different express companies and the shippers and handlers of live and dead birds, who form another section of those interested in the history of the wild pigeon, before the epicure meets him at the table. When the body of birds leave the South the local superintendents of the express companies are instructed to keep their eyes out for indications of a nesting, an the messengers generally are to report on their route. A correspondence of an inquisitive nature is carried on by every regular netter in order that he or his chums may strike the birds first' Schorger (1955, 146).

the absence of a certain precondition. In the Easter Island case, I imposed a restriction ensuring that the steady state was interior when tipping points were absent. Here, I imposed a restriction ensuring the existence of a steady state when feedbacks are zero. Proposition 2 then tells us that while feedback effects can introduce multiple equilibria (the proposition does not claim uniqueness), there will always remain at least one interior stable equilibria.[24] Importantly, the existence of this steady state does not rely on the strength or weakness of the feedback effect. For any $F > 0$, a stable interior steady state exists. This a very useful result.

With this result in hand, I now examine the role of positive feedbacks in propagating shocks. To do so I need a precipitating event. There are two obvious candidates: one is simply an increase in the size of the market as represented by an increase in the population size L (which may fit the passenger pigeon case); the other is a shift in demand towards the harvest good (which may fit the bison case). Given the Malthusian growth process I introduced earlier, it seems natural to choose a change in L.

Solving for the steady state impact of a change in the economy's size is straightforward. Setting net natural growth equal to harvesting, differentiating, and forming elasticities shows that

$$\frac{dS}{dL}\frac{L}{S} = -\frac{1}{1 - \epsilon_R - b_m F} < 0, \tag{13}$$

where $\epsilon_R = R'(S)S/R$ and the sign of the denominator is positive at any stable steady state.[25]

The sign of the result is not surprising, but let me rewrite it to focus on the role of feedback effects in determining its magnitude. In particular, I follow the lead of Roe and Baker (2007a,b) by dividing the full impact of a change in a forcing variable (here labour, there CO_2 concentrations) into a response absent feedbacks and a multiplier due to feedbacks. Let λ_0 be the change in the steady state resource stock arising from a 1% change in the endowment in the absence of feedback effects; then, letting $0 \leq f < 1$ measure the strength of feedback effects, I can rewrite (13) to obtain

$$\frac{dS}{S} = \frac{\lambda_0}{1 - f}\frac{dL}{L} < 0, \tag{14}$$

where $\lambda_0 = -1/(1 - \epsilon_R)$, $f = b_m F/(1 - \epsilon_R)$, and the ratio $1/[1 - f]$ is typically referred to as the gain from feedback.

This formalization leads to the useful graphical apparatus shown in figure 4. On the vertical axis I measure the (absolute value) change in the resource stock arising from a 1% increase in the labour force. The horizontal axis measures the

24 Uniqueness is guaranteed under further conditions; it is easy, for example, to prove uniqueness if we limit the strength of feedback effects.

25 The proof to proposition 3 in the appendix contains details regarding this comparative static.

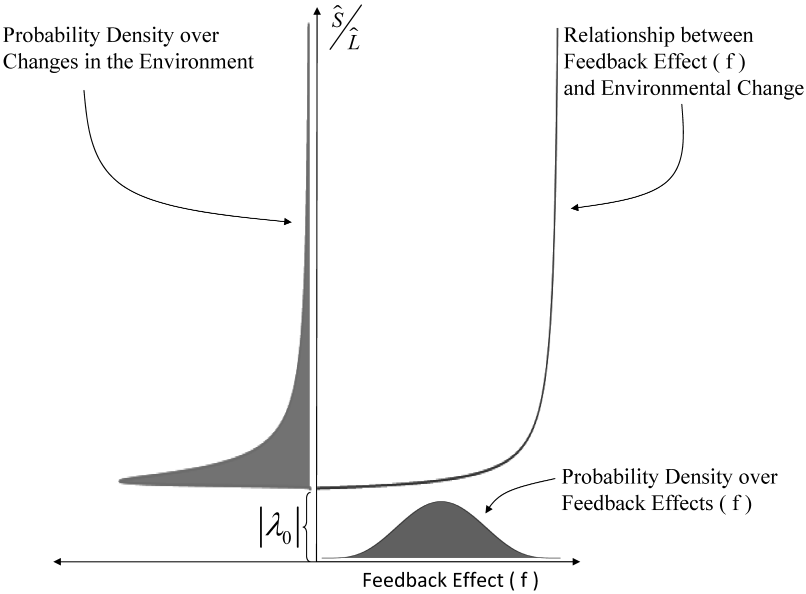

FIGURE 4

magnitude of the feedback effect, f. Since the magnitude of feedback effects is in many cases very uncertain, I follow Roe and Baker (2007a,b) by assuming that the parameter f has a probability density function as shown on the horizontal axis. Under the assumption that f is the only uncertain parameter, we can translate the probability density on the strength of feedbacks into the probability density on the resulting change in the environment.[26]

Suppose there was a zero feedback, then the resulting change in the environment arising from a 1% change in the endowment is relatively small and given by the vertical distance to the curve at $f = 0$; that is, it is $|\lambda_0|$. As positive feedbacks grow (f rises) the resulting change in the environment grows until we reach a limiting case imposed by stability $f < 1$. The graph is useful in showing how feedback effects magnify the impact of almost any change or stress put on the environment and how uncertainty over the strength of positive feedbacks has a tendency to magnify uncertainty over outcomes. In Roe and Baker's application, uncertainty is over the magnitude of f, and $\lambda_0/(1 - f)$ is the climate sensitivity parameter linking the (long run) predicted temperature change that would arise from a hypothetical doubling of CO_2 concentrations over pre-industrial levels. As Roe and Baker argue, and as the figure itself shows, even if the uncertainty over the feedback parameter is represented by a symmetric and fairly standard probability density, this uncertainty is translated – via the mechanism of a

26 See for example, Davidson and MacKinnon (1993, p. 489), for details regarding such a transformation.

positive feedback – into a rather long- and fat-tailed distribution over environmental outcomes.

The intuition is simple. Although the likelihood of a very high feedback f is very low and falling, the impact of such an f on the outcome variable is very high and rising. These two forces can offset each other to generate a very fat-tailed distribution over outcomes that are truly catastrophic.

These fat-tailed distributions are similar to those generated in Weitzman (2009a) although his method is quite different. Weitzman's work, and specifically his result showing the cost of climate inaction could be infinite, stimulated a critique by Nordhaus and subsequent replies. While their disagreements are many, the debate centres on the conditions required to obtain a fat-tailed distribution and the proper method of valuing the extreme outcomes they present (see Weitzman 2009a,b,c; Nordhaus 2009). My analysis here says nothing about valuation and nothing specific about their debate. However, the Crisis Model does imply something specific about the Roe and Baker mechanism for generating fat-tailed distributions. The analysis above varied f while keeping all other 'parameters' constant. Since f is related to the Crisis Model primitive F in a simple manner, I can vary f freely by varying my parameter F. The problem is that the remaining terms in the comparative static are not parameters but instead functions. This is also true in the Roe and Baker analysis, but they defend their constant parameter assumption by noting that their analysis comes from a first-order Taylor series approximation around a pre-existing climate equilibrium.[27] While this may be a reasonable method in terms of the climate system, in terms of the model developed here it is not. It should come as no surprise – given the hunting examples I mentioned earlier – that the impact of positive feedbacks should wane as the resource stock is diminished. Any change in the model's primitive measure of feedbacks, F, will surely alter the equilibrium around which any Taylor series approximation is taken, and this may, in fact, reduce the gain from feedbacks. This concern is, in fact, correct: positive feedback effects are, in an important sense, self-limiting.

PROPOSITION 3. *Positive feedbacks are self limiting: in particular,* $|(dS/dL)(L/S)|$ *is increasing in* F *for* F *small and decreasing in* F *for* F *large.*

Proof. See the appendix.

An increase in the strength of positive feedbacks F always lowers the resource stock because, all else equal, feedbacks raise harvesting. But as the resource stock shrinks, two compensating changes occur. First, nature compensates with faster percentage growth as the stock is reduced (recall $T = 0$ here). This dampens the response, as expected. Second, as the resource stock falls, the size of the M

27 In an set of appendix notes available at the Science website, Roe and Baker go further to consider second order approximations. They argue that any potentially offsetting effects are not empirically relevant.

sector falls, and this weakens the impact of feedback effects. Proposition 2 and 3 together imply that positive feedbacks can play only a supporting role in crisis creation. Proposition 2 tells us that the existence of an interior steady state is independent of feedback effects. Proposition 3 tells us that changes around this steady state must be bounded. In terms of figure 4, proposition 3 tells us that once we allow for endogenous changes in f, the long tail of the distribution over outcomes is truncated. Feedback effects will still tend to shift the mass of the distribution rightward, but the implication is clear: feedback effects, like tipping points, are not likely to be the sole author of environmental crises.

5. Speculating about the future

I now demonstrate how tipping points, poor governance, and positive feedbacks generate an environmental crisis. I do so by developing an analytical result showing that positive feedbacks and tipping points are complements in crisis creation, and then illustrate the result with a simulation of the model. Although in principle many changes to the economic system could act as the shock triggering a crisis, I limit myself to variations in the economy's endowment. To ensure that it is the combination of preconditions delivering the crisis, I restrict variations in the endowment to those that would keep the steady state S above the critical point $\sqrt{T}$ if positive feedbacks were absent. This is the restriction needed for a stable interior steady state in the Easter Island case. As well, I restrict variations in the endowment to those that would not generate extinction if positive feedbacks were present, but a tipping point absent. The purpose of these restrictions is obvious: by invoking them I make sure that any resulting crisis must be due to the simultaneous existence of both a tipping point and positive feedbacks. The analytical result I develop is described in proposition 4, below.

PROPOSITION 4. *If $T > 0$, $F > 0$, and $G = 1$, then there exists a critical stock level $S^C > \sqrt{T}$ and associated labour endowment L^C, which represent a steady state. For any L satisfying $L > L^C$, no stable interior steady state exists.*

Proof. See the appendix.

Proposition 4 tells us that tipping points and positive feedbacks are complements in creating crises.

Since the critical stock S^C is greater than $\sqrt{T}$, the point at which the economy falls into crisis is even further away from what might be viewed as the precipice of T. And while positive feedback effects are self-limiting, this property can be irrelevant if the economy exhibits a tipping point. A rather mundane change in the economy's endowment is magnified by positive feedbacks to push us into a basin of attraction from which we cannot recover. The basin of attraction is, in this case, a one-dimensional set of S, but the logic and intuition are similar to the workings of the Easter Island case.

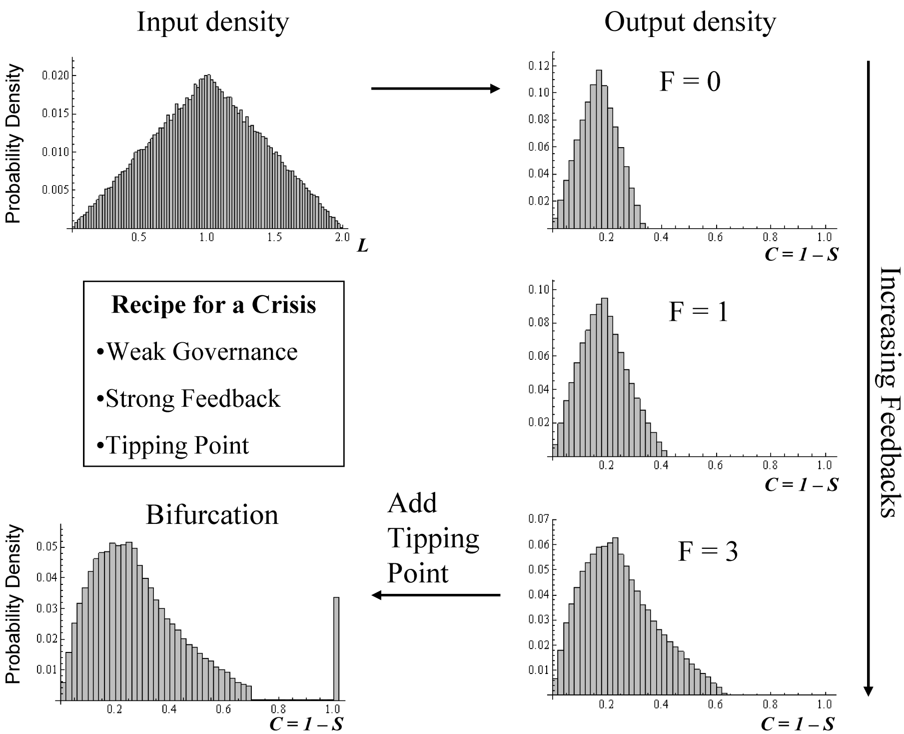

FIGURE 5

There are a variety of ways to illustrate this result. One method of illustration is contained in the set of panels in figure 5. The top left panel presents the sample probability density function for endowment shocks. It is constructed by drawing 100,000 endowment shocks from a simple triangular distribution with support from 0 to 2 and a mean of 1.[28] The triangular distribution was chosen for simplicity and ease of calculation. The remaining panels present the distribution of the model's steady state outputs in terms of the variable $C = 1 - S$ under a variety of conditions. The variable C is just a simple translation of S. I have presented results in terms of C, to reinforce the connection between these panels and those often presented in the debate over catastrophic climate change.

In the first panel on the right, the model has zero feedback effects and no tipping point. It is apparent that when the endowment is near its mean of 1, the climate is also near its mean of approximately 0.2. Both distributions are symmetric or nearly so.

28 The draws are on endowment levels that I then feed into the model to find the steady state output. I could however generate the same result by renormalizing and drawing zero mean additive shocks that in turn affect the endowment.

In the second panel on the right, the model has a small feedback effect, $F = 1$, and no tipping point. Positive feedbacks shift the distribution of outcomes rightward. The mean has increased, as has the variance.

In the bottom panel on the right, I increase the strength of feedbacks to $F = 3$, while retaining the no-tipping-point assumption. The distribution of outcomes again shifts to the right and its tail lengthens. Propositions 2 and 3 tell us, however, that regardless of how large F is driven, this distribution will never reach $C = 1$.

Finally, the panel in the bottom left shows the impact of introducing a small tipping point $T = .04$ while retaining the strong positive feedbacks of $F = 3$.

The key and surprising result is, of course, the separation of probability mass when a tipping point is introduced. It is important to note that introducing the tipping point to the model with either $F = 0$ or $F = 1$ does not produce this result. It is also important to note that when $T = .04$, then $\sqrt{T} = .2$. Therefore, the stock level where 'nature has been pushed too far' would equal $S = .2$, and this corresponds to $C = .8$ in the figure. As shown, the bifurcation occurs at a point well below $C = .8$, and this is, in effect, what proposition 4 claims – feedback effects magnify the impact of tipping points on the stability of the system.

The new distribution of outcomes contains a smooth almost continuous section that is skewed towards extreme C outcomes as in our earlier panel with strong feedbacks, but it also contains a mass point at $C = 1$ (i.e., $S = 0$). The mass point arises because the strength of positive feedbacks magnifies the endowment shock to such an extent that it pushes the economy past the critical stock level; as a result, the entire probability associated with endowment shocks above this point is then transferred to the extreme tail of the distribution by the action of the tipping point.

This characterization of a crisis is quite appealing. First, the probability of the crisis is not zero or approaching zero. It is a low probability event surrounded by a sea of zero probability. It is this separation, in fact, that makes it – and all other crises – unique. A crisis is after all not a low-probability event surrounded by other low-probability events – it is a unique event with no real points of comparison. There are several analogies to think about: the Great Depression was not just two or three times worse than a typical recession – it was a unique event; extinction is not in any real sense like a situation of excessive harvesting; and a nervous breakdown is not anything like even a really bad day at the office. Like pregnancy, you are either in a crisis or you are not – there is no middle ground – and this is exactly what the last panel of the figure is telling us.

We also know that crises are difficult to learn about and difficult to learn from. Crises are difficult to learn about because there are no observations near them. As economists, we connect variation in the economy's inputs/policies/shocks to variation in outcomes to learn about the economy. The vast majority of these observations would allow us to understand the bulk of the mapping from inputs to outputs shown in the panels, but knowledge of these connections may be of little use in understanding a crisis. For example, suppose we thought tipping points were just utter nonsense, but positive feedback effects were possible. Specifically, suppose our maintained hypothesis was the model with $F = 3$, but no tipping point. Then, if we observed the distribution of inputs in the top panel and the distribution of outputs in the bottom left panel, in the majority of cases we would find evidence in favour of our hypothesis. The vast majority of these observations look like those coming from a model with strong feedbacks and no tipping point. Therefore, we can understand 'normal time economics' quite well, but still be surprised by crises.

But even when we do experience a crisis, learning from it is difficult. A researcher who observes a crisis, or collects data on several of them is handicapped by the fact that much of the variation in the forcing variable – here endowment shocks – maps into the same outcome: a crisis. A researcher looking across time or space for examples of crises will find his vision obscured by the possibility that observed variation in the true forcing variable seems to have little to do with the onset of a crisis, since it varies widely across his/her sample.

Finally, we all know crises are difficult to reverse. Why is this? If the economy exhibits only positive feedbacks, then even a small improvement in governance will improve on outcomes. This is true because even marginal improvements in governance are effective in altering behaviour around a strictly interior steady state. But with a tipping point present, things can be very different. Once we are at the crisis point, the world has changed and so have the rules. Small changes in governance will have no effect whatsoever on outcomes, even though these same changes would have had large effects in the absence of crisis. This is true because the local dynamics at the crisis point are decidedly unfriendly to recovery. Returning to pre-crisis conditions requires Herculean efforts to move the economy not only back from the brink, but back far enough to escape the pull of the tipping point. It is for this reason that escaping from a crisis is so costly.

5.1. A future climate crisis?

Could climate change create a crisis in the future? A climate crisis could manifest itself in several ways: an abrupt change in the global mean temperature; a change in hydrological cycles creating extreme droughts over large areas; or rapid sea level rise. Many of these outcomes are correlated, but I have no wish to even speculate on the likelihood of a trifecta of catastrophes, so let me focus here on a large ($>3^\circ$C) temperature rise. First and most simply, are the preconditions for a crisis met?

Consider the governance problem. At present, 84 nations are signatories to the Kyoto Accord and 75 have ratified it; of these only 21 are on track to meet their emission targets. It is well known that the Kyoto framework is dead, and success at the COP15 meetings in December 2009 in Copenhagen is critical.[29] The governance problem is huge and will remain so in the foreseeable future, as large developing countries such as Brazil, China, and India have little incentive to

29 The information on emissions and ratification is taken from UNFCC (2008a,b) and (2009).

help solve a problem that they themselves did not create. Even if Copenhagen is a tremendous success, it is important to recognize that the climate system exhibits extreme inertia. If a ton of carbon is emitted today, 70% of it will remain in the atmosphere in 10 years time, 35% in 100 years, and almost 20% in 1,000 years.[30] Inertia plus difficulties in international negotiations make it quite likely that we will have very little control over the climate over the next 50 years.

Positive feedbacks are also present, but the most commonly mentioned ones differ from the economic/environment feedbacks I have developed here. I will mention just two possibilities that are especially relevant to Canadians. The melting of sea ice may or may not eliminate polar bears from Canada, but it does replace sea ice with open ocean, which tends to trap heat. By trapping heat rather than reflecting light, more sea ice melts and, presto, we have a positive feedback. A similar feedback comes from the drying of forests, which heightens the risk of forest fire and the release of sequestered carbon. Again, a positive feedback effect is created, but these two examples should, by my own definition, fall under the category of physical processes $P(S)$.

Are there any economic/environmental links that could generate the right kind of positive feedbacks? One possibility is that international trade could fuel a positive feedback by shifting energy-intensive production to countries with no emission controls. The shift in production would need to be large and the resulting increase in emissions in the non-participant country would need to trigger further efforts to curb emissions in the rest of the world. Given what we know about the extent of dirty-industry migration, this seems unlikely. Another possibility is an escalating game of chicken played between superpowers each of whom refuses to cut back for fear of its economic and hence military costs. While it is possible to think of other scenarios, in the end I don't find any of them very convincing, so it should be recognized that at least one precondition for crisis is currently weak.

Finally, tipping points: there are at least two truly frightening candidates for tipping points. One would be the uncontrollable melt of the Greenland ice sheet, which would raise sea levels worldwide by several metres. This in itself would constitute a crisis, but if the consequent freshening of the Atlantic led to major changes in ocean circulation, this too would represent a crisis (see Walker 2006; Schiermeier 2006; Barreiro et al. 2008). A second possibility is that as warming proceeds, higher ocean temperatures may lead to the release of methane trapped under the ocean floor. Methane is trapped in crystals called clathrates, and these are in super abundance. A release of even a fraction of this methane could produce run away changes in global temperatures.[31]

Many of these effects – especially the feedback effects – are highly uncertain. And the worst-case scenarios of ice sheet melt and degassification of the oceans are thought to be very low-probability events. Nevertheless, this is the nature

30 See Solomon et al. (2009) for an analysis of how future warming is affected by today's emissions. See Hoos et al. (2001) for time-in-the-atmosphere calculations.

31 See Overpeck and Cole (2006) for a discussion regarding abrupt climate change and methane-clathrate release.

of the exercise. By definition, crises are infrequent, low-probability events that appear unlikely ex ante. In the case of climate change this is true as well, but there are no laws of physics that rule out abrupt climate change in the next century, and many in the climate change community believe the probability of such an occurrence is far from zero.

The most compelling case for a serious consideration of catastrophic climate change comes not from a physical scientist, but from the economist Martin Weitzman at Harvard. In a series of papers, Weitzman makes a powerful argument for a reconsideration of climate change policies that suggest a slow ramp-up of emission cutbacks. His case rests on several pieces of empirical evidence plus theoretical work that produces what he refers to as the Dismal Theorem (see Weitzman 2009a,b,c). I find most compelling two pieces of empirical evidence that Weitzman cites. The first piece of evidence is the lengthy 800,000-year history of atmospheric concentrations of CO_2 drawn from ice cores. This record shows that carbon concentrations have varied within a relatively narrow band for the last 800,000 years. For example, CO_2 concentrations have varied from a low of approximately 170 to a high of 300 ppm. Currently, we are at 385 ppm. The second piece of evidence is that during the last 800,000 years variation in concentrations during any 1,000 years was always less than 25 ppm. The increase in concentrations during the last 10 years is almost 20 ppm; over the last 15 years 28 ppm.[32] From this and other evidence, Weitzman concludes that we are currently engaged in a risky experiment that injects massive quantities of carbon into the atmosphere on a time scale that is truly unprecedented. I find this argument difficult to ignore and its conclusion unsettling.

6. Conclusion

In this paper I have described how the interplay of positive feedback effects, poor governance and tipping points can generate an environmental crisis. To do so I started with the canonical model of renewable resource growth and then amended it to develop what I have called the Crisis Model.

The changes I introduced – positive feedbacks, a tipping point, and limited governance – all are quite natural. To demonstrate the strength of these three new forces, I examined them in isolation. To start, I showed how a tipping point creates a basin of attraction driving the economy towards catastrophic outcomes and linked the size of this basin of attraction to the strength of governance. If initial conditions place the economy within this basin of attraction, moving the economy away from the catastrophic outcome may take Herculean efforts.

32 See Lüthi et al. (2008). The data are available as supplementary material at the Nature website. Figure S2 in the supplementary material shows CO2 never rose more than 25 ppm in any thousand-year period. It should be noted, however, that the sampling interval for ice core data is far in excess of 10 years. The average sampling interval over the 800,000-year time period is over 700 years, with many intervals exceeding 1,000 years. Recent data are available at: http://cdiac.ornl.gov/ftp/trends/co2/maunaloa.co2.

Relatively large tipping points can produce catastrophic outcomes in virtually all cases, but whether this was the case on Easter Island is hard to judge. Back-of-the-envelope calculations suggest that only a sizeable tipping point could have generated the crisis, and this suggests that tipping points alone cannot be the author of all environmental crises.

In the second section I examined positive feedback effects. Positive feedbacks were shown to arise in a variety of circumstances and I linked the impact of feedbacks to technological and environmental determinants. Positive feedbacks can play a key role in crisis creation, since they tend to magnify shocks to the system. They played at least a supporting role in the 19th-century elimination of the passenger pigeon and the kill-off of the bison. More recently, uncertainty over the strength of feedback effects has been shown to generate a fat-tailed distribution over environmental outcomes. While feedbacks play a similar role here, I showed that the impact of feedback effects was limited by resource and other constraints in my simple general equilibrium model. As a result, positive feedbacks alone cannot create a crisis.

The final section combined these partial results. I demonstrated that tipping points and feedback effects are complements in crisis creation and then illustrated the result by drawing 100,000 endowment shocks to generate the model's probability density function over long-run environmental outcomes. The combination of tipping points and feedback effects creates a bifurcation in the density. The bifurcation occurs because natural variation in inputs is magnified by positive feedbacks and this pushes the economy past a critical threshold related to its tipping point. When the economy crosses this threshold, it is swept into crisis.

This depiction of crisis has several attractive features: a crisis is a low-probability event, it is unique, it is difficult to recover from, and it is difficult to predict. These are features of all crises, whether environmental, financial, or political.

With this theory of crisis in hand, I turn to speculate about the future and ask whether climate change is a potential crisis in the making. I find the three preconditions for crisis are met and argue that the run-up in CO_2 concentrations in the last 150 years can be viewed as a potential precipitating event. Do I think a future climate crisis is likely – no I do not – but then again, no crisis ever is.

Appendix

A.1. Feedback effects

From CES demand $[H/M] = [p_m/p_h]^\sigma$. Zero profit conditions give $p_m = wa_{lm}$ and $p_h = wa_{lh}$, where a_{lm} and a_{lh} are the unit labour requirements in H and M, respectively. Substitute and log differentiate to obtain $[\hat{H} - \hat{M}]^D = -\sigma[\hat{a}_{lh} - \hat{a}_{lm}]$, where $\hat{x} = dx/x$. From the supply side $H = L_h/a_{lh}$, $M = L_m/a_{lm}$ and $L = L_h + L_m$. Log differentiate to obtain $[\hat{H} - \hat{M}]^S = \hat{L}_h - \hat{L}_m + \hat{a}_{lm} - \hat{a}_{lh}$ and $\hat{L} = s_m\hat{L}_m + [1 - s_m]\hat{L}_h$, where $s_m \equiv L_m/L$. Equate the demand and supply variations, substitute for $\hat{L}_m$, and set $\hat{L}=0$ to find $\hat{L}_h = s_m[\sigma - 1][\hat{a}_{lm} - \hat{a}_{lh}]$. From the assumptions on production we know $\hat{a}_{lm} = -\epsilon_m \hat{S}$, and $\hat{a}_{lh} = -\epsilon_h \hat{S}$. It is also easy to show $s_m = b_m$, where b_m is the budget share of manufacturing. Define $F \equiv [\sigma - 1][\epsilon_m - \epsilon_h]$; then we obtain $\hat{L}_h/\hat{S} = -b_m F$ and $\hat{L}_m/\hat{S} = [1 - b_m]F$. These are the general equilibrium reallocations of labour that occur when S changes and prices adjust. Differentiating the production function for harvesting yields $\hat{H} = [\epsilon_h - b_m F]\hat{S}$. Therefore, the direct effect of a change in S is given by $\epsilon_h \hat{S}$; the feedback effect is $-b_m F\hat{S}$.

A.2. Easter Island with governance

Tastes are $u = H^{\beta} M^{1-\beta}$. From the demand side we have $[H/M]^D = [\beta/[1-\beta]][p_m/p_h]$. When $G = 1$, relative prices equal unit costs $p_m = wa_{lm}$ and $p_h = wa_{lh}$. Relative demand becomes $[H/M]^D = [\beta/[1-\beta]][a_{lm}/a_{lh}]$. Relative supplies are given by $H = L_h/a_{lh}$ and $M = L_m/a_{lm}$. Combine relative supply and demand to find $L_h = \beta L$ and $L_m = [1 - \beta]L$. Therefore, under open access, $H^O = \alpha\beta LS$ and $M^O = [1-\beta]L$ because $a_{lh} = 1/\alpha S$ and $a_{lm} = 1$ in the Easter Island case.

Under limited governance, regulation limits effort in harvesting to a fraction $G < 1$ of the open-access level. Hence, $H^G = G\alpha\beta LS$ and $M^G = [[1-\beta] + [1-G]\beta]L$. Inverting relative demand and substituting for the quantities of H and M produced yields

$$\frac{p_h}{p_m} = \frac{\beta}{1-\beta}\frac{M}{H} = \frac{1}{\alpha S}\frac{[1-\beta]+[1-G]\beta}{[1-\beta]G}.$$

Relative prices with harvest controls are proportional to relative prices under open access (at the same stock) with a factor of proportionality linked to the stringency of governance, G. That is, $[p_h/p_m] = [p_h/p_m]^O\Psi(G)$, where $\Psi(G) \equiv [[1-\beta] + [1-G]\beta]/[1-\beta]G$, where $\Psi(G) \geq 1$, $\Psi'(G) < 0$ and Ψ reaches its minimum at the open-access solution $G = 1$.

A.3. Proof of proposition 1

A.3.1. Dynamics

The dynamic system is given by $\dot{L} = L[b + \phi\alpha\beta GS]$, and $\dot{S} = r[S - T][1 - S] - \alpha\beta GLS$, where $K = 1$; the energetic reader can carry K and make the appropriate adjustments. The interior steady state is given by $S^* = -b/[\phi\alpha\beta G]$, $L^* = -[\phi/b]r[S^* - T][1 - S^*]$. Existence requires $T < S^* < 1$ or $\phi\alpha\beta G + b > 0$ and $b + \phi\alpha\beta GT < 0$, which I assume throughout. Note that, if $S = T$ and $L = 0$, we have another steady state, and $S = 1$ and $L = 0$ are also a steady state. The sustainability isocline starts at $\{S = T,\ L = 0\}$, rises to $\{S = \sqrt{T},\ L = [r/\alpha\beta G][1 - \sqrt{T}]^2\}$ and then falls to $\{S = 1,\ L = 0\}$. We can define the Jacobian from the dynamic system, as $J_{11} = b + \phi\alpha\beta GS$, $J_{12} = \phi\alpha\beta GL$,

$J_{21} = -\alpha\beta GS$, and $J_{22} = r[1 + T - 2S] - \alpha\beta GL$. Evaluating the characteristic equation at $\{S = 1, L = 0\}$ shows $\lambda_1 = b + \phi\alpha\beta\ g > 0$ and $\lambda_2 = r[T - 1] < 0$. It is a saddle point approached along the horizontal axis if $S > T$. At $\{S = T, L = 0\}$, the characteristic equation shows $\lambda_1 = b + \phi\alpha\beta gT < 0$ and $\lambda_2 = r[1 - T] > 0$. It is a saddlepoint. The approach must be from above, where $S > T$ and $L > 0$. The characteristic equation at $\{S^*, L^*\}$ is given by $\lambda^2 - \lambda J_{22} - J_{21}J_{12} = 0$. $J_{12} > 0$ and $J_{21} < 0$. $J_{22} = r[1 + T - 2S^*] - \alpha\beta L^* G$. J_{22} simplifies to $J_{22} = [r/S^*][T - S^{*2}]$. Therefore, $J_{22} > 0$ if $S^* < \sqrt{T}$; $J_{22} = 0$ if $S^* = \sqrt{T}$; and $J_{22} < 0$ if $S^* > \sqrt{T}$. Both roots are negative if $J_{22} < 0$ or $S^* > \sqrt{T}$ and the steady state is stable. Both roots are positive if $J_{22} > 0$ or $S^* < \sqrt{T}$ and the steady state is unstable. When $S = \sqrt{T}$, $J_{22} = 0$, and $\lambda^2 = J_{21}J_{12} < 0$, the steady state exhibits a limit cycle.

A.3.2. Boom and bust trajectories
Consider the trajectory from any small $L > 0$, $S = 1$. The trajectory must be positively sloped while $S > S^*$ and it remains above the sustainability isocline. It remains above the S-isocline because it can cross it only vertically. The trajectory therefore rises and peaks as it crosses the fertility isocline. It then turns downward. The trajectory cannot cross the S-isocline from above, as this would cut across the saddlepath entering $\{S = T, L = 0\}$ from this quadrant. Therefore, the trajectory must cross $S = T$ and escape to crash into the horizontal axis. Since $\dot{S} < 0$ at $S = 0$, it must do so in finite time.

A.3.3. Limit cycles
The proof used in Brander and Taylor (1998) does not hold under the assumptions of the Crisis Model. Instead, employ Dulac's extension to Bendixson's Negative Criterion. Proof is left to the reader.

A.4. Proof of proposition 2

A.4.1. Existence
Recall $H = \alpha LS/q(S)$, $R(S) = rS[1 - S]$, $q(S) = 1 + [S^{\epsilon_m}/\alpha S^{\epsilon_h}]^{\sigma-1} = 1 + uS^F$, where $u = \alpha^{1-\sigma}$. Setting natural growth equal to the harvest and cancelling S from both sides yield $\alpha L/q(S) = r[1 - S]$. The left-hand side of this expression equals αL at $S = 0$; equals $\alpha L/[1 + u]$ at $S = 1$; and is declining in S. The right-hand side equals r at $S = 0$; equals zero at $S = 1$; and is declining in S. Since $r > \alpha L$ by assumption and $\alpha L/[1 + u] > 0$, the two sides must equal at some $S \in (0, 1)$.

A.4.2. Stability
Let S^* denote an interior steady state. Then stability requires $\dot{S} > 0$ for $S < S^*$ and $\dot{S} < 0$ for $S > S^*$.$\dot{S} = R(S) - H(S)$ and hence stability requires $R'(S^*) - H'(S^*) < 0$. The slope of the harvest function near a zero stock simplifies as follows: $(dH/dS)|_{S=0} = (H/S)[1 - b_m F] = (H/S)[1] = \alpha L_h = \alpha L > 0$, where the second equality follows because the M sector vanishes when S is near zero and hence

$b_m = 0$; the third equality follows because $b_h = 1$ implies all labour is in harvesting; that is, $L_h = L$. It is possible to show $R'(0) = r$, and given the restriction that $r > \alpha L$, the harvest function must cut $R(S)$ from below; that is, $R'(S^*) - H'(S^*) < 0$ as required.

A.5. Proof of proposition 3

In a steady state $H(S, L) = R(S)$, where $H(S, L) = aLS/q(S)$. Totally differentiate and rearrange to obtain (in obvious notation) $dS/dL = -H_L/[H_S - R_S]$. Note $H_S S/H = [1 - b_m F]$ and $H_L L = H$. Define $SR'(S)/R = \epsilon_R$. Forming elasticities and substituting yields $\hat{S}/\hat{L} = -1/[1 - b_m F - \epsilon_R] < 0$, where $1 - b_m F - \epsilon_R > 0$ by stability. Let $\psi \equiv 1 - \epsilon_R(S) - I(S, F)$, where $I = b_m(S, F)F$. To prove the claim differentiate ψ to obtain $d\psi/dF = -\epsilon'_R(S)\,[dS/dF] - dI/dF$. To solve for dS/dF, write the steady state definition as: $H(S, F) = R(S)$, where $H(S, F) = aLS/q(S)$, $q(S) = 1 + uS^F$, $u > 0$. Totally differentiate and rearrange to find $dS/dF = -H_F/[H_S - R_S]$. Form elasticities to find $\hat{S}/\hat{F} = [b_m F \ln S]/[1 - b_m F - \epsilon_R] < 0$, where $1 - b_m F - \epsilon_R > 0$ by stability and $\ln S < 0$. Therefore, $dS/dF < 0$. It can be shown that $[dI/dF]|_{F=0} = b_m > 0$ and $\epsilon'_R(S) < 0$. Therefore, $d\psi/dF < 0$ for F near zero. To show $d\psi/dF > 0$ for F large, note: $dI/dF = F(\partial b_m/\partial S)[dS/dF] + (\partial b_m/\partial F)F + b_m$. Substituting and rearranging, I obtain: $d\psi/dF = [-\epsilon'_R(S) - F(\partial b_m/\partial S)]\,[dS/dF] + b_m\,[-(\partial b_m/\partial F)(F/b_m) - 1]$, where $\partial b_m/\partial S = F b_h b_m/S > 0$, $[\partial b_m/\partial F][F/b_m] = F b_h \ln S < 0$, and $\epsilon'_R(S) = -1/[1 - S]^2$. Recall S^* is bounded below by the level that would arise if $L_h = L$. Since $b_h > 0$, $\ln S < 0$, and F is arbitrarily large, the second term in brackets is necessarily positive. Note $\epsilon'_R(S)$ is small and finite for S small, while $b_h b_m F/S$ becomes large as F increases. Therefore, the first term in brackets is negative for F large. Since $dS/dF < 0$, this implies $d\psi/dF > 0$ for F large.

A.6. Proof of proposition 4

The critical stock and associated endowment must satisfy $N(S^c) = H(S^c, L^c)$ and $N'(S^c) = H'(S^c, L^c)$. Since H is strictly increasing in L, for any $L > L^c$ no steady state exists. The two conditions can be written as $\alpha LSb_h = r[S - T][1 - S]$ and $r[[1 + T] - 2S] = [1 - b_m F]\alpha L b_h$. Eliminate $\alpha L b_h$ to obtain one equation in S. After rearranging, this equation becomes $[S^2 - T] = b_m F[[S - T][1 - S]]$, which solves for S^c. If $F = 0$, we find $S^c = \sqrt{T}$. Since $b_m F > 0$ for any $F > 0$, $S^c > \sqrt{T}$ as required. L^c is given by: $L^c = (r/\alpha b_h)[1 - T/S^c][1 - S^c]$.

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