Food, Fuel, and the Spatial Economy

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Very Preliminary.
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Introduction

What role do Energy constraints play in determining the size, geographic location, and number of human settlements?

Theory focused on Energy constraints.

Empirics focused on Geographic features.

Application to Domesday England 1086.
When you can measure what you are speaking about, and express it in numbers, you know something about it; when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind; it may be the beginning of knowledge, but you have scarcely, in your thoughts, advanced to the stage of Science. (Lord Kelvin, 1883)

In contrast to early economics, much of modern economic analysis today largely ignores geography. In the spirit of Lord Kelvin, our new measures of spatial economic activity may give impetus to the reemerging geographic economics. (Nordhaus and Chen, 2009)
Why not Counties, Counties or Provinces?

Political Units often not the optimal unit of analysis:

Why? Geographic variables averaged over large and heterogeneous areas mean very little; weighting introduces endogeneity; aggregation can hide relationships; political boundaries are endogenous to geographic attributes.

Degrees of Freedom: 200 countries or 28,000 grid cells.

Because we can: Geo-Econ dataset and many others contain a vast treasure of data on economic activity, climate, geography, population, etc.
Our Goal is somewhat smaller

It's to understand the location, number and size of settlements in England in 1086.

Using a 3D diagram as Nordhaus has done, but with far less skilled graphic artists, we have:
Related Literature

- J. Macdonald and G.D. Snooks (1985, etc...) First researchers to evaluate Domesday census using econometric methods.

- J. Moreno-Cruz and M. S. Taylor (2012, 2013) Developed Only Energy model and extension to market economy to study energy’s role in agglomeration.

P. Krugman, V. Henderson, X. Gabaix, R. Baldwin, etc. For all the contributions to Economic geography, urban & regional economics
The Main Assumptions

- Energy is the only input; Energy collection is costly; Food and Fuel energy are consumed.

- Collection occurs across geographic space; consumption occurs in the economy’s core or settlement.

- Core size set endogenously; existence of cores is assumed;
The Main Assumptions

- Cores fill Landscapes; Landscapes fill countries.

- Landscapes are idealized: simple, connected, coastal, and edge.

- Population growth is Malthusian; consider only steady states.
The Main Assumptions

- Transport options are exogenous.
- Determinants of power density are random.
- Determinants are drawn from standard uniform distributions.
The Three Building Blocks
(From Previous Work)

- Scaling Law
- Low-cost Transport Multipliers
- The Determinants of Spatial Productivity
Scaling Law

\[ EX = \frac{W}{\Delta} \]

\[ 1 - \frac{c}{\Delta} R \]

\[ R = \frac{\Delta}{c} \]
Energy supply is cubic in spatial productivity

Size of exploitation zone is inversely proportional to the square of costs
Low-cost Transport Multipliers

City

θ

l

l_1

l_2

Energy Producer

River/Road
\[ t^* = \frac{\Delta}{\mu gd} \left\{ \begin{array}{ll}
((1 - \rho)^{1/2} \sin \theta + \rho \cos \theta)^{-1} & \text{if } \theta \leq \bar{\theta} \\
1 & \text{if } \theta \geq \bar{\theta}
\end{array} \right. \]

\[ W^S = 2 \times \left[ \int_{0}^{\bar{\theta}} \int_{0}^{i^*} \nu \left( \Delta - \frac{\mu gd}{\Delta} (i_1^* + i_2^*) \Delta \right) dv d\theta + \int_{0}^{\bar{\theta}} \int_{0}^{i^*} \nu \left( \Delta - \frac{\mu gd}{\Delta} \nu \Delta \right) dv d\theta \right] \]

\[ W^* = 1 \frac{\pi \tilde{\Delta}^3}{3 (\mu gd)^2} \]

\[ \tilde{\Delta} \equiv \Delta \left( (\pi + g(p)) / \pi \right)^{1/3} \]
CRS transportation makes life easy and neat for the theorist.

Figure 1: Exploitation Zones
The Determinants of Spatial Productivity

Power Density = \( \frac{\text{Joules/time}}{m^2} = \frac{\text{Watts}}{m^2} \)

\[
= \frac{1}{\text{time}} \times \frac{kgs}{m^2} \times \frac{\text{Joules}}{kgs}
\]

\( \Delta = \alpha \text{red} \quad \alpha > 0 \)
Food and Fuel

\[ p \left[ \Delta_0 - cr^* \right] = \left[ \Delta_e - cr^* \right] \]
Food and Fuel

\[ W_o^S = \frac{\pi \Delta_o^3}{c^2} \left( \frac{p - \frac{\Delta_e}{\Delta_o}}{p - 1} \right)^2 \left( 1 - \frac{2p - \frac{\Delta_e}{\Delta_o}}{3(p - 1)} \right) \]

\[ W_e^S = \frac{\pi \Delta_e^3}{c^2} \left( \frac{1}{3} - \left( \frac{p \frac{\Delta_o}{\Delta_e} - 1}{p - 1} \right)^2 \left( 1 - \frac{2p \frac{\Delta_o}{\Delta_e} - 1}{3(p - 1)} \right) \right) \]

\[ (W_o/W_e)^D = (W_o^S/W_e^S) \]
General Equilibrium

(b)
Per-Capita Incomes

\[ I(p, \Delta_o, \Delta_e) = \frac{\pi \Delta_e^3}{3c^2} \left( 1 + \frac{(p \frac{\Delta_o}{\Delta_e} - 1)^3}{(p - 1)^2} \right) \]

\[ I(p, \lambda \Delta_o, \lambda \Delta_e) = \lambda^3 I(p, \Delta_o, \Delta_e) \]
Malthus

\[ births = b = b_0 + b_1[Y/L] \]

\[ deaths = d = d_o - d_1[Y/L] \]
\[ L = M \cdot S \cdot V \]

\[ S = \frac{\pi \Delta_e^3}{3c^2} \quad \text{and} \quad V = \left( 1 + \frac{(p \frac{\Delta_o}{\Delta_e} - 1)^3}{(p - 1)^2} \right) / \beta(p) \]
From Core to Landscape Predictions

- Cores “fill” Landscapes
- Exploitation zones do not intersect
- Symmetry imposed by homogeneity
(c) Coastal

(d) Edge
Simple Landscapes

The extensive margin

\[ \bar{R}_{e_j} = \chi^j \Delta_e^0 / c_j \]

Filling the landscape

\[ N_j = \frac{A_j}{\pi \left[ \Delta_e^j / c_j \right]^2} = \frac{B_j}{\left[ \chi^j \Delta_e^0 \right]^2} \]

\[ B_j = A_j c_j^2 / \pi \]

In terms of Observables

\[ \Pi_{ij} = \frac{I_{ij}(p, \Delta_o, \Delta_e)}{A_{ij}} = \chi^j \frac{V \Delta_e^0}{3} \rightarrow \chi^j \Delta_e^0 = \frac{3 \Pi_{ij}}{V} \]
Predictions for Simple Landscape

\[ \log N_j = \Gamma_0 + \log A_j + 2 \log c_j - 2 \ln \Pi_{ij} \]

\[ \Gamma_0 = - \ln 9 - \ln \pi + 2 \ln V \]

\[ \log L_{ij} = \Gamma_1 + \log M_j + 3 \log \Pi_{ij} - 2 \log c_j \]

\[ \Gamma_1 = \log [9\pi/V^2] \]

\[ \ln L_j = \ln \Pi_{ij} + \ln A_j + \ln M_j \]
Connected, Coastal and Edge Landscapes

\[
\Delta^j_e = (1 + g(\rho))^{1/3} \lambda^j \Delta^0_e \quad \text{and} \quad \Delta^j_o = (1 + g(\rho))^{1/3} \lambda^j \Delta^0_o
\]

\[
g(\rho) = \frac{1}{\pi} \frac{\sqrt{1 - \rho^2}}{\rho} - \frac{\bar{\theta}}{\pi} \geq 0
\]

\[
c(\theta) = \begin{cases} 
c \Gamma[\theta, \rho] & \text{if } \theta \leq \bar{\theta} \\
c & \text{if } \theta \geq \bar{\theta}
\end{cases}
\]

\[
\Gamma[\theta, \rho] = ((1 - \rho^2)^{1/2} \sin \theta + \rho \cos \theta)
\]
Predictions for Coastal, Edge, and Connected

\[
\log N_j = \Gamma_0 + \log A_j + 2 \log c_j - 2 \log \Pi_{ij} - \log(1 + g_j(\rho_j))
\]

\[
\Gamma_0 = -\log 9 - \log \pi + 2 \log V
\]

\[
\log L_{ij} = \Gamma_1 + \log M_j + 3 \log \Pi_{ij} - 2 \log c_j + \log(1 + g_j(\rho_j))
\]

\[
\Gamma_1 = \log\left[9\pi/V^2\right]
\]

\[
\ln L_j = \ln \Pi_{ij} + \ln A_j + \ln M_j
\]
From Landscapes to Political Aggregates

Figure 4: Four Landscapes
- Country is composed of many equally sized cells
- Cells are large and contain many cores
- All cells share the same transport costs
- Individuals living in a country are similar
- Landscapes are groups of cells that are similar
- Spatial productivity determined by product of environmental factors that are random

- Random factors are iid. No spatial correlation.

- Random factors are symmetric. Uniformity.

- Random factors are essential. Allows for zeroes.
Defining a Country

\[ A_j = \sum_{\min \bar{x} + (j-1)\sigma}^{\min \bar{x} + j\sigma} a = n_j a = A f(\lambda_j) \]

\[ \Delta_j = \lambda_j \Delta^0_e = [g_{1j} g_{2j} \ldots g_{Kj}] \Delta^0_e \]

\[ f(\lambda) = \frac{(-1)^{K-1} (\ln(\lambda))^{K-1}}{(K-1)!} \quad 1 \geq \lambda \geq 0 \]
The Distribution of Population

\[ h(\omega) = \frac{3}{M_j V \Delta^0_e} f\left(\frac{3}{M_j V \Delta^0_e \omega}\right) \]

The Distribution of Settlements

\[ g(\nu) = \frac{1}{2} \frac{c}{(\pi \Delta^0_e)^{1/2} \nu^{-3/2}} f\left(\frac{c}{(\pi \Delta^0_e \nu)^{1/2}}\right) \]
The Distribution of Settlement Sizes

\[ L_{ij} = \frac{\pi MV (\Delta_e^0)^3}{3c^2} \chi_j^3 \]

\[ J(\ell) = P(L_{ij} < \ell) = F \left( \frac{3^{1/3} c^{2/3}}{(\pi MV)^{1/3}(\Delta_e^0)} \ell^{1/3} \right) \]

\[ j(\ell) = \frac{(c/3)^{2/3}}{(\pi MV)^{1/3}(\Delta_e^0)} \ell^{-2/3} f \left( \frac{3^{1/3} c^{2/3}}{(\pi MV)^{1/3}(\Delta_e^0)} \ell^{1/3} \right) \]
A Numerical Example

Figure 5: Distribution Land Productivity
Figure 6: Defining a country statistically
Figure 7: Possible Landscapes
- Capable of explaining huge variance in economic activity across space.

- Capable of explaining clustering on coasts and rivers.

- Capable of explaining economic deserts.

- Leads to precise empirical predictions for testing using Landscapes as the unit of analysis.
An Application to Domesday Data (Incomplete)
Counties as Unit of Analysis
Table 1: Summary Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pop. density</td>
<td>33</td>
<td>6.695</td>
<td>2.622</td>
<td>1.077</td>
<td>13.125</td>
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<tr>
<td>Rental</td>
<td>33</td>
<td>38.667</td>
<td>21.176</td>
<td>3</td>
<td>78</td>
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<tr>
<td>Num. settlements</td>
<td>33</td>
<td>400.212</td>
<td>349.580</td>
<td>62</td>
<td>1993</td>
</tr>
<tr>
<td>Population</td>
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<td>8103.636</td>
<td>5837.878</td>
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<td>26309</td>
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<tr>
<td>Roughness</td>
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<td>18957.57</td>
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<td>42922.43</td>
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Table 2: Correlations

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<th>Rental</th>
<th>Num. settlements</th>
<th>Population</th>
<th>Roughness</th>
</tr>
</thead>
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<tr>
<td>Rental</td>
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<td>Num. settlements</td>
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<td>-0.332</td>
<td>1.000</td>
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<tr>
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<td>0.471</td>
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<td>-0.423</td>
<td>0.444</td>
<td>-0.120</td>
<td>1.000</td>
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<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
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<tr>
<td>---------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
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<td></td>
<td>Dependent Variables</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>Pop. Density</td>
<td>Settlements Density</td>
<td>Avg. size</td>
<td></td>
<td></td>
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<td>-0.619</td>
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<td></td>
<td>(0.057)</td>
<td>(0.235)</td>
<td>(0.0447)</td>
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<td>-0.197</td>
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<td></td>
<td>(0.069)</td>
<td>(0.283)</td>
<td>(0.054)</td>
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<tr>
<td>Number Obs.</td>
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<td>33</td>
<td>33</td>
<td></td>
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<td>Adjust R²</td>
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<td>0.292</td>
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<td>F(2,30)</td>
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<td>112.79</td>
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</table>

Notes: Standard errors in parentheses. All variables are in logs.

\[
\log L_{ij} = \Gamma_1 + \log M_j + 3 \log \Pi_{ij} - 2 \log c_j
\]

\[
\Gamma_1 = \log[9\pi/V^2]
\]

\[
\log N_j = \Gamma_0 + \log A_j + 2 \log c_j - 2 \ln \Pi_{ij}
\]

\[
\Gamma_0 = -\ln 9 - \ln \pi + 2 \ln V
\]
- Promising, but....

- Are counties the right unit of analysis?

- Endogenous income

- Measurement error

- Etc.
How much variation is there within counties?

If very similar county = landscape

Our theory says they could be very different even with a small number of factors we find:

\[
Gini = 2 \times \int_0^\infty (F(\lambda) - \lambda) d\lambda = 1 - 2^{-(n-1)}
\]
Domesday Data 565 Places Within Sussex
### Within Country variation in Spatial Productivity

#### Table 1: Distribution Rents Between and Within Counties

<table>
<thead>
<tr>
<th>COUNTY</th>
<th>NUM</th>
<th>MEAN</th>
<th>STD</th>
<th>GINI</th>
<th>COUNTY</th>
<th>NUM</th>
<th>MEAN</th>
<th>STD</th>
<th>GINI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Berkshire</td>
<td>356</td>
<td>8.52</td>
<td>6.27</td>
<td>0.56</td>
<td>Lincolnshire</td>
<td>2378</td>
<td>2.97</td>
<td>2.72</td>
<td>0.65</td>
</tr>
<tr>
<td>Buckinghamshire</td>
<td>507</td>
<td>6.10</td>
<td>3.34</td>
<td>0.57</td>
<td>Middlesex</td>
<td>113</td>
<td>11.5</td>
<td>10.3</td>
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<tr>
<td>Cambridgeshire</td>
<td>638</td>
<td>2.95</td>
<td>2.01</td>
<td>0.61</td>
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<td>2953</td>
<td>1.08</td>
<td>1.03</td>
<td>0.69</td>
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<tr>
<td>Cheshire</td>
<td>561</td>
<td>1.40</td>
<td>1.42</td>
<td>0.66</td>
<td>Northamptonshire</td>
<td>670</td>
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<td>Cornwall</td>
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<td>0.67</td>
<td>Nottinghamshire</td>
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<tr>
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<td>4.80</td>
<td>3.94</td>
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<td>0.49</td>
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<td>4.62</td>
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</table>
Conclusions

Built a spatially explicit model of economics and geography relying on energy constraints as the key organizing principle.

Generates explicit and testable empirical predictions for size, number and location of economic concentrations.

Application appears promising but much work remains to be done.