How to crochet a hyperbolic plane

You can make a beautiful piece of hyperbolic geometry using only the most basic of crochet skills! These instructions were developed by mathematics professor Daina Taimina, who has a wonderful book entitled “Crocheting Adventures with Hyperbolic Planes”.

Any yarn will work, but Daina recommends a yarn that will not stretch too much to get the best results. She also recommends choosing a hook size smaller than the one recommended on the yarn label. Crochet as tightly and evenly as you can to make a good looking model. This is a great project for using up left-over yarn!

1) Chain a number of stitches for the starting row. Anything between 10 and 20 stitches is good.

2) For the first stitch in each row insert the hook into the 2nd chain from the hook. Take yarn over and pull through chain, leaving 2 loops on the hook. Take yarn over and pull through both loops. One single crochet stitch has been completed.

3) For the next 3 stitches proceed exactly like the first stitch except insert the hook into the next chain (instead of the 2nd).

4) For the 4th stitch proceed as before except insert the hook into the same loop as the 3rd stitch.

5) Repeat Steps 3 and 4 until you reach the end of the row.

6) At the end of the row before going to the next row do one extra chain stitch.

7) When you have the model as big as you want, you can stop by just pulling the yarn through the last loop.

You will find that your model starts to grow very quickly after the first few rows. This is called exponential growth, as the number of stitches in each row is 4/3 times the previous row. Your model will try to curl up on itself to keep its curvature as even as possible. It is the same reason why plant leaves and petals are curly, or why coral reefs have their intricate shape.

Try experimenting with different rates of increase: instead of creating a new stitch after every 3 stitches, why not try 2 or 4 and see what happens?
You can also make similar models with knitting, using yarn-overs or other increase methods, but the number of the stitches will quickly get too big for your needles to handle!

For more information, visit Daina’s website at: [www.math.cornell.edu/~dtaimina/](http://www.math.cornell.edu/~dtaimina/).

### The Maths of Hyperbolic Planes

In school, all the geometry we are taught is on a flat plane, like a piece of paper or a blackboard. But shapes are often curved, like the planet that we live on or the organic shapes of plants or animals. There are two ways that a surface can curve.

In **spherical geometry** there is less material as you move away from the centre, or equator. It takes far less time to walk around the axis of the Earth if you are standing at the North Pole than if you are standing at the equator! Spherical geometry is great for surfaces that want to maximise their volume while minimising their surface area, like a bubble or a water droplet. When you are knitting or crocheting something spherical, like a hat, you will decrease stitches as you move away from the equator.

In **hyperbolic geometry** there is more material as you move away from the centre. A piece of hyperbolic geometry maximises surface area while minimising volume, which is perfect for leaves which want a large surface area to capture the most sunlight, or a coral reef which wants to collect the most nutrients. This is why, in hyperbolic crochet, we keep adding in more stitches, and why it looks very organic.

Results about geometry which you learn in school, such as that the angles in a triangle sum to 180°, or Pythagoras’ Theorem, are only true in flat space and fail in both spherical and hyperbolic spaces. On a sphere, the angles in a triangle always add up to more than 180°, and in hyperbolic geometry they add to less than 180°.

One of the most important of these basic results is called the **parallel postulate**, which claims that if you draw a straight line and a point somewhere else on the page, then there is exactly one straight line through this point parallel to the first line. This was thought to be self-evidently true and was considered an axiom of geometry by the ancient Greek mathematician Euclid in his classic book *The Elements*. Mathematicians spent hundreds of years trying to show that it followed from other more basic facts, and it was not until the 19th century that hyperbolic geometry was discovered, which demonstrated that the parallel postulate was not always true.

The three types of geometry, flat, spherical and hyperbolic, also occur in 3-dimensional shapes. Scientists are still trying to decide on the shape of our own universe, because the type of geometry it has will determine its eventual fate. Einstein’s theory of relativity says that matter causes space-time to curve. If there is too much matter in the universe, its geometry will be spherical and it will eventually collapse back in on itself. If there is not enough matter, its geometry will be hyperbolic and it will expand forever. And if there is just the right amount of matter, its geometry will be flat. We currently believe that the universe is flat within a 0.4% margin of error. Find out more at [http://map.gsfc.nasa.gov/universe/uni_shape.html](http://map.gsfc.nasa.gov/universe/uni_shape.html).