Patterns of Nonlinear Shear Stiffness Degradation of Reconstituted Clay with Different Stress Histories

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This paper describes patterns of nonlinear shear stiffness degradation with respect to the stress history of clay. An experimental study using undrained triaxial compression tests was conducted on specimens cut from reconstituted clay samples of kaolinite. The nonlinear pattern of stiffness degradation was analyzed within the frameworks of both the conventional overconsolidation ratio (OCR) and the stress path rotation angle. The experimental data were subsequently interpreted based on the concept of the kinematic sub-yield surface. The pattern of stiffness degradation is more relevant to the rotation angle of the current stress path than the OCR value. The sizes of the sub-yield surfaces are variable. Results show that the kinematic movements of sub-yield surfaces within the overall bounding surface may provide an insufficient tool to fully describe the pattern of stiffness degradation.

Keywords: seafloor, sediments, surveys

Introduction

The nonlinear degradation of shear stiffness of soil in the small strain range, generally from 0.0001% to 0.1%, is an essential consideration for reliable analyses and predictions of ground deformations and soil-structure interactions, as evidenced by several case studies (Addenbrooke et al. 1997; Burland 1989; Corral and Whittle 2010; Jardine et al. 2005; Zdravkovic and Potts 2010). Past studies have shown that the stiffness nonlinearity (i.e., the degree of stiffness degradation) of clay is affected by various factors (Jardine 1995; Santagata 2008). Among these factors, the stress history is particularly important, as most engineering designs do not include an
appropriate retracing of the in-situ stress history and because few of the numerical models used in engineering account for this.

Although the stiffness at very-small strain (less than 0.0001%) has been well-correlated to the overconsolidation ratio (OCR) of axial stress (Choo et al. 2011; Hardin and Black 1968; Hardin and Drnevich 1972; Viggiani and Atkinson 1995), the stiffness degradation at following strain regime has shown incongruent patterns with the OCR. The experimental investigations by Koutsoftas and Fischer (1980) and Kokusho et al. (1982), conducted on various undisturbed marine clays under isotropic confinement, reached the same conclusion that the stiffness nonlinearity is not affected by the OCR. Later experimental results by Vucetic and Dobry (1988) supported the independence of stiffness degradation on the OCR. On the other hand, the results from internal strain measurements in undrained triaxial tests on K₀-consolidated resedimented Boston Blue Clay (RBBC) (Santagata 1998) demonstrated that the stiffness nonlinearity of normally consolidated (NC) RBBC clearly decreased once overconsolidated, whereas that of overconsolidated (OC) RBBC became similar regardless of the OCR value. Amorosi et al. (1999) also performed K₀ triaxial compression and extension tests on reconstituted Vallericca clay, demonstrating that the compression tests showed a tendency similar to that of RBBC, though extension tests showed a somewhat different one. Santagata (2008) later showed that the stiffness nonlinearity of OC RBBC was affected by the pre-shear path regardless of whether it was unloading or reloading.

Besides the OCR, the stress history of soils can also be characterized by a sudden change in the direction of the stress path. Atkinson et al. (1990) showed that the rotation angle of stress path, θ, required to follow the new stress path has a significant impact on the subsequent stiffness degradation. Stallebrass and Taylor (1997) summarized experimental findings on the effect of the rotation of stress path: the strong dependence of soil stiffness on the rotation angle at small-strain decreases as the soil is sheared, becoming negligible after a certain change in stress. Finno and Cho (2011) showed that the angle dictates the variations in the stiffness degradation of compressible Chicago clay observed from stress-probe tests, postulating that the differences in the initial stiffness stem from the accuracy limit of strain measurements. Despite the possibility that the OCR and the rotation angle of stress path are interwoven in the stress history, there have been few (if any) quantitative descriptions that considered both factors simultaneously.

The nonlinear degradation of soil stiffness is often described by the conceptual model proposed by Jardine (1992a). This conceptual model consists of a series of yield surfaces: the Y₁ surface within which the soil exhibits linear elasticity; the Y₂ surface within which the soil exhibits nonlinear behavior; and the Y₃ surface (or bounding surface), beyond which the overall plastic strains are developed. Based on this framework, three-surface kinematic hardening (3SKH) models (Grammatikopoulou et al. 2008; Stallebrass and Taylor 1997) consider the influence of stress path rotation by the eccentric alignment of sub-yield surfaces about the local stress origin, assuming that the kinematic movements of the Y₁ and Y₂ surfaces within the Y₃ surface without isotropic hardening (i.e., the sizes of the Y₁ and Y₂ surfaces are invariable). While this assumption appears plausible, the limited improvement shown in settlement predictions of OC clays (Zdravkovic and Potts 2010) implies that the validity of underlying assumptions in 3SKH models deserves further speculation.
As demonstrated, important issues remaining in advancing the predictive capabilities of stiffness degradation include i) impacts of the OCR and the rotation angle of stress path on the stiffness nonlinearity particularly when they are combined, and ii) the applicability of kinematic sub-yield surfaces for tracing stress history. This study aims to gain insight into these issues by examining the pattern of shear stiffness degradation of clay with respect to the stress history. A series of undrained shearing tests using reconstituted kaolinite samples values was conducted, differentiating the OCR and the pre-shear path. The nonlinear pattern of stiffness degradation was analyzed within the frameworks of both the conventional OCR concept and the stress path rotation angle. The experimental data were subsequently interpreted based on the concept of the kinematic sub-yield surface, from which the role of the sub-yield surface in simulating the nonlinear degradation of the soil stiffness is discussed.

Apparatus and Material

An automated triaxial testing system with small-strain stiffness measuring devices was used in this study. The triaxial testing system with automated stress path controls was equipped with two digital controllers to control the water pressure and volume. To minimize the error in the measured load associated with friction between the piston and bushing, an internal submersible load cell capable of measuring an axial load of \( \pm 4 \) kN with a resolution of 0.06 N was used. For reliable measurement of small strains, miniature linear variable differential transformers (LVDTs) that measured a length of \( \pm 2.5 \) mm with a resolution of 0.00008 mm were attached directly onto the surface of the specimen. Two axial LVDTs and one radial LVDT were attached to the middle part of the specimen in the axial and circumferential directions. Two pairs of bender elements were used to measure the shear wave velocities and anisotropic elastic moduli in the vertical and horizontal directions. Vertical and radial LVDTs and a pair of horizontal bender elements were mounted using specially designed brackets that fit into the curved surface of a cylindrical specimen. To attach the brackets tightly onto the rubber membrane, both silicon-based adhesive and small steel pins were used. To eliminate end friction during the tests, lubricated ends were used for both top and bottom of the specimen. Figure 1 shows the final testing specimen fully equipped with the bender elements and the local LVDTs.

Vertical and horizontal elastic shear moduli, \( G_{vh(BE)} \) and \( G_{hh(BE)} \), at a strain level of less than 0.001% were determined by elastic shear wave velocities obtained from bender element tests. According to Viggiani and Atkinson (1995), the distance between the tips of the bender elements was assumed as the travel distance of the shear wave. As suggested by Lee and Santamarina (2005), the zero after the first bump in the wave signature plot was selected as the arrival time of the shear wave. Details pertaining to the bender element tests are described in Choo et al. (2011). The wave propagation theory for a semi-infinite, homogeneous and elastic soil mass yields the following equation of the elastic shear modulus of soil:

\[
G_{(BE)} = \rho V_{S(BE)}^2
\]

where \( G_{(BE)} \) is the elastic shear modulus of the soil mass as determined by the bender elements, \( \rho \) is the bulk density of the specimen, and \( V_{s(BE)} \) is the shear wave velocity measured by the bender elements.
Reconstituted kaolinite samples were used for the triaxial experiment. Table 1 summarizes the index properties of the reconstituted kaolinite samples. For the reconstitution, the kaolinite powder was initially mixed into slurry at water content of 110%, approximately double the liquid limit. To obtain a flocculated clay

![Triaxial specimen with attached local LVDTs and bender elements.](image)

**Figure 1.** Triaxial specimen with attached local LVDTs and bender elements. (Color figure available online.)

<table>
<thead>
<tr>
<th>Index property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific gravity</td>
<td>2.59</td>
</tr>
<tr>
<td>Liquid limit (%)</td>
<td>53</td>
</tr>
<tr>
<td>Plasticity index (%)</td>
<td>20</td>
</tr>
<tr>
<td>Percentage finer than #200 sieve (%)</td>
<td>98</td>
</tr>
<tr>
<td>Initial water content (%)</td>
<td>45.9</td>
</tr>
<tr>
<td>Initial void ratio</td>
<td>1.19</td>
</tr>
<tr>
<td>Preconsolidation yield stress (kPa)</td>
<td>140 kPa</td>
</tr>
<tr>
<td>Unified Soil Classification System</td>
<td>MH</td>
</tr>
</tbody>
</table>

**Table 1.** Index properties of the testing material
structure and eliminate any microbe that may alter the chemical properties of the soil, a 16 normality of salt and sodium azide were added to the slurry. The kaolinite slurry was mechanically mixed in a mixing chamber for 2 to 3 days. During the mixing process, the chamber was vacuumed to assure the complete saturation of the slurry. After the slurry was completely mixed, it was placed into a consolidation chamber with a diameter of 340 mm and a height of 600 mm. To minimize any potential wall friction, 1-mm thick silicone grease was spread over the inner surface of the consolidation chamber. After allowing sedimentation in the consolidation chamber for 1 to 2 days, the slurry was incrementally consolidated until the axial stress reached 140 kPa.

**Triaxial Experiment**

The reconstituted kaolinite block was hand-trimmed into a number of cylindrical specimens measuring 140 mm in height and 70 mm in diameter. The specimens were initially saturated at a back pressure of 240 kPa for approximately 24 hours. Full saturation of the specimen was subsequently accomplished by verifying that the B value was greater than 0.98.

The specimens were then consolidated under either a $K_0$ or an isotropic stress condition until the axial effective stress reached 220 kPa, approximately 1.5 times the maximum past pressure to eliminate possible disturbance effects. Managing the difference between the isotropic and $K_0$ stress conditions allowed control of the radial stress during consolidation. During $K_0$ consolidation, the radial stress was continuously adjusted to constrain the specimen from lateral deformation by a feedback loop of cell pressure from internal radial measurements. During isotropic consolidation, the lateral effective stress was applied at the same loading rate as the axial stress.

The axial stress loading schemes were identical in terms of both the isotropic and the $K_0$ consolidation processes. Based on the preliminary study of reconstituted kaolinite performed by Kim (2004), loading rates of 1.5 kPa/hr for virgin loading phase and 3.0 kPa/hr for the unloading and reloading phases were carefully adopted to prevent the generation of excess pore water pressure during consolidation.

After consolidation with an axial effective stress of 220 kPa (OCR = 1), the specimen was sheared under an undrained condition. Great care was taken such that the sample presumably did not experience major mechanical deterioration during the undrained shearing process.

During the undrained shearing tests, the deviator stress, mean effective stress, local axial strain, local radial strain and shear strain were calculated using the force, pressure, and displacement measured by the internal load cell, pressure controllers and local axial/radial LVDTs. Under the triaxial stress condition, the mean normal effective stress, $p'$, and the deviator stress, $q$, were defined as

$$ p' = \frac{(\sigma'_a + 2\sigma'_r)}{3}, $$

$$ q = \sigma'_a - \sigma'_r, $$

where $\sigma'_a$ and $\sigma'_r$ are the axial effective stress and the radial effective stress, respectively.
The triaxial shear strain, $\varepsilon_s$, is given by:

$$
\varepsilon_s = \frac{2(\varepsilon_a - \varepsilon_r)}{3},
$$

(4)

where $\varepsilon_a$ and $\varepsilon_r$ are the axial strain and the radial strain, respectively. The excess pore pressure, $u$, was measured at the bottom of the specimen using a pore pressure transducer. The changes in the stress, strain and pore pressure from the initial value at the beginning of the undrained shearing are indicated by $\Delta$ (e.g., $\Delta\sigma'$, $\Delta p'$ and $\Delta u$).

To avoid unfavorable variability in different specimens, a multistage testing approach in which the shearing of the specimen was strictly limited until the axial strain was no greater than 0.1% was adopted during the undrained shearing process. After each undrained shearing, the excess pore pressure was neutralized by a back-pressure equalization technique (Kim et al. 2011). Hird and Pierpoint (1997), Santagata (1998) and Gasparre et al. (2007) also employed similar techniques to minimize the effect of sample variability on their evaluations of the deformation characteristics of soils. Preliminary checks on the multistage testing conducted on the specimen cuts from reconstituted kaolinite showed similar results to the validation by Santagata (1998).

After the first undrained shearing at OCR = 1, the axial stress increased to 340 kPa in both the K$_0$ and isotropic conditions. To establish the stress state that satisfied the OCR of 4, the axial stress was reduced to 85 kPa, at which the second undrained shearing was conducted. After the second undrained shearing, the axial stress was increased again to 480 kPa and subsequently decreased to 240 kPa for the third undrained shearing at an OCR value of 2. Table 2 summarizes the loading sequence in the test program. Because the axial stress was controlled to produce the different stress conditions, the OCR value became a primary parameter to record the stress history.

Two tests with six undrained shearing phases in total — three phases under the isotropic stress condition and three phases under the K$_0$ stress condition — were performed. Figure 2 shows the stress paths during the K$_0$ and isotropic consolidation conditions and the stress points at which the undrained shearing was performed, and Figure 3 shows the void ratios – mean effective stress relationships. Prior to each undrained shearing test, a drained creep condition was imposed for at least 30 hours until the rate of the axial strain decreased at 0.005%/day.

Based on the isotropic elasticity, the secant shear modulus can be given as

$$
G = \frac{\Delta q}{3\Delta \varepsilon_s},
$$

(5)

where $\Delta q$ and $\Delta \varepsilon_s$ are the increments of the deviator stress and triaxial shear strain, respectively. Here, the use of the shear modulus in the isotropic elasticity is intended to present the data in a conventional manner and not to imply that the soil was either an isotropic or an elastic material. A quantitative comparison was made by plotting the variation of the shear moduli normalized by the maximum value against the triaxial shear strains.

For a reasonable interpretation of the scattered data of the maximum shear modulus at a very low level of strain, a moving average technique was employed. Figure 4 compares the averaged variation and the raw values of the shear moduli.
<table>
<thead>
<tr>
<th>Test Condition</th>
<th>OCR1 (1st undrained shearing)</th>
<th>Loading Sequence</th>
<th>OCR4 (2nd undrained shearing)</th>
<th>Loading Sequence</th>
<th>OCR2 (3rd undrained shearing)</th>
<th>Loading Sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stress</td>
<td>$\sigma'_o = 220 \text{ kPa}$, $p' = 149 \text{ kPa}$</td>
<td>$\sigma'_o$: $220 \rightarrow 340 \text{ kPa}$</td>
<td>$\sigma'_o$: $340 \rightarrow 85 \text{ kPa}$</td>
<td>$\sigma'_o$: $85 \rightarrow 480 \text{ kPa}$</td>
<td>$\sigma'_o$: $480 \rightarrow 240 \text{ kPa}$</td>
<td></td>
</tr>
<tr>
<td>Condition</td>
<td>Isotropic loading until</td>
<td>Isotropic loading</td>
<td>Isotropic reloading</td>
<td>Isotropic unloading</td>
<td>Isotropic reloading</td>
<td></td>
</tr>
<tr>
<td>Stress</td>
<td>$\sigma'_o = 220 \text{ kPa}$, $p' = 220 \text{ kPa}$</td>
<td>$\sigma'_o$: $220 \rightarrow 340 \text{ kPa}$</td>
<td>$\sigma'_o$: $340 \rightarrow 85 \text{ kPa}$</td>
<td>$\sigma'_o$: $85 \rightarrow 480 \text{ kPa}$</td>
<td>$\sigma'_o$: $480 \rightarrow 240 \text{ kPa}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$p'$: $340 \rightarrow 85 \text{ kPa}$</td>
<td>$p'$: $85 \rightarrow 480 \text{ kPa}$</td>
<td>$p'$: $480 \rightarrow 240 \text{ kPa}$</td>
<td></td>
</tr>
</tbody>
</table>
The maximum shear moduli, $G_{\text{max}}$, was defined as the maximum value of the shear moduli in the region of shear strains ranging between 0.0005% and 0.001%. Scatter in this strain range is inevitable because the accuracy limit of internal LVDT for shear stiffness is around 0.002%, as demonstrated by Finno and Cho (2011). To validate the reliability of $G_{\text{max}}$ determined by moving average, the value was compared with the elastic shear moduli, $G_{vh(BE)}$ and $G_{hh(BE)}$, measured by the vertical and horizontal bender element tests at the beginning of the undrained shearing. Table 3 compares the values of $G_{\text{max}}$, $G_{vh(BE)}$ and $G_{hh(BE)}$, at the different initial stress levels. The values

Figure 2. Stress paths for $K_0$ and isotropic consolidation. (Color figure available online.)
of $G_{\text{max}}$ range from 96 to 105% of the values of $G_{vh(BE)}$: this similarity between the moduli related to the same vertical shear stiffness corroborates the interpretation of $G_{\text{max}}$ from the stress-strain relationship. In addition, the larger value of $G_{hh(BE)}$ indicates that the structure of the specimens have a horizontally preferred fabric.

**Patterns of Stiffness Nonlinearity Due to Different Stress Histories**

Figure 5 compares the rotation angle, $\theta$, from the previous stress path to the current stress path. The current stress path was defined as the initial linear portion of the smoothened stress path prior to the generation of excess pore water pressure. The

![Figure 3](image-url)  
**Figure 3.** Void ratio – mean effective stress relationships of $K_0$ and isotropic consolidation.

![Figure 4](image-url)  
**Figure 4.** Interpretation of the stiffness degradation curve.
The previous stress path was carefully selected to represent the last linear portion of the approaching stress path. For the isotropic consolidation series, the previous paths were set as a constant $q$ line (i.e., $q = 10$ kPa). The value of $\theta$ ranges from $27^\circ$ to $138^\circ$. Note that the data during the drained creep phase prior to the shear condition

<table>
<thead>
<tr>
<th>Test Condition</th>
<th>OCR</th>
<th>Notation</th>
<th>Void Ratio</th>
<th>$p_0'$ (kPa)</th>
<th>$q_0$ (kPa)</th>
<th>$G_{\text{max}}$ (MPa)</th>
<th>$G_{\text{vh}(BE)}$ (MPa)</th>
<th>$G_{\text{hh}(BE)}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_0$ Stress</td>
<td>1</td>
<td>K$_0$-OCR1</td>
<td>1.06</td>
<td>148.93</td>
<td>108.60</td>
<td>51.49</td>
<td>53.63</td>
<td>59.02</td>
</tr>
<tr>
<td>Condition 4</td>
<td>2</td>
<td>K$_0$-OCR2</td>
<td>0.96</td>
<td>169.78</td>
<td>53.06</td>
<td>84.47</td>
<td>80.48</td>
<td>101.06</td>
</tr>
<tr>
<td>Isotropic Stress</td>
<td>1</td>
<td>ISO-OCR1</td>
<td>1.04</td>
<td>209.92</td>
<td>10.21</td>
<td>76.76</td>
<td>76.26</td>
<td>82.45</td>
</tr>
<tr>
<td>Condition 4</td>
<td>2</td>
<td>ISO-OCR2</td>
<td>0.95</td>
<td>240.09</td>
<td>10.02</td>
<td>98.94</td>
<td>100.16</td>
<td>103.82</td>
</tr>
<tr>
<td>Isotropic Stress</td>
<td>4</td>
<td>ISO-OCR4</td>
<td>1.02</td>
<td>78.21</td>
<td>10.14</td>
<td>56.53</td>
<td>55.35</td>
<td>60.25</td>
</tr>
</tbody>
</table>

Table 3. Initial conditions of the undrained shearing tests

Figure 5. Determination of stress path rotation angle for the tests.
are not presented for clarity. The notation and the initial stress condition of each undrained shearing are described in Table 3.

Figure 6 shows the variation of the maximum shear stiffness, $G_{\text{max}}$, which generally increases as the stress level (mean effective stress, $p'$) increases. Normalizing $G_{\text{max}}$ by the initial mean effective stress, $p'_0$, gives better insight into an analysis of the factors influencing the shear stiffness. The increase of the elastic shear stiffness depending on the stress level is shown in Figure 5a. Figure 5b shows that the values of $G_{\text{max}}$ normalized by $p'_0$, which presumably excludes the influence of the stress level, correlates well with the OCR value. However, there is no apparent correlation between $G_{\text{max}}/p'_0$ and $\theta$, which implies that the rotation angle is not a major factor in determining the elastic shear stiffness. In regard to the elastic shear stiffness, the stress-level and OCR are the influencing factors, as in previous studies (Hardin and Black 1968; Hardin and Drnevich 1972) have suggested.

Figure 7 compares the degradation of the secant shear stiffness, $G$, normalized by $G_{\text{max}}$. The rates of the degradation during undrained shearing are not constant but

![Figure 6](https://example.com/figure6.png)

**Figure 6.** Variation of $G_{\text{max}}$: (a) Variation of $G_{\text{max}}$ versus $p'_0$; (b) Variation of normalized $G_{\text{max}}$ versus OCR; (c) Variation of normalized $G_{\text{max}}$ versus the rotation angle.
vary depending on the different initial stress conditions. Unlike $G_{\text{max}}/p_{0}$, a certain pattern in the relationship between $G/G_{\text{max}}$ and $\theta$ exists: the more the current stress path rotates from the previous stress path, the slower the degradation of the stiffness. The K₀-OCR1 test, which has the lowest value of $\theta$, gives the lower bound of the stiffness degradation curves, while the K₀-OCR2 test, which has the largest value of $\theta$, gives the upper bound. The order of the degradation curves consistently matches the order of $\theta$. The consolidation stress condition and the value of OCR do not appear to influence the order of the degradation curves.

The order of the degradation curve can be better quantified by defining a stiffness degradation ratio (SDR) as follows:

$$SDR(\%) = \frac{G(0.1\%) - G(0.001\%)}{G(0.1\%)} \times 100$$ (6)

Here, $G(0.1\%)$ and $G(0.001\%)$ are the shear moduli at the shear strains of 0.1% and 0.001%, respectively. The SDR is used based on the fact that that the apparent degradation of the soil stiffness is initiated approximately at a shear strain of 0.001%, and the clayey soils tend to exhibit overall yielding at a strain level of 0.1% (Jardine 1992b). A lower SDR represents higher degradation of the shear stiffness during small-strain deformation. Figure 8 presents the relationship between the SDR and $\theta$. Apparently, the SDR increases linearly with the value of $\theta$. The linearity was well established in the data of the K₀ tests, while some deviations from the linear regression line were observed in the data of the isotropic stress tests. The function of the linear regression line is also given in Figure 8. When the current stress path progresses toward the reverse direction of the previous path (i.e., $\theta = 180^\circ$), the SDR becomes 47%, thus implying that the secant shear stiffness decreases by approximately 50% relative to the overall amount of yielding. When the current stress path progresses towards the same direction of the previous path (i.e., $\theta = 0^\circ$), the SDR is only 4.2%. This linear pattern in the relationship between the SDR and $\theta$ is useful to scale the influence of the stress path change upon a reduction of the stiffness of the soil.

Figure 7. Normalized stiffness degradation curves with estimated rotation angle.
In summary, the maximum shear stiffness depends mainly on the stress level and OCR, but it does not exhibit an explicit relationship with the rotation angle of the stress path. However, the rotation angle is the major factor influencing the stiffness degradation. The early work of Atkinson et al. (1990) showed that the highest stiffness at small strains is observed for the case of $\theta = 180^\circ / C_{14}$, which constitutes a complete reversal in the stress path direction, whereas the lowest stiffness is observed when $\theta = 0^\circ$. Conversely, Finno and Cho (2011) suggested a hypothesis that the dependence of $G_{\text{max}}$ on stress path rotation manifests due to the accuracy limit of internal strain measurement (0.002%): if the limit is small enough to meet the shear stiffness measured by bender element tests, the $G_{\text{max}}$ will not be affected by the stress path rotation. The experimental results discussed in this section may confirm the hypothesis of soil nonlinearity suggested by Finno and Cho (2011).

**Patterns of Stiffness Nonlinearity Due to Multiple Yielding**

Figure 9a illustrates the configuration of the $Y_1$ and $Y_2$ yield surfaces under the $K_0$ and isotropic stress conditions prior to an instance of undrained shearing. The $Y_1$ and $Y_2$ surfaces are dragged differently along different approaching paths (i.e., $K_0$ and isotropic unloading paths), toward the same stress point at which the undrained loading initiates. By definition, the rotation angles $\theta$ are different in the $K_0$ and isotropic unloading paths, and thus the magnitude of the stiffness degradation is also different for different values of $\theta$. This occurs because the relative position between the current stress point and the yield surface determines the plastic deformation, which degrades the stiffness of the soil. Even with the same amount of undrained shearing, the different eccentric alignments of the $Y_1$ and $Y_2$ surfaces result in different movements of the yield surfaces, as manifested in the rotation angle in the nonlinear shear stiffness. In addition, due to relatively small $Y_1$ surface, the larger $Y_2$
surface practically determines the degree of stiffness nonlinearity as well as the relative influence of the stress path rotation.

According to Tatsuoka et al. (1997), after a prolonged creep period, the Y1 and Y2 surfaces are less dependent on the previous stress history and thus the surfaces become concentrically aligned. Recent observations by Clayton and Heymann (2001) suggested that if, at the end of the previous stress path, enough time is allowed for creep, the recent stress history does not have an effect on the stiffness of the soil. Diminishing the effect of the stress path rotation can be explained by the time-dependent concentric alignment of the sub-yield surfaces during the creep. As shown in Figure 9b, for the concentric yield surfaces, the directional difference in the distance between the current stress point and yield surface becomes less influential.

Similar to the work of Gasparre et al. (2007), Y1 and Y2 stress points were chosen as the points where the $e_a - \Delta \sigma'_a$ and $\Delta u - \Delta q$ relationships start to deviate from their initial linear parts, respectively. Figures 10 and 11 present how the Y1 and Y2 stress points were determined. To locate the Y1 points clearly in Figure 9, the axial stress, $\sigma'_a$, was normalized by the initial mean effective stress, $p'_0$. As shown in Figure 11, the increment of the shear stress, $\Delta q$, which initiates the Y2 yielding, is approximately $4 \sim 7$ kPa.

Figure 12 compares the Y1 and Y2 points in the $\Delta p' - \Delta q$ space. The distance from the origin to the yield point can alternatively measure the size of the yield surface. Figure 12a shows the location of the Y1 points, which align slightly to the right of the origin, thus indicating the initial stiffness anisotropy in the elastic region within

Figure 9. Configuration of kinematic sub-yield surfaces under $K_0$ and isotropic consolidation and unloading (a) immediately after unloading and (b) after the drained creep.
the $Y_1$ surface (Graham and Houlsby 1983). Such anisotropy appears diminished because the $Y_2$ points and corresponding stress path directions were aligned vertically.

Based on the conceptual model of Jardine (1992a), the $Y_1$ and $Y_2$ surfaces exhibit kinematic hardening. Hence, the size of each yield surface is presumably invariable. However, Figure 12 suggests that the sizes of the $Y_1$ and $Y_2$ surfaces vary with respect to the initial stress, OCR, and the consolidation stress conditions. The assumption of invariable $Y_1$ and $Y_2$ surfaces and the eccentric alignments may not be

Figure 10. Determination of $Y_1$ points. (Color figure available online.)
valid, as the yield points of the isotropic stress conditions, in which case the values of \( \theta \) (i.e., 84\(^\circ\), 97\(^\circ\), and 100\(^\circ\)) are similar, do not locate closely in the \( \Delta p' - \Delta q \) space. Furthermore, the close proximity between the yield points of the K\(_0\)-OCR4 and ISO-OCR4 tests, of which the undrained loadings were initiated practically at the same stress point, cannot be explained by the multiple yield surface framework, in which different rotation angles of \( \theta \) should result in different yield points at the same loading path. Therefore, the eccentric alignments of the yield surfaces are not valid in an explanation of the experimental data. This is also confirmed by considering that a prolonged period of drained creep elapsed prior to the undrained loading. For further analysis of the data, we assumed that the sizes of the concentric Y\(_1\) and Y\(_2\) surfaces are variable.

The sizes of the Y\(_1\) and Y\(_2\) surfaces are generally proportional to the magnitude of \( p'_0 \). As shown in Figure 12, the outermost yield points belong to the yield surfaces of the K\(_0\)-OCR2 and ISO-OCR2 tests, which have the largest values of \( p'_0 \). However, the order of the size of the remaining surfaces does not precisely correspond to the
order of $p'_0$. Alternatively, Figure 13 compares the yield points in the stress space normalized by the initial mean effective stress, $p'_0$. Unlike the earlier results shown in Figure 11, the yield stress points locate in pairs according to the OCR. The sizes of the $Y_1$ and $Y_2$ surfaces are likely proportional to the magnitude of $p'_0$ as well as the OCR values. Under a normally consolidated condition, the size of the yield surface is proportional to the value of $p'_0$. Under the overconsolidated condition, the size of the yield surface can increase proportionally according to the OCR value.

**Figure 12.** $Y_1$ and $Y_2$ points in $\Delta p' - \Delta q$ space. (Color figure available online.)
Unlike the stiffness degradation, the sizes of the yield surfaces do not show any apparent pattern with respect to the rotation angle, $\theta$.

To investigate the overconsolidation effect further, the $Y_1$ and $Y_2$ yield stresses were normalized by the pre-consolidation yield stress, $\sigma'_c$, as shown in Figure 14. Scattering of the yield points in Figure 12 is diminished in the normalized stress spaces. Under the $K_0$ stress conditions, both the $Y_1$ and $Y_2$ stress points collapse into a single point. Under the isotropic stress conditions, the $Y_1$ points are marginally grouped into one point. This implies that the size of the yield surfaces is primarily

Figure 13. $Y_1$ and $Y_2$ points normalized by $p'_0$. (Color figure available online.)
proportional to the pre-consolidation yield stress under the $K_0$ stress conditions. As shown in Figure 14b, however, the $Y_2$ points of the isotropic stress conditions are scattered, which indicates that the stress level influences the $Y_2$ surface under the isotropic stress condition more than the $K_0$ stress condition.

Although the concept of the $Y_2$ surface — a surface beyond which the plastic strains become pronounced — is very useful to explain the nonlinear behavior of soil

**Figure 14.** $Y_1$ and $Y_2$ points in $p'$-$q$ space normalized by the preconsolidation yield stress. (Color figure available online.)
under low levels of strain, defining an explicit $Y_2$ surface is not informative as such. There are several methods that can be used to locate the $Y_2$ yield stress point (e.g., that of Gasparre et al. (2007)), whereas a quantitative explanation of soil behavior using these $Y_2$ surfaces has achieved limited success. Table 4 summarizes the stress increments and shear strains required to reach the $Y_1$ and $Y_2$ yield points. The average shear strains at the onset of $Y_1$ and $Y_2$ yielding are 0.0012% and 0.0026%, respectively. As shown in Figure 7, the shear strain of 0.0026% for the $Y_2$ yielding approximates to a plastic threshold strain, $e_{crit}$, beyond which rapid changes in the secant and tangent shear stiffness levels occur (Jardine 1995). However, solely defining the $Y_2$ surface cannot sufficiently and fully explain the pattern of stiffness degradation when it varies with the recent stress history.

The average stress increments initiating the $Y_1$ and $Y_2$ yielding are 1.93 and 4.65 kPa, respectively. Figure 15 compares the $Y_2$ surfaces as a series of circles while

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**Table 4. Stress increment and shear strain at $Y_1$ and $Y_2$ points**

<table>
<thead>
<tr>
<th>Test Condition</th>
<th>OCR</th>
<th>$\epsilon_s$ (%)</th>
<th>$R_d$ (kPa)</th>
<th>$\epsilon_s$ (%)</th>
<th>$R_d$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_0$ Stress Condition</td>
<td>1</td>
<td>$9 \times 10^{-3}$</td>
<td>0.009</td>
<td>$2.2 \times 10^{-3}$</td>
<td>0.019</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>$1.3 \times 10^{-3}$</td>
<td>0.015</td>
<td>$2.2 \times 10^{-3}$</td>
<td>0.032</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>$1.4 \times 10^3$</td>
<td>0.023</td>
<td>$2.8 \times 10^{-3}$</td>
<td>0.051</td>
</tr>
<tr>
<td>Isotropic Stress Condition</td>
<td>1</td>
<td>$1.2 \times 10^{-3}$</td>
<td>0.008</td>
<td>$3.1 \times 10^{-3}$</td>
<td>0.022</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>$1.2 \times 10^{-3}$</td>
<td>0.011</td>
<td>$3.1 \times 10^{-3}$</td>
<td>0.031</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>$0.9 \times 10^{-3}$</td>
<td>0.020</td>
<td>$2.3 \times 10^{-3}$</td>
<td>0.046</td>
</tr>
</tbody>
</table>

*Note: Stress increment inducing $Y_1$ or $Y_2$ yielding $R_d = \sqrt{(\Delta \rho')^2 + (\Delta q)^2}$.*

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**Figure 15.** Assumed $Y_2$ and $Y_3$ surfaces.
assuming a radius of 4.65 kPa. The assumed $Y_3$ surfaces — which are the yield surfaces of the modified Cam clay (MCC) model passing through pre-consolidation yield stresses with the slope of the critical state line of 1.0 — are plotted together in Figure 15. When compared to the size of the $Y_3$ surface, the $Y_2$ surface is comparatively small such that only its movement appears insufficient in a description of the various patterns in the stiffness degradation. The $Y_2$ surface successfully defines the boundary of the plastic threshold strains, whereas an overall description of the stiffness degradation may require more than the kinematic movement of the $Y_2$ surface. Therefore, a reasonable formulation to scale the moving yield surfaces to the stiffness degradation is needed.

In summary, the sizes of $Y_1$ and $Y_2$ yield surfaces are not constant but depend on the stress level as well as the OCR. The rotation angle of the current stress path does not explicitly affect the sizes of the $Y_1$ and $Y_2$ yield surfaces. This result would imply that during plastic deformation, the $Y_1$ and $Y_2$ surfaces not only translate as a rigid body but also expand or contract isotropically.

Conclusions

An experimental study using undrained triaxial compression tests was conducted on specimens cut from reconstituted clay samples of kaolinite to investigate the pattern of nonlinearity in shear stiffness. Prior to the undrained shearing process, the specimens were differently consolidated under a $K_0$ or an isotropic stress condition, thus preparing different stress histories. Based on the results from and analyses of the data presented herein, the following conclusions are drawn regarding the pattern of stiffness nonlinearity of reconstituted clay:

1. The pattern of stiffness nonlinearity is more relevant to the rotation angle of the current stress path, $\theta$, compared to the OCR value. However, the effect of the stress history on the maximum shear stiffness is well-described by the conventional OCR values. The degree of stiffness degradation is linearly proportional to the rotation angle, $\theta$.

2. The sizes of the $Y_1$ and $Y_2$ yield surface can vary. Under a normally consolidated condition, the size of the yield surface is proportional to the value of $p_0$. Under the overconsolidated condition, the size of the yield surface can increase proportionally according to the OCR value. The effect of the rotation angle of the current stress path on the size of the sub-yield surfaces is limited. This is probably due to the concentric alignment of the sub-yield surfaces after prolonged drained creep prior to undrained shearing. The $Y_1$ and $Y_2$ surfaces not only translate kinematically but also expand or contract isotropically.

3. Defining the $Y_2$ surface, beyond which the plastic strains become dominant in soil deformation, may provide an insufficient tool to fully describe the patterns of stiffness nonlinearity when they vary with the rotation angle. The size of the $Y_2$ surface is comparatively small such that it may be difficult to describe the pattern of stiffness nonlinearity completely via only its kinematic movement within the overall $Y_3$ yield surface. A reasonable formulation to scale the moving yield surfaces to the degree of stiffness degradation is necessary.

References


