Size-scaling limits of impulsive elastic energy release from a resilin-like elastomer

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Elastically-driven motion has been used as a strategy to achieve high speeds in small organisms and engineered micro-robotic devices. We examine the size-scaling relations determining the limit of elastic energy release from elastomer bands with mechanical properties similar to the biological protein resilin. The maximum center-of-mass velocity of the elastomer bands was found to be size-scale independent, while smaller bands demonstrated larger accelerations and shorter durations of elastic energy release. Scaling relationships determined from these measurements are consistent with the performance of small organisms which utilize elastic elements to power motion. Engineered devices found in the literature do not follow the same size-scaling relationships, which suggests an opportunity for improved design of engineered devices.

Many organisms use impulsive, elastically-driven motion to exceed the power limitations of muscle [1, 2]. For example, mantis shrimp store elastic bending energy in mineralized bars within their raptorial appendages, which upon release drives their appendages at velocities up to 30 m/s, allowing them to crush shells or spear prey [3–6]. Although large organisms make some use of elastic structures (e.g. tendon), elastic energy storage and release can be crucial for small organisms (typically < 10 cm in length) to achieve rapid movement [1, 7]. These small organisms - such as mantis-shrimp, trap-jaw ants, locusts, and fleas - use muscle contractions to load energy into elastic structures, and can incorporate a latch to separate the storage and release of elastic energy into distinct phases [8]. This strategy enables these organisms to achieve astonishing kinematic performance (high velocities, large accelerations, and short durations of movement), and perhaps most remarkably, to perform these motions in a repeatable manner sustained by their metabolic processes.

Organisms that store and release elastic energy have served as inspiration for recent robotics research [9–16]. Several research groups have taken a biomimetic or bioinspired approach in an attempt to match (or exceed) biological performance using an engineered device. This approach has led to new techniques for robotic manipulation [10, 13, 16, 17], the ability to move robots on difficult terrain [9, 14, 15], and has been used to test scientific hypotheses about locomotion [11, 13, 14]. However, these engineered devices are typically larger than biological organisms, and the fastest organisms have a greater kinematic performance than currently achievable by small robots using elastic elements to perform repeatable motions [8]. In this work, we address the gap in performance between biological and synthetic systems, by examining the role of size-scale and materials properties for elastic energy storage and release.

Kinematic performance in these elastically-driven systems depends on the properties of an elastic element (spring). Although springs are often assumed to be ideal, the materials properties and geometry of a spring can constrain its kinematics. Along with the spring, there are three other major components of an elastically-driven system: (1) a motor (in organisms, muscle) that generates sufficient work to load the elastic element, (2) an energy-efficient latch to store and release the elastic element without significant dissipation, and (3) a load mass that is moved by the elastic element and that is not actively involved in elastic energy release. To understand the limits of elastic energy release due to only spring properties, we take a reductionist approach by examining the dynamics of a freely-retracting spring in isolation - externalizing the motor and latch used to store energy in the spring, such that the spring carries no additional load mass. From an initially uniform uniaxial extension, we release long thin bands of polyurethane elastomer - which have similar mechanical properties to resilin, an elastomeric protein found in some arthropods and important for elastically-driven motion in locusts [18, 19], which dissipates only a small fraction of stored energy (>90% resilience [20–23], see also Supplementary Information Fig. S1-S3). Building upon recent work [24–27], we track the full displacement field of the retracting elastomer. We use the displacement field to determine the center-of-mass motion of the band which allows for a functional determination of the scaling relations that define the limits of impulsive elastic performance. We focus on the size-scale and materials properties of a spring and how these factors affect its elastically-driven performance by examining three key kinematic parameters often used to assess performance in biology and micro-robotics [1–3, 9, 28–31]: maximum center-of-mass velocity ($v_{max}$), maximum center-of-mass acceleration ($a_{max}$), and duration of elastic energy release ($\Delta t$).

Based on heuristic arguments and previous work on elastomer recoil, we can rationalize expected scaling rela-
tions for these three measures of kinematic performance. First, for the duration of energy release, an elastic wave is needed to propagate along the entire length of the band, and so the duration should scale as the ratio of the length of the band \( L_0 \) to the elastic wavespeed of the material \( c_0 \)

\[
\Delta t \sim \frac{L_0}{c_0},
\]

where here we are using the zero strain limit of both the elastic wavespeed and length of the band. Secondly, the maximum center-of-mass velocity might be expected to follow the same scaling as the maximum velocity of the free end of the band determined in previous work \cite{24–26, 32–34}, which was found to increase with the initial strain loaded into the band before release \( \epsilon_{\text{in}} \) and is proportional to the elastic wavespeed of the material. This gives the expected scaling relation for the center-of-mass velocity as

\[
v_{\text{max}} \sim c_0 \epsilon_{\text{in}}.
\]

Finally, the maximum center-of-mass acceleration scaling can be estimated by taking a ratio of \( v_{\text{max}}/\Delta t \), as the band reaches its maximum velocity on the timescale of the duration of energy release, which results in

\[
a_{\text{max}} \sim \frac{c_0^2 \epsilon_{\text{in}}}{L_0}.
\]

**PROCEDURE**

To experimentally verify these scaling relations, a commercially available pre-fabricated polyurethane elastomer sheet (McMaster-Carr, 8716K61, durometer 40A, 1.6 mm thick) was sectioned into long, thin bands using a razor blade. For the narrowest bands (width, \( w_0 < 2 \) mm), a laser cutter (Universal Laser Systems) was first used to create shallow grooves to guide the razor blade, to reduce variations in the width. Samples were cut to ensure a uniaxial geometry \( (L_0 >> w_0) \), with 1.6 mm \( \leq w_0 \leq 27 \) mm and 17 mm \( \leq L_0 \leq 267 \) mm. The mechanical properties of the material were characterized by using a tensile testing apparatus (Instron 5564), by performing cyclic loading/unloading of the bands at low strain-rate \( (\dot{\epsilon} < 0.01 \text{ s}^{-1}) \). The polyurethane has a zero strain elastic wavespeed of \( c_0 = 40 \pm 2 \text{ m/s} \), a resilience of \( > 97\% \) at up to 300\% strain, and similar mechanical properties to resilin (see Supplemental Information for details of the material characterization and comparison to resilin). Free retraction measurements were performed by initially loading a band clamped between two pneumatic grips to a given initial strain using the tensile testing apparatus, and then releasing one of the grips [Fig. 1A]. Upon release, the band rapidly contracts, and the motion was recorded using a high speed camera (Photon Fastcam SA3, frame rate 20–75 kfps). A macro zoom lens (Nikon AF Nikkor 24–85mm) was used to maximize the image of the band to cover the full 1024 pixel CCD of the camera along the direction of motion \( (x\text{-axis}) \), giving a pixel resolution of \( 33 \sim 420 \mu\text{m} \) depending on the band length and initial strain. Markings, which had been placed along the band (Sharpie® marker, metallic silver),

\footnote{Formally, we define \( c_0 \) in terms of stress \( (\sigma) \), strain \( (\epsilon) \), and density \( (\rho) \) as \( c_0 = \lim_{\epsilon \to 0} \sqrt{\frac{\partial \sigma}{\partial \epsilon} \rho} \), measured at low strain-rate.}
were then digitized from the high speed videography using a custom MATLAB script to determine the position \( x \) of each point of the band as a function of time \( t \) [Fig. 1B]. To generate velocity, acceleration, and higher order derivatives of the position with respect to time, the digitized position data was fit to free knot splines [35, 36]. Combining the motion of each section of the band, the center-of-mass kinematics were then deduced, allowing for the determination of \( v_{\text{max}} \) and \( a_{\text{max}} \). The duration was determined as the time between the onset of the propagating elastic wave (determined by a minimum on-set threshold of jerk) until the kinetic energy of the band reached its maximum (which occurs at \( v = v_{\text{max}} \)).

**RESULTS**

The kinematic performance of 13 different bands with varying geometry (varying \( L_0 \) and \( w_0 \)) was measured (Fig. 1 shows an example of one these measurements) as a function of the strain energy loaded into the band (between 1-8 values of \( \epsilon_{\text{in}} \) for each band, for a total of 57 unique measurements). For the uniaxial geometry used, we found no dependence of the center-of-mass kinematic performance on \( w_0 \). The maximum center-of-mass velocity increased linearly with initial strain and was independent of the length of the band [Fig. 2A], in agreement with the scaling relation of Eq. (2) with a slope of \( 20 \text{ m/s} = c_0/2 \) [dashed line in Fig. 2A]. The maximum center-of-mass acceleration increases with increasing initial strain and decreasing band length [Fig. 2B], confirming the scaling relation [Eq. (3)] with a slope of \( 800 \text{ m}^2/\text{s}^2 = c_0^2/2 \) [dashed line in Fig. 2B, bottom panel]. The duration of elastic energy release increased with increasing band length [Fig. 2C]. The predicted scaling of Eq. (1) matches the data in the limit of small initial strain [dashed line in Fig. 2C, bottom panel]; however, the duration weakly increases at larger initial strains which can be attributed to a strain dependence of an unloading elastic wave propagating through the band [24, 33, 34].

The size-scaling limits of the model elastomer for repeatable, elastic energy release (Fig. 3, dashed lines) are determined by setting the initial strain to \( \epsilon_{\text{in}} = 3 \) in the scaling relations from Fig. 2 (for \( \epsilon_{\text{in}} > 3 \) failure of the polyurethane was often observed). The maximum velocity of the polyurethane elastomer recoil is size-scale independent [Fig. 3A], while the maximum acceleration and duration of movement depend on size [Fig. 3(B,C)]. The dashed lines in Fig. 3 represent the kinematic performance of this particular material choice of polyurethane elastomer, under a specific loading geometry (uniaxial extension), and driving zero added load mass. Although the kinematic performance of elastic energy release depends on those three factors (material, geometry, and...
load mass), in the next two paragraphs we justify two specific claims about the recoil scaling limits shown in Fig. 3: (1) the size-scaling of kinematic performance is independent of these three factors, and (2) the dashed lines in Fig. 3 are approximate upper bound for the particular material choice used in this study, independent of geometry and load mass.

First, the size-scaling of kinematic performance (summarized in the first column of Table I) should be independent of the specific choice of materials, geometry, and load mass. Changing the elastic material would alter the specific pre-factors in the scaling limits through changing $c_0$, failure properties, and resilience, without altering the fundamental trade-offs with size-scale [37]. A different geometry (e.g. using a cantilevered beam as a spring) or adding load mass to the system would alter the absolute kinematic performance of the system. However, if the relative size of elements all change with system size, then this simply introduces a lengthscale-independent pre-factor to the scaling relations. As a specific example, for a cantilevered beam driving a heavy load mass the scaling relations shown in Fig. 2 still hold, but with added coefficients that depend on two dimensionless parameters: the aspect ratio of the beam (length to thickness), and the ratio of the spring mass to load mass. Since these are independent of size-scale when relative size proportions are held constant, the scaling of kinematic performance with characteristic length shown in Fig. 3 are robust descriptions of the size-scale dependence of elastically-driven motion.

The second claim above — that for the specific polyurethane used in this study the dashed lines in Fig. 3 are an approximate upper bound to elastically-driven performance — is also related to the geometry and load mass. In both of these cases, changing geometry or adding load mass, the net effect is a decrease in the system’s kinematic performance, and do not change the scaling argument in Fig. 3. Intuitively, adding load mass to the system would decrease the kinematic performance compared to the unloaded elastomer bands used here. The uniaxial geometry used in this work ensures a nearly uniform strain energy density in the material. Other geometries (such as bending) result in a non-uniform strain energy density, and material failure will likely occur at a lower average strain energy density than for uniaxial extension. And although geometries which introduce a mechanical advantage in the system through a lever arm can amplify displacement, they also increase inertial load. As a result, a longer lever arm does not improve performance of the three kinematic parameters in Fig. 3. Therefore, changing geometry or load mass would shift the polyurethane scaling to lower performance (lowering the intercepts in the plots of Fig. 3), without altering the size-scaling relationship (the slopes in Fig. 3).

DISCUSSION

We interpret our results by comparing the size-scale dependence of the kinetic performance of the model elastomer with the performance of organisms and engineered devices that incorporate elastic elements (Fig. 3). The limits of kinematic performance for the polyurethane elastomer shows a similar size-scaling to elastically-driven organisms, which is in contrast to the engineered devices (Table I). Specifically, the maximum acceleration scales inversely with characteristic lengths for both elastic recoil measurements and organisms, yet the maximum acceleration of current engineered devices is dependent more weakly on size-scale. We are cautious in the interpretation of this result as each organism or engineered device in this dataset represents a unique embodiment of materials properties and geometry of elastic energy release, and the engineered devices span a narrower range of lengthscales than the organisms. However, the connection between size-scale dependence of the recoil performance and elastically-driven organisms suggests a possible universality to the size-scaling limits of elastic energy release.

Another notable feature that emerges from Fig. 3 is the ability for examples from biology to match the performance of the synthetic elastomer system. The scaling limits of kinematic performance for the elastomer recoil is similar to the performance of hydra, trap-jaw ants, and mantis shrimp. This is impressive for three reasons. First, compared to our isolated polyurethane elastomer, we would expect a diminished performance for organisms since having the motor and latch “on-board” adds additional load mass and constrains the performance of these elements. Second, dissipation is likely much more significant at the lengthscales of these organisms [7], and remarkably, both the hydra and mantis shrimp achieve their kinematic performance under water in a viscous environment. Finally, since performance of organisms in the lab is often inferior to that in nature [31, 38], the kinematic performance of these organisms could potentially be higher in a natural setting. The remarkable performance of hydra, trap-jaw ants, and mantis shrimp despite these hindering factors, suggests that the mate-

| TABLE I. Dependence of velocity, acceleration, and duration on the characteristic lengthscale ($L_c$) for recoil measurements along with two parameter power law fits to the organisms and engineered devices in Fig. 3. Here we report the power law exponent $\alpha$, obtained by fitting to $AL_c^\alpha$, where both $A$ and $\alpha$ are adjustable fitting parameters. |
|---|---|---|
| Recoil | Organisms | Devices |
| Velocity | $\sim L_c^0$ | $\sim L_c^{-0.1}$ | $\sim L_c^{0.2}$ |
| Acceleration | $\sim L_c^{-1}$ | $\sim L_c^{-0.9}$ | $\sim L_c^{0.5}$ |
| Duration | $\sim L_c^1$ | $\sim L_c^{1.1}$ | $\sim L_c^{0.9}$ |
Material properties of the biological springs are likely critical to their kinematics. While resilin is often discussed as an energy store (going back to refs. [20] and [39]), many arthropods also use the much harder chitin as a primary material to store energy, as is the case for chitinous springs in locusts [18, 40], froghoppers [41, 42], planthoppers [42], mantis shrimp [43], and trap-jaw ants [44]. Chitin, having an elastic modulus orders of magnitude larger than resilin [45], may account for the ability of arthropod systems to surpass the maximums observed in our experiments which use a resilin-like elastomer as the primary energy store.

Material properties of elastic elements have been shown previously to play an important role in elastic energy storage and release in synthetic systems. Work on engineered devices has noted the importance of using spring materials with a high elastic energy density capacity, such as elastomers [37, 46]. Even though metals typically have a significantly higher elastic wavespeed, the large strain to failure of elastomers allows them to reach velocities that are often greater than that of metal springs [47]. However, typical elastomers dissipate a significant fraction of the stored elastic energy (low resilience), so one might expect that elastomeric materials with high resilience, such as elastin or resilin found in some organisms [21–23], would serve as ideal candidates for the quick release of elastic energy. Understanding the trade-offs between resilience, elastic wavespeed, and maximum strain in biological materials employed by organisms undergoing elastically-driven motion could provide insight into the ultimate limits of elastic energy release. Recent evidence suggests that similar trade-offs persist in biological systems. While resilin and elastin are highly resilient materials, their capacity for elastic energy storage is low. This suggests that the coupling of resilient and stiff materials commonly found in biological systems may offset these inherent trade-offs [41]. The weak size-scale dependence of the engineered devices (Table I) and their diminished performance compared to biological organisms suggests that there are opportunities for improved design. For small-scale devices, performance enhancements could be developed from a bioinspired approach, utilizing composite elastic materials with both resilient and stiff components. Depending on the desired function of the device, this work suggests some advantage to engineering devices at smaller size-scales to maximize performance (e.g., maximizing acceleration or minimizing duration might be a goal). A deeper understanding of spring performance in the context of an on-board motor, latch, and load mass could further reveal important design principles currently limiting engineering design.

The impact of size-scale on kinematic performance is complicated by the choice of using either absolute performance, or scaling the performance relative to body size (relative performance). Relative performance of running (body lengths per second) and jumping (jump height per body length) have been used to characterize both biological organisms [29, 48–50] and engineered devices [9, 30, 51, 52]. In biology, relative size has been used to standardize for size differences between animals in the same species [48] or across several species [49], and it has been suggested that relative performance is more ecologically relevant as it correlates well with the ability to evade predators [53, 54]. Drag forces were suggested as the reason for a decrease in absolute jump height for insects [7], therefore using relative jump height would be a way to normalize for this effect. In contrast to the prevalent use of relative performance, we find that absolute velocity is a size-scale independent quantification of elastic performance for the lengthscales probed in the current

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**FIG. 3. Scaling relations from recoil experiments have a similar size scaling as organisms which use elastically-driven motion and show similar performance limits.** The kinematic performance for organisms and engineered devices (from Table 1 of ref. [8]) are compared with the limits of the recoil measurements (this work) in terms of velocity (panel A), acceleration (panel B), and duration (panel C). The dashed line is taken using the scaling relations found in Fig. 2 (using $\epsilon_m = 3$).

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### Table 1

<table>
<thead>
<tr>
<th>Species</th>
<th>Maximum velocity [m/s]</th>
<th>Maximum acceleration [m/s²]</th>
<th>Duration of movement [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydra</td>
<td>$10^2$</td>
<td>$10^{-1}$</td>
<td>$10^1$</td>
</tr>
<tr>
<td>Trap-jaw Ants</td>
<td>$10^2$</td>
<td>$10^{-1}$</td>
<td>$10^1$</td>
</tr>
<tr>
<td>Mantis Shrimp</td>
<td>$10^2$</td>
<td>$10^{-1}$</td>
<td>$10^1$</td>
</tr>
<tr>
<td>Froghoppers</td>
<td>$10^2$</td>
<td>$10^{-1}$</td>
<td>$10^1$</td>
</tr>
<tr>
<td>Planthoppers</td>
<td>$10^2$</td>
<td>$10^{-1}$</td>
<td>$10^1$</td>
</tr>
</tbody>
</table>

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*Characteristics of various invertebrates showing a comparison of maximum velocity, maximum acceleration, and duration of movement.*
work. Higher relative velocities (along with higher accelerations or shorter durations) can be achieved simply by reducing the size of an elastic element. This suggests that comparing the performance of systems which are orders of magnitude different in size-scale requires caution. Biologists examine kinematics of organisms both in an absolute sense (a cheetah runs more quickly than an ant) and in a relative sense (relative to body size, some ants are faster than cheetahs). However, when considering the contribution of elastic elements to the actuation of movement, their absolute size is central to their performance regardless of the size of the organism in which they are located.

In summary, we have measured the kinematic performance of elastic energy release for an elastomer with similar mechanical properties to the protein resilin. In agreement with expected scaling relations, the maximum center-of-mass velocity of a freely retracting band is independent of length, and depends only on the initial strain at which the band was released and the elastic wavespeed of the material. The maximum center-of-mass acceleration and duration of elastic energy release were found to depend on the length of the elastomer band, with an improved performance at smaller size-scales. Previously reported measurements of kinematic performance in elastically-driven organisms show similar size-scaling limits to the elastomer studied here, whereas the acceleration of engineered micro-robotic devices varies more weakly with size-scale. The current results, which probe the upper bound of elastically-driven kinematics of a resilin-like material, show a similar performance to some of the fastest biological systems. Future work which seeks to delineate the role of elastic wavespeed, maximum strain, and resilience in elastic biological systems could lead to a foundational understanding for improved engineering design. Specifically, the mechanical properties of resilin, chitin, and resilin/chitin composites would be of great importance to compare to engineered systems.

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