Does Eighth-Grade Mathematics Teaching in the United States Align With the NCTM Standards? Results From the TIMSS 1995 and 1999 Video Studies

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Debates about the future of school mathematics in the United States often center on whether standards-based instruction is improving or undermining students’ achievement. Critical for making progress in these debates is information about the actual nature of classroom practice in U.S. classrooms. This article focuses on one key element of classroom practice—teaching—and presents the results of two studies of randomly selected, nationally representative U.S. eighth-grade mathematics lessons that were videotaped as part of the TIMSS 1995 and 1999 Video Studies. Analyses compare features of teaching found in these lessons with pedagogical recommenda-
tions for middle school teachers in the *Principles and Standards for School Mathematics (Principles and Standards)* in order to examine the extent to which teaching in U.S. eighth-grade classrooms is standards-based. Results show that typical mathematics teaching, in both 1995 and 1999, is more like the kind of traditional teaching reported for most of the past century (Cuban, 1993; Fey, 1979; Weiss, Pasley, Smith, Banilower, & Heck, 2003; Welch, 1978) than the kind of teaching promoted in *Principles and Standards*.

*Key words:* Middle grades, Policy issues, Reform in mathematics education, Survey study, Teaching (role, style, methods), and Teaching practice

There is currently a lively debate about how mathematics should be taught in U.S. classrooms. Some educators argue for more active student involvement, increased emphasis on conceptual understanding, and greater attention to flexible problem-solving skills; others argue for more emphasis on developing basic mathematical skills. At the center of the debate are the series of documents (hereafter the *Standards*) published by the National Council of Teachers of Mathematics (NCTM) designed to set school mathematics goals for students and teachers (NCTM, 1989, 1991, 1995, 2000). Some researchers report that students in schools or districts that follow “standards-based” mathematics teaching or that use new curricula aligned with the *Standards* learn more than students in conventional classrooms (e.g., Alternatives for Rebuilding Curricula, 2003; Fuson, Carroll, & Drueck, 2000; Huntley, Rasmussen, Villarubi, Songtong, & Fey, 2000; Knapp, Shields, & Turnbull, 1992; McCaffrey, et al., 2001). Critics argue that a shift to standards-based teaching has led to lower student achievement levels, and that a “back-to-the-basics” approach is in order (e.g., “Erosion of Basic Math Skills,” 2002; Klein, 2001; Loveless, 2001).

With the future direction of mathematics education in the United States at a crossroads, what kind of data is relevant for informing the debate and the policy decisions that lie ahead? First, although student achievement data can be useful, it is impossible to divine recommendations for practice from achievement data alone. Of most relevance is information about classroom practices that might explain the achievement. What is actually happening *inside* mathematics classrooms? Second, because the debate about the future of mathematics education is a national-level debate, it must be informed by nationally representative data. Educators and policymakers need to know what classroom practice looks like across the country, not just in specific locations receiving special interventions.

There are many aspects of classroom practice that might affect students’ achievement and that will be relevant in future policy discussions, including the content presented and the nature of classroom teaching. In this article, we focus on teaching. We present the results of nationally representative, randomly selected samples of U.S. eighth-grade mathematics lessons that were part of the Third International Mathematics and Science Study (TIMSS) 1995 and 1999 Video Studies. Our goal is to paint a national-level picture of eighth-grade mathematics teaching and to identify changes in classroom teaching during a time of active national-level debates and calls for substantive change in classroom practice.
We compare the classroom teaching found in the TIMSS national samples of lessons in the United States with some of the recommendations contained in the NCTM Standards. Are teachers, in average classrooms around the country, implementing these suggestions from the Standards? Or is classroom practice unaffected by the Standards? Describing classroom practice in relation to these highly visible policy documents provides a way of benchmarking the nature of teaching, looking for changes in teaching over time, and providing information on what is happening inside classrooms that can be used to make informed choices for the future.

WHAT IS KNOWN ABOUT THE INFLUENCE OF THE STANDARDS ON CLASSROOM TEACHING?

In 1992, Ferrini-Mundy and her colleagues visited seventeen school sites throughout the United States in order to gauge the impact of the 1989 Curriculum and Evaluation Standards for School Mathematics (Ferrini-Mundy & Schram, 1997; Ferrini-Mundy, Graham, Johnson, & Mills, 1998). In their project, Recognizing and Recording Reform in Mathematics Education (R3M), the researchers found that although the teachers, principals, and superintendents at these schools were largely supportive of the Standards and professed a commitment to their implementation, progress toward this end was only in the very early stages. When the R3M researchers observed classes in these schools, they noticed a wide discrepancy between the “intended” (or adopted) curriculum and the “implemented” curriculum.

A decade later, Weiss and her colleagues conducted systematic observations of 66 mathematics lessons from middle school classrooms throughout the United States (Weiss et al., 2003). Each lesson was rated on a number of 5-point scales. For example, observers assessed the degree to which lessons reflected “current standards.” A lesson was given a score of 1 if it was “not at all reflective of current standards” and a score of 5 if it was “extremely reflective of current standards.” Ratings were generally quite low; in this case, the modal rating was 2. The study’s authors conclude, “Observations conducted for the Inside the Classroom study suggest that the nation is very far from the idea of providing high quality mathematics . . . education for all students” (p. xiii).

Similarly, Spillane and Zeuli (1999) reported that in their classroom observations of 25 elementary and middle school mathematics teachers, only 4 taught in a way that was consistent with reform ideals. What makes these results particularly dramatic is that the 25 teachers were selected based on their responses to a survey inquiring about reform-oriented practice. These teachers scored in the top 10% of a sample of 283 teachers who completed the survey, indicating that they had a strong sense of themselves as reform teachers. Based on the research just reviewed, it appears that the Standards have not had a large effect on the nature of school mathematics teaching, even in some classrooms where teachers believe they are implementing the Standards’ recommendations. We now consider whether the nationally representative samples of TIMSS lessons paint a similar or different picture of
mathematics teaching at the eighth-grade level in the United States and whether the picture changes over time.

**METHOD**

This article draws on data collected in the United States for the video portions of the TIMSS 1995 and TIMSS 1999 studies. The 1995 sample included 81 randomly selected eighth-grade mathematics lessons, and the 1999 sample included 83 randomly selected eighth-grade mathematics lessons. In both Video Studies, videotaped lessons from other countries were collected as well. In the 1995 study, lessons were collected from Germany and Japan, and in the 1999 study mathematics lessons1 were collected from Australia, the Czech Republic, Hong Kong Special Administrative Region of the People’s Republic of China, the Netherlands, and Switzerland. For complete results of the 1995 study, see Stigler, Gonzales, Kawanaka, Knoll, and Serrano (1999), and for the 1999 study, see Hiebert et al. (2003). In this article we present new analyses that focus only on the U.S. lessons and that search for changes in teaching practices from the mid- to the late-1990s.

Classrooms videotaped for the TIMSS 1995 Video Study were a random subsample of the full TIMSS 1995 sample of eighth-grade mathematics classes included in the achievement study (see Stigler et al., 1999). The TIMSS 1999 Video Study drew a separate sample from that used for achievement but followed the sampling standards and procedures implemented for the TIMSS 1999 achievement study (see Jacobs et al., 2003; Martin, Gregory, & Stemler, 2002). Internationally, sampling for the TIMSS 1999 Video Study was conducted in two stages. In the first stage, a stratified nationally representative sample of schools was selected, and in the second stage, mathematics classes were randomly selected from the eighth-grade mathematics classes in the sampled schools. In both video studies, the school sample was a Probability Proportionate to Size (PPS) sample. In a PPS sample, the probability of selection assigned to each school is proportional to the number of eligible students in the eighth grade in schools countrywide. One eighth-grade mathematics class per selected school was sampled at random, and no substitutions of any sort (i.e., of teachers or class periods) were allowed.

In both studies, each eighth-grade mathematics class was videotaped once, in its entirety, without regard to the particular mathematics topic being taught or type of activity taking place. The only exception was that class lessons were not videotaped on days that a test was scheduled for the entire class period. Teachers were asked to do nothing special for the videotaped session and to conduct the class as they had planned.

Data collection in the United States for both studies took place over the school year in order to include a typical range of topics and classroom conditions. Data were collected for the 1995 study from November 1994 through May 1995 (7 months) and for the 1999 study from January 1999 through May 2000 (17 months)—5 school

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1 In the 1999 TIMSS Video Study, science data were also collected from five countries.
years later. Additional sampling and data collection details can be found in Stigler

A series of codes were developed during the TIMSS 1999 Video Study by a team
of individuals that included bilingual representatives from each participating
country as well as specialists in mathematics and mathematics education.\(^2\) This team
worked closely with two advisory groups: national research coordinators repre-
senting each of the countries in the study and a steering committee consisting of
five North American mathematics education researchers. The main goal of the code
development process was to identify variables that represented features of teaching
that were meaningful in all the participating countries and that contained theoret-
cal or empirical connections to students’ mathematics learning. An iterative process
of searching the research literature, viewing the videotapes, and soliciting sugges-
tions from research team members, international coordinators, and steering
committee members generated potential features of teaching that were transformed
into codes. For each code applied to the data, intercoder reliability of at least 85%
was established. Reliability was calculated both prior to coding and midway through
the coding process.

An important example of the code development process pertains to working on
mathematics problems. It became apparent early in the process, from all sources,
that a key descriptor of teaching is the nature of mathematics problems and the way
that they are worked on during the lesson. If the research team could mark the begin-
ning and ending of every problem (or set of problems) worked on during the
lesson, then the characteristics of problems could be examined with a variety of
secondary codes. This would provide a more precise description of teaching than
general descriptors of the lesson as a whole. But defining what counts as a problem
is not simple, especially when the definition must be reliably applied across a team
of coders from seven countries. Mathematics problems come in a bewildering
array of shapes and sizes. Eventually, the research team constructed a definition that
was clear enough to achieve reliability. This breakthrough enabled a large number
of additional codes to characterize the nature of the problems and how they were
worked on with students.

The TIMSS 1995 video data were recoded using the coding scheme developed
for the 1999 study and by the same group who coded the 1999 data in order to ensure
that identical and reliable measures would be compared across the two samples. In
addition, a team of U.S. mathematicians and teachers of postsecondary mathematics
was commissioned to evaluate a randomly selected subset of lessons from both
datasets. This team examined the mathematical reasoning that was explicit in each
lesson and reached their judgments by consensus.

To help understand and interpret the videotaped lessons, teacher questionnaires
were collected in both Video Studies. Although the questionnaires from the two
studies differed somewhat, both contained items addressing teachers’ knowledge
and perceived implementation of “current ideas” in mathematics teaching and

\(^2\) All of the U.S. lessons were coded by native English speakers.
learning. The percentage of U.S. teachers who completed the questionnaire was 98% in the 1995 study and 100% in the 1999 study.³

For the majority of the analyses presented in this article, teaching in the United States in 1995 and 1999 was compared statistically. The lesson was the unit of analysis in all cases. All statistical comparisons were conducted using data with survey weights. Weights reflect the overall probability of selection for each classroom, and provide unbiased estimates of national means and distributions. The analyses were conducted using ANOVAs and two-tailed t tests at the .05 level. All analyses used unrounded estimates and standard errors, which also were computed for each estimate. It is important to keep in mind that the data collected were not longitudinal; different teachers were videotaped in 1995 and 1999.

The coding scheme was not developed specifically to measure the link between the videotaped instruction and the NCTM Standards. Consequently, the data do not provide a systematic test of whether each principle or standard is being implemented. However, the coding scheme was developed by an international team well versed in current theories of mathematics learning and teaching in each of the participating countries. Some of the codes do capture the gist of some of the standards, especially those described as Process Standards for Grades 6–8 in Principles and Standards (NCTM, 2000): Problem Solving, Reasoning and Proof, Communication, Connections, and Representation. For all these Process Standards, at least one code was applied to the data that, to some extent, captures the intent of that standard.

Although the data were collected before Principles and Standards was published so classrooms could not have been affected directly by this document, we are interested in the extent to which classroom teaching in 1995 and 1999 aligned with these recommendations, perhaps because of the influence of the prior NCTM Standards or other factors. By aggregating the results across the particular features of teaching that can be addressed by the TIMSS video data, it is possible to develop an incomplete but suggestive picture of the ways in which eighth-grade classroom teaching during the 1995 and 1999 sampling periods was (and was not) consistent with Principles and Standards.

RESULTS

Teachers’ Knowledge of the NCTM Standards

An initial question to ask when investigating the extent to which teaching is consistent with the NCTM Standards (as presented in Principles and Standards or in other NCTM documents) is whether teachers even know about the recommendations. In both Video Studies, teacher questionnaires asked the eighth-grade mathematics teachers about their familiarity with “current ideas” in mathematics education. As Table 1 indicates, a considerable number of teachers in both samples

³ Copies of the teacher questionnaires used in the 1995 and 1999 studies can be found in Appendix C of Stigler et al. (1999) and in Appendix A of Jacobs et al. (2003), respectively.
reported some degree of familiarity with current ideas. In 1995, virtually all of the participants stated that they were at least “somewhat aware” of current ideas. In 1999, over three fourths of the participants stated that they “keep up” with current ideas. Because the questionnaire items were worded differently, teachers’ responses between 1995 and 1999 were not compared statistically. However, teachers’ interpretations of “keeping up” with current ideas may be most similar to interpretations of being “very aware” of current ideas.

Of course, teachers might understand the phrase “current ideas” in a variety of ways. Therefore, as an additional probe, open-ended questions in both questionnaires asked teachers how they had heard about current ideas in mathematics teaching and learning and to identify materials they had read that described the ideas. Half of the teachers in 1995 and almost two thirds of the teachers in 1999 listed NCTM conferences and/or documents, including NCTM Standards, journals, and newsletters (see Table 1). The difference between the two samples was not statistically significant.

Table 1
Teacher Reports on Their Knowledge of Current Ideas About Teaching and Learning Mathematics: 1995 and 1999

<table>
<thead>
<tr>
<th>1995 Questionnaire Item</th>
<th>% of teachers</th>
<th>1999 Questionnaire Item</th>
<th>% of teachers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Awareness of current ideas</td>
<td>Keep up with current ideas</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Very aware</td>
<td>39</td>
<td>Agree</td>
<td>76</td>
</tr>
<tr>
<td>Somewhat aware</td>
<td>57</td>
<td>No opinion</td>
<td>17</td>
</tr>
<tr>
<td>Not very or not at all aware</td>
<td>4</td>
<td>Disagree</td>
<td>7</td>
</tr>
<tr>
<td>Read NCTM literature</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yes</td>
<td>50</td>
<td>Yes</td>
<td>64</td>
</tr>
<tr>
<td>No</td>
<td>50</td>
<td>No</td>
<td>36</td>
</tr>
</tbody>
</table>

Note. The first item (“How aware do you feel you are of current ideas about the teaching and learning of mathematics?”/“In general, I feel I keep up with current ideas in mathematics teaching and learning.”) was not compared statistically across the two samples because the wording of the question and response options were different. No statistically significant differences across the samples were detected in the second item (“What written materials are you aware of that describe current ideas about the teaching and learning of mathematics? Please list up to three, and indicate whether you personally have read each one.”).
teachers said they were “knowledgeable” or “very knowledgeable” about the 1989 NCTM Curriculum and Evaluation Standards for School Mathematics. Almost three fourths of the teachers (74%) reported that they had participated in a professional development activity that provided them with strategies for implementing these Standards, such as a local workshop or a regional or national NCTM meeting. This question was not asked in the 1995 Video Study.

**Teachers’ Perceived Implementation of the NCTM Standards**

In both Video Studies, teachers were asked, “To what extent do you feel that the lesson you taught today is in accord with current ideas about the teaching and learning of mathematics?” Significantly more teachers responded “a fair amount or a lot” in 1999 (86%) than in 1995 (70%) (see Figure 1). The percentage of U.S. teachers that reported that their lesson was “not at all” in accord with current ideas was only 1% in 1999 and 7% in 1995.

![Figure 1](image)

*Figure 1. Teacher reports on the degree to which their lessons reflect current ideas about teaching and learning mathematics: 1995 and 1999*

Teachers then were asked to note what part of their lesson reflected these ideas, and explain why. A wide range of categories were needed to classify teachers’ diverse responses. Table 2 presents categories describing teachers’ perceptions of how and why their lessons reflected current ideas about teaching and learning math-
ematics. U.S. eighth-grade mathematics teachers in both samples pointed to external observable practices as well as internal cognitive and affective factors to signal the influence of the NCTM Standards on their teaching. Incorporating technology, real-world problems, and group work in the classroom are observable practices that might, or might not, capture the intent of the Standards. Actively involving students in thinking critically about mathematical problems, and basing instruction on how students learn, seem more likely to represent the intent of the Standards. A similar percentage of teachers in 1995 and 1999 identified each of these categories, with the exception of “use of technology.” More teachers in 1999 reported that their lessons exemplified current ideas by using technology (particularly calculators).

### Teachers’ Classroom Practices and the NCTM Standards

Do teachers’ practices align with selected recommendations contained in the NCTM Standards, especially those in Principles and Standards? The results from teachers’ questionnaire responses indicate that a majority of the U.S. teachers claimed to be aware of current ideas of teaching, attributed their knowledge to the NCTM Standards, and believed that their videotaped lesson illustrated these ideas. Although these data are informative, both the questionnaire inquiries and the teachers’ responses can be subject to multiple interpretations. It is well understood that self-reports can be misleading (Frykholm, 1999; Hiebert & Stigler, 2000). The video data allow us to move beyond self-reports and explore more specifically, and with more consistency across teachers, what actually was happening in the classroom.

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6 Categories mentioned by fewer than 5% of the U.S. respondents from both datasets are not shown.

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Table 2

<table>
<thead>
<tr>
<th>How the lesson exemplifies current ideas(^a)</th>
<th>(%) of teachers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1995</td>
</tr>
<tr>
<td>Use of technology*</td>
<td>4</td>
</tr>
<tr>
<td>Actively involves students</td>
<td>14</td>
</tr>
<tr>
<td>Includes real-world or other recommended types of problems</td>
<td>12</td>
</tr>
<tr>
<td>Classroom discussion</td>
<td>6</td>
</tr>
<tr>
<td>Students construct their own knowledge</td>
<td>8</td>
</tr>
<tr>
<td>Collaborative group work</td>
<td>11</td>
</tr>
<tr>
<td>Independent work</td>
<td>4</td>
</tr>
<tr>
<td>Teaches critical thinking or problem-solving skills</td>
<td>14</td>
</tr>
</tbody>
</table>

\(\%\) Teachers were asked, “Please describe one part of the videotaped lesson that you feel exemplifies current ideas about the teaching and learning of mathematics and explain why you think it exemplifies these ideas.” Categories mentioned by fewer than 5% of the U.S. respondents from both datasets are not shown.

* Indicates a significant difference between 1995 and 1999.
In this section, we lay the coded results of teaching practices against relevant recommendations found in *Principles and Standards*. Most of the data are in the form of frequencies—the number of times a feature of teaching occurred. In many cases, a feature of teaching recommended by *Principles and Standards* was observed but with low frequency. The persistent question for the reader will be whether the degree of frequency meets the level expected when judging alignment or consistency with recommendations. The question has no easy answer. We leave the interpretation of the alignment of many individual features to the reader but offer some observations in this regard in summary sections and in the discussion section of the article. The order in which the results are presented is guided by the chronology of the Grade 6–8 Process Standards section of the *Principles and Standards* document (NCTM, 2000, pp. 256–285).

**Problem Solving**

The first Grade 6–8 Process Standard is Problem Solving. The authors of *Principles and Standards* note, “Through problem solving, students can experience the power and utility of mathematics” (NCTM, 2000, p. 256). A number of codes applied to the TIMSS 1995 and 1999 video data relate to problem solving as it is described in *Principles and Standards*.

It is important to know that in both the 1995 and 1999 samples the vast majority of lesson time was spent working on mathematics problems. On average, students worked on problems 81% of the lesson time in 1995 and 85% of the lesson time in 1999. Does this large allocation of lesson time devoted to mathematics problems reflect “problem solving” as the authors of *Principles and Standards* had in mind? Not by itself. The code “mathematics problem” was defined too broadly to indicate what kinds of activities and processes were occurring during this time. The following sections further explore the alignment of recommendations found in *Principles and Standards* regarding problem solving with a variety of teachers’ classroom practices.

**Individual and group work.** *Principles and Standards* suggests that teachers should allow their students some time to solve problems both individually and in groups. “Middle-grades students . . . will benefit from frequent opportunities for both independent and collaborative problem-solving experiences” (NCTM, 2000, p. 256). In other words, not all of every mathematics lesson should be devoted to working as a whole class; students should be given time to work by themselves and with their peers. In their questionnaires, some teachers in both 1995 and 1999 pointed to collaborative group work as a hallmark of standards-based teaching. How often did individual and group experiences occur in the classroom?

In the Video Studies, lesson time was parsed into public interaction (when the whole class was engaged in a shared activity) and private interaction (when students were working on their own or in small groups). U.S. eighth-grade mathematics teachers devoted, on average, about one third of the lesson time (37% and 32%, respectively) to private interaction. During 70–80% of the private interac-
tion time, students worked by themselves. Virtually all of the lessons in both datasets (89% and 95%, respectively) designated some class time to working individually. About one third of the lessons provided an opportunity for students to work collaboratively (34% and 30%, respectively). As these numbers suggest, students spent the majority of private interaction time working individually rather than in pairs or groups. Multiplying the private interaction time by the percentage of this time spent in group work shows that, on average, 10% and 6% of lesson time in 1995 and 1999, respectively, was devoted to collaborative group work. The two samples did not differ statistically on any of these indicators.

**Private work assignments.** What were students given to do when they worked on their own? To address this question, private work assignments were classified into one of two categories: (1) repeating procedures that had been demonstrated earlier in the lesson or learned in previous lessons, or (2) doing something other than repeating learned procedures for at least some of the time. “Something other” might have been developing solution procedures that were new for the students or modifying solution procedures they already had learned.

In both Video Studies, at least three fourths of the private (individual or group) work time was spent repeating procedures (78% of the time per lesson on average in 1995 and 75% in 1999). On average, students worked on an assignment containing at least one problem that involved something other than repetition during 8% of the time per lesson in 1995 and 9% in 1999. These differences were not significant.

**Number and length of problems.** *Principles and Standards* urges teachers to assign challenging tasks. Students should “engage profitably in complex investigations, perhaps occasionally working for several days on a single problem and its extensions” (NCTM, 2000, p. 256). This recommendation implies that “complex” problems should be assigned in some lessons, and they should be worked on for a relatively long period of time. How many problems were assigned and worked on (either with the whole class or individually) in the videotaped lessons? On average, a total of 33 problems were assigned and worked on per lesson in 1995, and 32 problems in 1999 (see Table 3). As noted earlier, the definition of mathematical problem applied to the videotaped lessons was quite broad and included routine exercises as well as challenging problems.

To investigate further the nature of the mathematics problems presented during the lesson, codes for three types of problems were defined: independent, concurrent, and answered-only. Independent problems were presented as single problems and worked on for a clearly definable period of time during the lesson. Concurrent problems were presented as a set of problems (e.g., from a worksheet or textbook)
and were worked on, at least in part, during private work time. It was not possible to measure the amount of time spent on each concurrent problem. Answered-only problems were most often taken from homework or an earlier test and had already been completed prior to the lesson; only their answers were shared.

How much time, on average, was spent on a given problem? Based on the video data, the most accurate answer to this question can be provided by examining the time spent on independent problems. An independent problem took an average of 3 minutes to solve in 1995 and 5 minutes in 1999, not a statistically significant difference. On the other hand, in 1999 the percentage of mathematics problems worked on for 45 seconds or longer (an arbitrary marker) was significantly lower (61%) than in 1995 (73%).

Procedural complexity. As noted above, Principles and Standards calls for solving complex mathematics problems. Determining the complexity or mathematical challenge of a problem from a videotape can be difficult, because it depends on many factors, including the experience and capability of the student. Because the TIMSS video data contains only one lesson from each classroom, the only valid and reliable measure of complexity that could be defined was one of procedural complexity. A form of procedural complexity that can be measured independent of the student depends on the following two factors: the number of steps or decisions required to solve a problem using common solution methods and the existence of subproblems. Based on these two factors, three levels of procedural complexity were defined:

- **Low complexity:** Solving the problem using conventional procedures requires four or fewer decisions or steps by the students. The problem contains no subproblems, or tasks embedded in larger problems, that could themselves be coded as problems. An example is, “Solve the equation: \(3(x - 5) = 4\).”

- **Moderate complexity:** Solving the problem using conventional procedures requires more than four decisions by the students and/or contains one subproblem. An example is, “Solve the set of equations for \(x\) and \(y\): \(2y = 3x - 4\); \(2x + y = 5\)”

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Table 3

<table>
<thead>
<tr>
<th>Type of problem</th>
<th>Number of problems</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1995</td>
</tr>
<tr>
<td>Total problems</td>
<td>33</td>
</tr>
<tr>
<td>Independent problems</td>
<td>9</td>
</tr>
<tr>
<td>Concurrent problems</td>
<td>21</td>
</tr>
<tr>
<td>Answered-only problems</td>
<td>3</td>
</tr>
</tbody>
</table>

*Note. The total number of problems may not equal the sum of the three types because of rounding. No statistically significant differences across the samples were detected on types of problems.*

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\(^9\) Examples of steps include removing parentheses in an equation, representing a situation with an equation, and substituting a value into an expression.
• **High complexity:** Solving the problem using conventional procedures requires more than four decisions by the students and contains two or more subproblems. An example is, “Graph the following linear inequalities and find the area of intersection: \( y \leq x + 4; x \leq 2; y \geq -1. \)"

As seen in Figure 2, the majority of problems per lesson in both samples were of low procedural complexity, averaging 78% of problems in 1995 and 67% of problems in 1999. High complexity problems were relatively rare in both samples, accounting for averages of 7% of the problems per lesson in 1995 and 6% in 1999.

![Figure 2. Average percentage of problems per lesson at each level of procedural complexity: 1995 and 1999](image)

*Note:* Percentages may not sum to 100 because of rounding.

*Indicates a significant difference between 1995 and 1999.

**Real-life connections and real-world objects.** *Principles and Standards* mentions incorporating real-life contexts into middle school mathematics classes. The authors note, “Many interesting problems can be suggested by everyday experiences” (NCTM, 2000, p. 256). To what degree were mathematics problems presented within a real-life context rather than using only mathematical language with written symbols? In both datasets, about one fifth of the problems per lesson, on average, were connected to real life (20% and 22%, respectively). But significantly more lessons in 1999 (66%) included at least one such problem compared to 1995 (45%). Teachers might also choose to incorporate real-world objects such as boxes and cans,
maps, dice, or newspapers in their mathematics lessons. However, such objects were not frequently seen in either 1995 or 1999 (8% and 15% of the lessons, respectively), and there was no significant difference between the samples.

*Exploring alternative solution methods.* Principles and Standards suggests that teachers should present students with problems that can be solved in different ways. “Such problems help students develop and use a variety of problem-solving strategies and approaches, and sharing these methods within the classroom affords students opportunities to assess the strengths and limitations of alternative approaches” (NCTM, 2000, p. 257). Three codes were applied to problems in the videotaped lessons that capture information about the extent to which alternative solution strategies were encouraged in mathematics classrooms. First, problems were marked if students were given a choice in how they wanted to solve them. Second, problems were marked if multiple solution methods were publicly presented. Third, problems were marked if both of these features were present (i.e., students had a choice, and multiple solution methods were presented) and the class critiqued, examined, or compared the methods.

Student choice included either of the following events: (1) the teacher (or textbook) explicitly stated that students were allowed to use whatever method they wished to solve the problem, or (2) two or more solution methods were identified and students were explicitly asked to choose one of the identified methods. A solution method was defined as a sequence of mathematical steps used to produce a solution. In just under half of the lessons in 1999 (45%), teachers allowed students to choose a solution method for at least one problem. This represents a significantly higher proportion of lessons compared to just 16% of the lessons in 1995. But the average percentage of problems per lesson with this feature was less than 10% in both samples (4% in 1995 and 9% in 1999; not a significant difference).

The picture is similar when looking at how often alternative solution methods were shared. To count as an alternative solution method, each method needed to (1) be distinctly different from other methods presented, (2) include enough detail so that an attentive student could follow the steps and use the method to produce a solution, and (3) be accepted by the teacher as a distinct and legitimate method (not a correction or elaboration of another method). The difference in the percentages of lessons in 1995 and 1999 that included alternative solution methods for at least one problem (25% and 37%, respectively) was not statistically significant. In both samples, 5% or less of the problems per lesson involved sharing alternative solution methods (3% in 1995 and 5% in 1999; not a significant difference).

Problems characterized as “examining methods”—which included student choice, presenting alternative solution methods, and actively discussing these methods—probably come closest to matching the intent of the NCTM recommendations for challenging and open problem-solving activities. The percentage of lessons that included at least one examining methods problem rose from 3% in 1995 to 17% in 1999 (a significant difference). As with student choice and alternative solution methods, however, “examining methods” problems constituted only a small percentage of the problems per lesson in each sample (1% in 1995 and 2% in 1999; not a significant difference).
Problems with multiple answers. Principles and Standards recommends that teachers “challenge students with problems that have more than one answer” (NCTM, 2000, p. 258). In other words, students should learn that some mathematics problems have multiple correct answers and that solving a mathematics problem could mean finding more than one answer. The number of (correct) solutions presented for each problem\(^{10}\) was measured in both of the Video Studies. An example of a problem for which multiple answers are possible is the following: “Name a possible combination of angles within a triangle. Give your answer in degrees.” In the videotaped lessons, if the teacher and/or students publicly presented two or more correct answers for a problem, the problem was coded as containing multiple answers. Significantly more lessons in 1999 (22%) included at least one such problem compared to 1995 (9%); however, such problems were quite rare (2% in 1995 and 3% in 1999; not a significant difference).

Use of technology. An appropriate use of technology, especially computers and graphing calculators, is recommended in Principles and Standards. The authors explain, “The availability of technology—in the form of computers and scientific or graphing calculators—allows middle-grades students to deal with ‘messy,’ complex problems” (NCTM, 2000, p. 258).

Lessons in the 1995 and 1999 samples that used computers, graphing calculators, or computational calculators (i.e., those not equipped with a graphing capability or whose graphing capability was not used) at any point were marked as such. Lessons were not included if technology was present in the classroom but not used. Four percent of the lessons in 1995 involved computers, and the same percentage involved graphing calculators. In 1999, only 1% of the lessons involved computers and 6% involved graphing calculators. More common was the use of computational calculators; these were used in 32% of lessons in 1995 and 39% in 1999. As these percentages indicate, frequency of technology use did not differ significantly between the 1995 and 1999 samples.

Summary. Did the eighth-grade classrooms in the United States engage in problem solving as recommended in the Grades 6–8 section of Principles and Standards? This question is difficult to answer because, like most pedagogical features, the presence of problem solving is a matter of degree, and its enactment depends on the interaction of multiple features. To further complicate the question, Principles and Standards does not suggest that every mathematics problem present an extended and authentic challenge for the students. The real question is whether classroom practices reveal the degree and nature of problem solving recommended in Principles and Standards and whether this has changed over time. With regard to change, the results are mixed. For several indicators, including the connection to real-life contexts, the use and examination of alternative solution methods, and problems with multiple solutions, the percentage of lessons in which they appeared was

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\(^{10}\)The data do not address whether problems with multiple answers and/or multiple forms of the answer were posed. Low numbers might reflect a choice not to publicly discuss these issues, or a lack of opportunity to do so.
higher in 1999 than in 1995. In each of these cases, however, the percentage of problems per lesson in which they appeared remained the same. Perhaps a greater percentage of teachers are incorporating these features into their lessons but only on occasion. We will let the reader judge how closely these practices are aligned with the Standards.

Reasoning and Proof

According to Principles and Standards, a variety of mathematical reasoning skills should be developed in the middle grades. For example, “students should sharpen and extend their reasoning skills by . . . using inductive and deductive reasoning to formulate mathematical arguments . . . [and] students should . . . formulate generalizations and conjectures about observed regularities” (NCTM, 2000, p. 262). To this end, appropriate tasks include those that “require the generation and organization of data to make, validate, or refute a conjecture” (NCTM, 2000, p. 265).

Deductions, rationales, generalizations, and counterexamples. A group of mathematicians and teachers of postsecondary mathematics judged the degree to which mathematical reasoning took place in the lessons. This group previously analyzed a randomly selected subsample of 30 U.S. lessons from the 1995 sample (Manaster, 1998). For the 1999 study, they analyzed a randomly selected subsample of 20 U.S. lessons. In both cases, written records of the lessons were used to judge the nature and quality of their mathematical content. Written records were used rather than videotapes because the mathematics expert group and the research team wished to have judgments made that were “blind” to the country of origin and unbiased by cultural and nonessential contextual conditions. Consequently, written records were prepared of each subsample lesson with all country-specific references removed or disguised.

One kind of mathematical reasoning of immediate interest is deductive reasoning, a hallmark of mathematics. The group of mathematics experts defined deductive reasoning as deriving conclusions from stated assumptions using a logical chain of inferences. According to the group’s judgments, none of the 1995 lessons and just two of the 1999 lessons contained deductive reasoning. In an effort to uncover other forms of mathematical reasoning that might be found in the 1999 sample of eighth-grade mathematics lessons, the group defined three additional reasoning skills that they had not looked for in the 1995 sample:

- Developing a rationale: Explaining or motivating, in broad mathematical terms, a mathematical assertion or procedure.
- Generalizations: Recognizing that several examples share more general properties.
- Counterexamples: Finding one example that does not work to prove that a mathematical conjecture cannot be true.

None of the U.S. lessons in the 1999 Video Study showed evidence of developing a rationale, making generalizations, or using counterexamples, by the teacher or the students.
Proofs. All the mathematics problems in the complete 1995 and 1999 samples of lessons were examined by the research coding team for whether they involved proofs. A problem was coded as involving a proof if the teacher or students verified or demonstrated that the result must be true by reasoning from the given conditions to the result using a logically connected sequence of steps. Each potential proof was checked by a mathematics expert who was familiar with the coding definition.\textsuperscript{11} None of the U.S. lessons in either the 1995 or 1999 Video Study contained problems considered to be proofs.

Summary. Results on the engagement of students in mathematical reasoning indicate that special forms of mathematical reasoning did not occur in the U.S. lessons. No mathematics problems in any lesson involved proofs and, in the subsample examined by the group of mathematics experts, most special forms of reasoning were absent. Based on these findings, it is unlikely that the kind of mathematics reasoning recommended in \textit{Principles and Standards} for Grades 6–8 is occurring in typical U.S. eighth-grade mathematics classrooms.

Communication

\textit{Principles and Standards} recommends that middle school students take an active role in mathematical communication during lessons, such as presenting and discussing alternative solution methods. “Each student should be expected not only to present and explain the strategy he or she used to solve a problem but also to analyze, compare, and contrast the meaningfulness, efficiency, and elegance of a variety of strategies” (NCTM, 2000, p. 268). As described earlier, problems with alternative solution methods presented were relatively rare in both samples. Each problem of this type was analyzed further to determine whether students suggested any of the solution methods. When alternative solution methods were presented, students suggested at least one of the methods in just under half (46\%) of these problems in 1995, and in approximately two thirds (67\%) of these problems in 1999. The difference was not statistically significant. However, there was a significant increase in the percentage of lessons containing at least one problem with a student suggested alternative solution method—from 11\% to 26\%.

The 1999 results, in particular, indicate a relationship between encouraging or accepting alternative solution methods for problems and engaging students in presenting such methods. Students presented at least one of these methods for the majority of problems for which alternative methods were presented.

Connections

\textit{Principles and Standards} recommends that teachers help students make connections among the mathematical ideas presented in a lesson. The document explains that “as middle-grades students encounter diverse new mathematical content, they

\textsuperscript{11} Several members of the mathematics code development team had advanced degrees in mathematics and were considered to be mathematics experts.
have many opportunities to use and make connections” (NCTM, 2000, p. 274).

Relationships among problems. One way to assess the mathematical connections that might be addressed during a lesson is to examine the relationships among the mathematics problems presented during the lesson. If the problems are related in mathematically important ways, then the potential exists for connections to be made as teachers and students work through the problems. Because mathematics problems carried the bulk of the mathematical content in both 1995 and 1999 (more than 80% of lesson time was spent working on problems), exploring the mathematical relationships among problems within a lesson captures connections in the flow of the content through the lesson.

Each problem was coded as mathematically related, thematically related, repetition, or unrelated as defined below.

- **Mathematically related**: The problem was related to a preceding problem in the lesson in a mathematically significant way. This included using the solution to a previous problem to solve this problem, extending a previous problem by requiring additional operations to solve this problem, highlighting some operations of a previous problem by considering this problem as a simpler example, or elaborating a previous problem by solving this similar problem in a different way.

- **Thematically related**: The problem was related to a preceding problem only by virtue of it being a problem of a similar topic or a problem treated under a larger cover story or real-life scenario. This could occur, for example, when finding various measures of central tendency (mean, median) or when finding the volume and surface area of a geometric figure. If the problem was mathematically related as well, it was coded only as mathematically related.

- **Repetition**: The problem was the same, or mostly the same, as a preceding problem in the lesson. It required essentially the same operations to solve although the numerical or algebraic expression might be different.

- **Unrelated**: The problem was none of the above. That is, the problem required a completely different set of operations to solve than previous problems and was not related thematically to any of the previous problems in the lesson.

Figure 3 shows that in both 1995 and 1999 the majority of the problems per lesson were repetitions (75% and 68%, respectively). Relatively few problems were related in mathematically significant ways. Most lessons, however, contained at least a pair of problems that were related in a mathematically significant way (80% and 82%, respectively).

These results indicate that the relatedness among problems in a lesson seems to be largely achieved through repetition. Of course, other significant relationships might have been contained within individual problems or injected by the teacher (or the students). But the results above suggest that significant mathematical relationships rarely were built into the sequence of the problems that were presented.
Application problems. Individual mathematics problems can support students’ construction of mathematical connections by asking them to apply what they learn in one context to another context. Principles and Standards recommends helping sixth- through eighth-grade students make connections by solving relevant, applied problems. “Rich mathematical tasks prompt students to use and develop mathematical understandings and connections. Challenging problems encourage students to think about how familiar concepts and procedures can be applied in new situations” (NCTM, 2000, p. 275).

In the TIMSS 1995 and TIMSS 1999 video samples, problems were identified as applications if they asked students to apply procedures learned in one context to a (at least slightly) different context. Application problems required students to make decisions about how and when to use procedures. In this sense, applications were, by definition, more conceptually demanding than routine exercises following a prescribed set of procedures. In both samples, about a third of the problems per lesson (34% at both time points), on average, were applications. This means that a majority of mathematics problems presented during a lesson required students to execute prescribed procedures in a familiar manner. However, over half of the lessons in 1995 and 1999 included at least one appli-
cation problem (63% and 68%, respectively), and the difference was not statistically significant.

Processes involved in presenting and working on problems. The two indicators of mathematical connections considered to this point have relied on the inferences that can be made from the mathematical problems as presented during a lesson. A more direct measure of whether connections were made among mathematical ideas, facts, and procedures, or between mathematics and other areas was obtained by analyzing the way in which problems actually were worked on publicly by the teacher and students. Specifically, did the teacher or students publicly discuss mathematical relationships?

The benefit of analyzing teaching practice rather than just inferring such practice from teachers’ self-reports or from written curricula materials is seen most vividly in this kind of analysis. Mathematics problems can appear to suggest the development of mathematical relationships, but not necessarily be worked on or implemented in this way. For example, teachers can transform challenging problems into rather routine exercises. Conversely, while working on a routine exercise teachers can pose questions that transform the exercise into one that encourages students to construct important mathematical relationships.

The way in which connections get developed or become hidden when solving problems is revealed by following the life of problems that are publicly discussed during the lesson (so that full information about the problem is available). To capture this potential transformation of problems, each problem that was discussed publicly was coded twice, once for its apparent intent when presented to students and once for the way in which it was worked on with students. First, the problem was classified into one of the following three categories based on the mathematical processes that were implied by the problem statement:

- **Using procedures:** Problem statements that suggested the problem would be solved by applying a known procedure or set of procedures. An example is, “Calculate the length of the hypotenuse of a right triangle given the length of two sides.”
- **Stating concepts:** Problem statements that called for a mathematical convention or an example of a mathematical concept. An example is, “Draw a polygon that is not convex.”
- **Making connections:** Problem statements that implied the problem would focus on constructing relationships among mathematical ideas, facts, or procedures. An example is, “Graph the equations \( y = 2x + 3 \), \( 2y = x - 2 \), and \( y = -4x \), and examine the role played by the numbers in determining the position and slope of the associated lines.”

In both 1995 and 1999, the majority of problem statements in a lesson, on average, implied that students would solve the problem simply by using procedures (60% and 69%, respectively; not a significant difference) as opposed to stating concepts or connecting mathematical ideas (see Table 4). Those problem statements that did not imply using procedures were about evenly split between stating concepts and making connections.
These same problems (that were discussed publicly) then were classified into one of four categories based on the mathematical processes that were made explicit during the problem-solving phase:

- **Giving results only**: The public work consisted solely of stating an answer to the problem without any discussion of how or why it was attained.
- **Using procedures**: The problem was completed algorithmically, with the discussion focusing on steps and rules rather than underlying mathematical concepts.
- **Stating concepts**: Mathematical properties or definitions were identified while solving the problem, with no discussion about mathematical relationships or reasoning.
- **Making connections**: Explicit references were made to the mathematical relationships and/or mathematical reasoning involved while solving the problem.

As shown in Table 4, a small percentage of problems were implemented in a way that made mathematical connections visible for the students. The majority of problems in both samples fell into the categories of giving results only or using procedures (75% and 91%, respectively). In other words, when solving most problems, the focus was not on the underlying mathematical concepts or relationships, but rather on the algorithms and procedural steps involved.

Table 4

<table>
<thead>
<tr>
<th>Type of problem statement and implementation</th>
<th>Percentage of problems</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type of problem statement</strong></td>
<td></td>
</tr>
<tr>
<td>Using procedures</td>
<td>60</td>
</tr>
<tr>
<td>Stating concepts</td>
<td>17</td>
</tr>
<tr>
<td>Making connections</td>
<td>23</td>
</tr>
<tr>
<td><strong>Type of problem implementation</strong></td>
<td></td>
</tr>
<tr>
<td>Giving results only*</td>
<td>18</td>
</tr>
<tr>
<td>Using procedures</td>
<td>57</td>
</tr>
<tr>
<td>Stating concepts*</td>
<td>23</td>
</tr>
<tr>
<td>Making connections</td>
<td>3</td>
</tr>
</tbody>
</table>

* Indicates a significant difference between 1995 and 1999.

Even problems whose initial statement implied making connections were rarely implemented in this manner. In other words, problem statements that implied a focus on mathematical connections and relationships were completed by applying procedures, stating concepts, or giving results only (see Figure 4). The percentage of problems stated as making connections and then solved by making connections decreased significantly from 8% in 1995 to 0% in 1999, whereas those solved by giving results only rose significantly from 15% to 33%.
Summary. The picture that emerges from the indicators on the mathematical connections afforded during the U.S. lessons suggests that little structure is provided, and few opportunities exist, for students to construct relationships among mathematical ideas. Mathematics problems are mostly repetitions that require executing familiar procedures. The emphasis during lessons is on practicing skills. Even problems that appear to request some attention to conceptual underpinnings are worked on so that such connections are not made explicit for the students.

Representations

*Principles and Standards* encourages middle school teachers to incorporate a variety of mathematical representations in their lessons. “Representation is central to the study of mathematics. . . . Representations—such as physical objects, drawings, charts, graphs, and symbols—also help students communicate their thinking” (NCTM, 2000, p. 280).

For each problem in both the 1995 and 1999 sample of lessons, the presence or absence of the following representations was coded: physical materials, drawings
or diagrams, tables, and graphs. Physical materials included measuring instruments (e.g., rulers, protractors, compasses), special mathematical materials (e.g., tiles, tangrams, base-ten blocks), geometric solids, and cut-out plane figures. Papers, pencils, calculators, and computers were not included in this analysis. The materials must have been used or manipulated by the teacher or student(s) when presenting or solving the problem, not simply present in the classroom. A drawing or diagram must have included information relevant for solving the problem. Excluded were motivational diagrams that lacked such information (e.g., a photo of an Olympic runner that accompanied a story problem on race times). A table was defined as an arrangement of numbers, signs, or words that exhibited a set of facts or relations in a definite, compact, and comprehensive form. Typically, a table contained rows and/or columns that were labeled and had borders. Graphs included statistical displays such as bar graphs and line graphs.

The majority of lessons in both samples included at least one problem that incorporated physical materials, drawing/diagrams, tables, or graphs. Significantly more lessons in 1999 (87%) contained at least one problem with a representation than in 1995 (60%). However, in both 1995 and 1999, similarly small proportions of problems per lesson incorporated physical materials, tables, or graphs (see Table 5). Significantly more problems included drawings or diagrams in 1999 (26%) than in 1995 (13%).

Table 5
Average Percentage of Problems Per Lesson That Incorporated Each Type of Representation: 1995 and 1999

<table>
<thead>
<tr>
<th>Type of representation</th>
<th>Percentage of problems</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1995</td>
</tr>
<tr>
<td>Physical materials</td>
<td>7</td>
</tr>
<tr>
<td>Drawings or diagrams*</td>
<td>13</td>
</tr>
<tr>
<td>Tables</td>
<td>13</td>
</tr>
<tr>
<td>Graphs</td>
<td>8</td>
</tr>
</tbody>
</table>

* Indicates a significant difference between 1995 and 1999.

DISCUSSION

The majority of the eighth-grade teachers who participated in the Video Studies responded in questionnaires that they were familiar with “current ideas,” and also had specific knowledge of the NCTM Standards. Most teachers noted that they had attended NCTM conferences or read its publications. These responses are consistent with findings from the larger TIMSS studies conducted in 1995 and 1999, showing that more than 80% of U.S. eighth-grade mathematics students were taught by teachers who reported being “fairly familiar” or “very familiar” with the NCTM Standards, with no significant changes over time (Burian-Fitzgerald, McGrath, & Plisko, 2003). The 2000 National Survey of Science and Mathematics Education provides confirming evidence of teachers’ familiarity with and support for the
Standards (Whittington, 2002). Like the teachers in the Video Studies, most of these respondents also claimed that they implement the Standards, at least to some extent.

The picture drawn from the Video Studies suggests, however, that classroom practice is not consistent with the Grades 6–8 Process Standards of Principles and Standards. Although some teaching practices recommended by these standards were observed in some lessons, the typical eighth-grade classroom displays teaching at odds in many respects with the recommendations. Below is a summary of the findings from the U.S. classrooms in the Video Studies with respect to each of the five Process Standards:

- **Problem Solving.** Working on mathematics problems was the primary classroom activity; some time was devoted to individual and small-group work and some time to whole-class work; the majority of private work time was spent practicing already learned procedures; a relatively large number of problems were solved in each lesson, many quite quickly; most problems were of low procedural complexity; a number of lessons contained an occasional connection to real life and included a few instances of presenting and analyzing alternative solution methods; and technology did not play a major role in enriching problem solving opportunities.

- **Reasoning and Proof.** Deductive reasoning and other special forms of mathematical reasoning were rarely evident.

- **Communication.** When alternative solution methods were presented, students often participated in presenting them. This occurred in a number of lessons but on a relatively small percentage of problems.

- **Connections.** Relatedness among problems presented in a lesson was achieved largely through repetition; most problems were routine exercises rather than applications; and most problems were stated and solved with an emphasis on executing procedures and obtaining the correct result rather than making connections, even those whose problem statements implied an emphasis on making connections.

- **Representations.** Most problems did not incorporate representations other than conventional mathematical symbols, although a majority of lessons did include at least one problem involving physical materials, drawings or diagrams, tables, or graphs.

Judging the alignment of classroom practices with the recommendations in Principles and Standards is difficult because such judgments require decisions about degree or frequency of occurrence. But in our judgment, and, we suspect, that of most readers, the practices reported here are not well aligned with the recommendations. In fact, it would be a mistake to assume that Principles and Standards, at least with regard to the Grades 6–8 Process Standards, describes the mathematical experiences of most U.S. eighth graders. The nature of classroom mathematics teaching observed in the videotapes reflects the kind of traditional teaching that has been documented during most of the past century (Cuban, 1993; Fey, 1979; Hoetker
& Ahlbrand, 1969; Welch, 1978), more so than the kind of teaching recommended in *Principles and Standards*. Traditional teaching, as described in the cited reports, is characterized by brief demonstrations of mathematics procedures followed by practice on many similar problems. Particularly striking in the 1995 and 1999 videotaped mathematics lessons was the lack of emphasis on mathematical reasoning and conceptual development, and the relatively strong emphasis on applying familiar procedures to a repetitive series of similar problems.

What has changed between 1995 and 1999? Evidence that classroom teaching was more aligned in 1999 than in 1995 with the Grades 6–8 Process Standards in *Principles and Standards* was found on 14% of the measures presented in this article. These differences are summarized in Table 6. As seen in Table 6, a number of the significant differences between 1995 and 1999 occurred at the lesson level, meaning that more lessons included at least one problem with a particular characteristic (such as connected to real life). In general, similar differences were not found in the percentages of problems. The data indicate that teachers might be changing specific features of their practices, perhaps by occasionally

<table>
<thead>
<tr>
<th>Table 6</th>
<th>Significant Differences in Teachers’ Implementation of the NCTM Standards Between 1995 and 1999</th>
</tr>
</thead>
<tbody>
<tr>
<td>Higher percentage in 1999</td>
<td>Lower percentage in 1999</td>
</tr>
<tr>
<td>Problem solving</td>
<td></td>
</tr>
<tr>
<td>Moderately complex problems</td>
<td>Problems worked on for at least 45 seconds</td>
</tr>
<tr>
<td>Lessons with a problem connected to real life</td>
<td></td>
</tr>
<tr>
<td>Lessons with a problem with student choice of solution methods</td>
<td></td>
</tr>
<tr>
<td>Lessons with an examining methods problem</td>
<td></td>
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<tr>
<td>Lessons with a problem with multiple answers presented</td>
<td></td>
</tr>
<tr>
<td>Communication</td>
<td></td>
</tr>
<tr>
<td>Lessons with a problem with a student suggested alternative solution method</td>
<td></td>
</tr>
<tr>
<td>Connections</td>
<td></td>
</tr>
<tr>
<td>Problems implemented by giving results only</td>
<td>Problems implemented by stating concepts</td>
</tr>
<tr>
<td>Problems stated as making connections and implemented by giving results only</td>
<td>Problems stated as making connections and implemented by making connections</td>
</tr>
<tr>
<td>Representations</td>
<td></td>
</tr>
<tr>
<td>Problems incorporating a drawing or diagram</td>
<td></td>
</tr>
<tr>
<td>Lessons with a problem incorporating a representation</td>
<td></td>
</tr>
</tbody>
</table>

*Note.* In the table above, “lessons” refers to the percentage of lessons with at least one of the designated problem types, and “problems” refers to the average percentage of mathematical problems per lesson of the designated type.
incorporating a new type of mathematical problem into their lessons by, for example, connecting it to a real-life situation or by asking for multiple (correct) answers. However, such changes were seen only occasionally and only on some indicators. Whether these patterns reflect a trend that will continue to grow over time can be answered only by collecting national-level classroom data at regular intervals in the future.

Not all differences found between the 1995 and 1999 lessons brought classrooms more in line with the Grades 6–8 Process Standards in Principles and Standards. In some respects, teachers moved away from recommendations found in Principles and Standards. For example, students in the 1999 lessons worked on more problems that took less than 45 seconds to complete and solved more problems by simply giving the result. A possible explanation is that teachers are making trade-offs in order to occasionally try something new in their lessons, such as allowing students to present an alternative solution method.

In our judgment, many of the features of teaching that show some alignment with Principles and Standards, whether in 1995 or in 1999, are features being implemented at the margins of teaching rather than at its core. Indicators such as the percentage of mathematics problems that are embedded in real-life contexts or that use physical materials, whether students present alternative solution methods, and the percentage of problems presented with the apparent intent of making connections among mathematical facts, procedures, and ideas, can signal deeper changes in teaching or changes at the margins. We interpret the evidence to suggest that, to this point, the changes are at the margins. The nature of mathematical thinking and reasoning, and the conceptual mathematical work, remain unaligned with the intent of Principles and Standards. This is not surprising. Changes in U.S. classrooms usually are at the margins (Cuban, 1993).

Although the fact that U.S. eighth graders still spend most of their time practicing routine skills might not be surprising in light of historical trends and other research reviewed earlier, this information has implications for U.S. policy debates. As educators search for explanations for students’ levels of mathematics achievement, perceived by many to be too low, it is important to recognize what is happening in typical classrooms. If changes are recommended, they must address the realities of current practice.

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