Precautionary Saving and the Transmission of
Monetary Policy

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Abstract
A majority of households in the US does not hold interest bearing assets and their saving behavior is counter-cyclical. Using the Survey of Consumer Finances, I document precautionary saving in the economy with a new measure, \textit{checking account balance to income} ratio. I find that (a) an augmented medium-scale NK model with a precautionary saving motive can match this ratio well; (b) precautionary saving lowers the relative importance of the direct effect of monetary policy; (c) the precautionary saving mechanism leads to lower inflation during economic recoveries; and (d) an extension with downward rigid wages is able to produce an asymmetric response of the economy to monetary policy in line with the recent literature.

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1 Introduction

This paper explores the importance of two empirical facts for monetary policy. First, only a minority of households own interest bearing, liquid investments such as bonds, stocks or money market mutual funds. Second, for these same income groups, money demand is counter-cyclical and the checking account serves as their main savings vehicle. Together, these two facts contradict the dominant transmission channel of monetary policy in standard macro models.

Typically, in a representative agent New Keynesian model, monetary policy works through the direct intertemporal substitution channel: Kaplan, Moll and Violante (2016) find that this channel is responsible for up to 95% of the total effect of policy changes. Furthermore, economists have long known that representative agent models are inconsistent with both aggregate data and household behavior. For example, Campbell and Mankiw (1990) show that a model in which half of all agents are hand-to-mouth consumers aligns best with aggregate data on consumption, income and interest rates. Vissing-Jørgensen (2002) provides evidence that households who do not own bonds or stocks do not react to interest rate changes. These results have not gone unnoticed and most major models used by central banks and the OECD feature some version of hand-to-mouth or borrowing-constrained agents.\footnote{Lindé, Smets and Wouters (2016) provide an overview of commonly used NK models by institutions and lessons learned from the fit of these models during and after the Great Recession.}

Yet, the clear counter-cyclical savings behavior of a large subset of households who also do not own financial wealth suggests that simply adding hand-to-mouth consumers is not the answer. Intuitively, a given monetary policy intervention is less effective in an economy in which a subset of agents does not react to interest rate changes but at the same time, adjusts consumption and saving in the opposite direction. For example, precautionary saving amplifies recessions when households cut back consumption in response to...
higher labor market risk. A monetary policy intervention that lowers interest rates will then not change this response directly if households are not exposed to short-term interest rates.

Therefore, I ask four questions: How prevalent is precautionary saving in the data? How well does a New Keynesian model augmented with households that save in response to labor market conditions match this data? To what degree does the inclusion of these households change the transmission of monetary policy? And, in an application, can these models address the recent issues of a persistently low inflation rate despite a near zero target interest rate and thus seemingly low effectiveness of monetary policy?

Measuring precautionary saving is not straightforward. Kaplan and Violante (2014), Kaplan, Moll and Violante (2016) and many others solve non-linear models to match the wealth distribution with different asset categories. One drawback is that these models have a limited state space and the contribution of portfolio reallocation, asset prices and precautionary saving over the business cycle is hard to disentangle (as discussed in Kaplan, Moll and Violante (2016)). Generally, these models also focus on the stationary equilibrium or short time frames, such as the Great Recession.

Carroll, Sommer and Slacalek (2012) estimate a partial equilibrium model of bufferstock saving to match the time series of the aggregate saving rate in the US economy and separate the contribution of unemployment risk, wealth shocks and credit accessibility. They find that unemployment risk is important for business cycle variation in the savings rate while wealth and credit accessibility explain its long run trend.

I propose as a measure of precautionary saving that closely tracks voluntary saving over the business cycle the checking account balance to income ratio. Disaggregated data from the Survey of Consumer Finances from 1989 to 2013 shows that most households do not own financial assets such as

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3Kaplan and Violante (2014) fix the inflation rate, for example, though their goal is to estimate the MPC in response to tax refunds and not monetary policy.
bonds or stocks, but primarily have access to a checking account. For these households, movements in this ratio over time will likely reflect a true saving motive. Sorting households by income quintile, I find that this ratio divides the US population into three groups: The top 20% who do not display any business cycle pattern; the middle 30% whose ratio moves strongly counter-cyclically; and households with a fairly stable, slightly pro-cyclical ratio, the bottom 40%.\(^4\) The checking account balance to income ratio is a simple quantitative statistic that can be used to validate the performance of heterogeneous agent models over long business cycle frequencies.

Another recent study that aims to quantify precautionary saving is Krueger, Mitman and Perri (2016). They use the change in consumption over the change in income between 2006 and 2010 as an indicator for saving by income quintile. A negative change in this ratio, however, is not the same as precautionary saving and can reflect involuntary saving, such as forced deleveraging or debt with variable interest rates, two common occurrences over that period.

How well estimated general equilibrium models track precautionary saving is an open question. Bufferstock or precautionary saving\(^5\) amplifies business cycle movements (Challe and Ragot (2016), Ravn and Sterk (2017), Krueger, Mitman and Perri (2016)), is able to solve the forward guidance puzzle in NK models (Mckay, Nakamura and Steinsson (2016)) and has strong implications for the transmission of monetary policy (Werning (2015), Aucclert (2017), Kaplan, Moll and Violante (2016)). Yet, of the two models that are closest to this paper, Challe and Ragot (2016) use the ratio of consumption of the bottom 60% to the top 40% as an estimation target and assume

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\(^4\)The lowest 10% have very volatile income and account balances and were excluded from the sample.

\(^5\)For the purpose of this paper these terms are interchangeable though technically they represent two separate types of saving. The focus of this paper are models linearized around the steady state and therefore saving is of the bufferstock type. The key determinant of saving, however, is income / labor market uncertainty and intuitively saving can be called precautionary.
that agents save just enough to cover one period of consumption, whereas Ravn and Sterk (2017) compare the vacancy rate in the model to the data and find it to be a good match, but do not track saving explicitly.

To test how well these models of precautionary saving match the data, as my benchmark case, I augment a standard medium scale New Keynesian model in the mold of Smets and Wouters (2007) and Christiano, Eichenbaum and Evans (2005) with the precautionary saving mechanism developed by Carroll and Toche (2011). Wages are sticky and follow an exogenous wage rule that creates involuntary unemployment. Agents are of two types: traders who are insured against income loss through unemployment and limited agents that face an absolute income loss upon permanent unemployment. This risk then gives rise to the precautionary saving motive for limited agents.

This benchmark model abstracts from any additional amplification mechanism such as wage bargaining, hiring frictions or government stabilizers. Investigating the effect of an amplification mechanism is a worthwhile research question in itself, yet any such extension should also be able to explain observed saving behavior. Comparing the savings ratio generated by the benchmark model to the data will therefore provide important guidance for further extensions.\(^6\)

I find that saving in the benchmark model qualitatively tracks the data well, and provides a quantitative match until about the mid 2000s. In the run up to and aftermath of the Great Recession the implied saving ratio is too large by several percentage points. This gap between observed and implied saving shows (as many have before) how unique the last decade has been from the perspective of the business cycle literature.

Next, I turn to the transmission mechanism of monetary policy in the benchmark model. As a general result for models with incomplete markets, Werning (2015) derives conditions that determine how the total effect of

\(^6\)The two papers mentioned before incorporate a similar saving mechanism, but Challe and Ragot (2016) do not discuss monetary policy, and Ravn and Sterk (2017) only use labor as a factor in production.
monetary policy varies in comparison to a representative agent model\textsuperscript{7}. Yet, the focus of this paper is not the total effect but the relative importance of channels other than intertemporal substitution for the transmission of monetary policy. A recent contribution to this literature is Auclert (2017) who evaluates the amplification of monetary policy through three redistribution channels if agents have heterogeneous marginal propensities to consume.\textsuperscript{8}

The estimated benchmark model showcases a strong labor market channel of monetary policy and the contribution of the direct effect of monetary policy is reduced compared to an estimated representative agent version of the model. Furthermore, the benchmark model relies less on nominal and real frictions. In particular, capital adjustment cost and habit persistence, two parameters that are necessary to prohibit excessive consumption smoothing in a standard NK model are much lower. When the the labor market is kept fixed at its steady state values, I find that the response of uninsured agents to a monetary policy shock is several times smaller than in the benchmark model with time varying unemployment risk. For insured agents, I find no significant change in their response, in line with the criticism of Kaplan, Moll and Violante (2016).

The benchmark model features segmented asset markets and is related to the literature of Baumol-Tobin style models with monetary policy. These models are able to generate persistent liquidity effects following interest rate shocks even in the absence of nominal frictions (Alvarez, Atkenson and Edmond (2009) and Khan and Thomas (2014)). While the saving mechanism is fundamentally different from models with bufferstock saving, the addition of segmented asset markets solves to a large degree the empirical issue raised by Canzoneri, Cumby and Diba (2007), namely that representative agent model

\textsuperscript{7}For example, if income, liquidity and borrowing constraints of heterogeneous agents are proportional to output, the total effect of monetary policy will be the same, a result that is true in my model.

\textsuperscript{8}Another paper, McKay, Nakamura and Steinsson (2016), shows that models of bufferstock saving can resolve the forward guidance puzzle, the excessive response of representative agent models to forward guidance over long time horizons.
implied real interest rates are negatively correlated with aggregate, observed real interest rates. I show that the real interest rate in the benchmark model is markedly different from the implied aggregate rate and moves in the opposite direction at the onset and end of recessions. The resulting graph looks remarkably similar to the figure in Canzoneri, Cumby and Diba (2007).

As an application, I investigate to what degree the benchmark model can explain the occurrence of persistently low inflation since the Great Recession despite a low interest rate and good economic growth. Compared to a representative agent version of the model, a negative aggregate demand shock that both lowers output and inflation implies a lower interest rate and longer transition of inflation back to the steady state in the benchmark model. I further show that this result is strengthened in an extension of the benchmark model with downward rigid nominal wages.\(^9\) In addition, investment responds less to monetary policy in the extended model, a finding that is in line with recent evidence on asymmetric monetary policy in Tenreyro and Thwaites (2016).

The paper proceeds as follows: Section 2 discusses precautionary saving in the data. The model is described in Section 3. The estimation results and answers to the questions posed in the introduction are in Section 4. Section 5 concludes.

\(^9\)I choose this extension because the literature has mostly focused on hiring frictions and downward rigid nominal wages are one of the defining features of the last decade, see Schmitt-Grohé and Uribe (2013).
2 Household Data

Designing New Keynesian models with heterogeneous agents raises two empirical questions: What fraction of households displays a precautionary savings motive? And, what metric can be used to evaluate whether such a model captures the precautionary saving behavior of these consumers?

I use disaggregated data from the Survey of Consumer finances to answer these questions. Three facts emerge: First, few households own assets with interest rates that are closely linked to the Federal Funds Rate. Second, the checking account is their main vehicle for liquid saving, and third, the savings behavior of a large fraction of households, about 30% of the population, is strongly counter-cyclical, especially during the Great Recession.

The triennial Survey of Consumer Finances (SCF) is a representative survey of the financial position of all households in the United States. I use data from the years 1989 to 2013, discard the top 5% of households by net worth to remove outliers and restrict the sample to households age 22 to 79\(^{10}\). There are two benefits to exploring the question of precautionary saving using SCF data. First, survey questions are consistent going back to 1989 and therefore yield a longer time series than other data sets. Second, the SCF is very comprehensive and splits assets and liabilities into many different categories such as checking account balance, bonds, stocks, car loans and credit card debt. This disaggregation is not available in the Consumer Expenditure Survey of the BLS nor the PSID.

In my analysis, I propose the average checking account balance to income ratio as a measure of saving as it is more likely to track actual saving. Changes in the consumption to income ratio, for example, do not necessarily imply precautionary saving but can also reflect other forms of saving such as involuntary deleveraging. Intuitively, households are less likely to increase their checking account balance relative to income when they are

\(^{10}\text{I follow Kaplan and Violante (2014) in deciding on how to restrict the data.}\)
forced to pay down debt or have to fulfill other financial obligations that vary counter-cyclically with the business cycle. On the contrary, one would expect households to decrease their checking account balance under those circumstances. I therefore assume that an increase in the average checking account balance to income ratio reflects voluntary saving out of current income. Nevertheless, my findings are consistent with Krueger, Mitman and Perri (2016) who use PSID and Consumption Expenditure Survey data to measure household saving between 2006 and 2010 as changes in the ratio of consumption over income.

2.1 Ownership of financial assets

Table 1 shows the percentage of households in the SCF that own either bonds, stocks or invest into money market mutual funds by income quintile in the year 2001\textsuperscript{11}, the year with the highest fraction of asset ownership for the top four quintiles, see figure 1. Column two lists the medium income in each quintile, and column three the number of households that hold any of these investments in percent. The next two columns show the average investment size for each quintile as well as the median investment. The last column shows the amount invested by the 90th percentile within each quintile.

Two facts stand out: First, investment into bonds, stocks or money market mutual funds increases with income, but even in the highest income quintile, only about 50% of households hold these investments. Second, for all but the highest income group, the amount invested even at the top within each quintile is not very large. For example, in the lowest income quintile, the average investment is $1697; but even the 90th percentile has an investment of $0. Looking further up the income distribution, in the fourth quintile, of which 31% of households hold investments, the amount invested by the highest ranked households in that quintile is relatively modest and just roughly half of the quintile median income. Only for the highest income quintile do

\textsuperscript{11}Most of this data was collected in the year 2000
Table 1: Asset Holding by Income Quintile in 2001

<table>
<thead>
<tr>
<th>Quint</th>
<th>Med Inc</th>
<th>Perct Owning</th>
<th>Mean Asset</th>
<th>Med Asset</th>
<th>90th Ptile</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8741.14</td>
<td>6</td>
<td>1696.86</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>20858.90</td>
<td>14.1</td>
<td>4470.19</td>
<td>0</td>
<td>3400</td>
</tr>
<tr>
<td>3</td>
<td>34852.51</td>
<td>21.6</td>
<td>8252.77</td>
<td>0</td>
<td>12000</td>
</tr>
<tr>
<td>4</td>
<td>53451.36</td>
<td>31</td>
<td>12911.43</td>
<td>0</td>
<td>25000</td>
</tr>
<tr>
<td>5</td>
<td>94567.78</td>
<td>47</td>
<td>29704.99</td>
<td>0</td>
<td>90000</td>
</tr>
</tbody>
</table>

The percentage of households that hold investments in bonds, stocks or money market mutual funds increases with income quintile, but is very low. Mean total investments in these three categories never exceeds the median income in each category. Data is taken from the SCF 2001.

Financial investments represent a meaningful fraction of median income.

The data does not suggest that households who do not own financial investments are not exposed to any interest rate at all. They have mortgages, car loans, credit card debt and retirement accounts. The interest rates on these common liabilities, however, move slowly and do not adjust quickly. For example, it took several years at the zero lower bound for the average credit card or mortgage interest rate to decrease by just one percentage point, as shown in figures A.3 to A.5 in the Appendix. Secondly, retirement accounts are often subject to penalties and households rarely use these for consumption smoothing.\footnote{12Kaplan and Violante (2014) discuss why retirement accounts are not an important vehicles of short-term saving.}

### 2.2 Checking account balance

Virtually all households in the SCF own a checking account. Absent financial investments, this account is the main savings vehicle. For the years 1989 to 2013, I compute the ratio of checking account balance to income for all income groups except the bottom 10% who hold very little money in their checking accounts.
account and have volatile income.

I expect the movement of this ratio by income group over time to capture whether or not households voluntarily save or dissave, as argued above. For hand-to-mouth households, this ratio should stay roughly constant. Income groups that engage in precautionary saving should see counter-cyclical movements in this ratio, higher ratios around recessions and lower ratios during expansions. For the highest income groups that hold financial investments, the movement in this ratio is ambiguous since these households are more likely to reallocate their portfolio regularly.

The SCF does not disclose payment frequency and I use total quarterly income (reported total yearly income divided by 4), the usual time period in NK models as denominator. Checking account balance is the average
Using the average checking account balance to quarterly income ratio, all remaining households in the SCF can be divided into three groups by income: the bottom 40% households, the middle 30% and the top 20%. These are shown in figure 2. The only movement in the checking account balance to quarterly income ratio that all the groups have in common is a strong upward movement since the Great Recession.

Apart from this common movement, these three groups behave very differently. The bottom 40% have a fairly stable ratio. In contrast, the middle 30% of households display a ratio that moves counter-cyclically. It is low in the 1990s and increases towards and after the recession and jobless recovery of the early 2000s. Then, during the economic and housing boom of the mid 2000s, their ratio drops sharply, only to increase again in the year 2010 and
2013, following the Great Recession. Lastly, for the top 20% of the income distribution this ratio is continuously increasing, albeit at a faster pace since the financial crisis. This result is robust to the exclusion of households that own financial investments, see figure A.2 in the Appendix.

The advantage of using disaggregated SCF data becomes clear when liquid assets are added to checking account balances, the only category of liquid investment in the PSID, for example. Figure A.1 in the Appendix shows how the few households that own these investments dominate the ratio after aggregation by income group. The variation in the stock market, especially around the year 2000, and thus asset value to income ratio for those households who own stocks at that time dwarfs the movement of savings in checking accounts of all other households. The information on the saving behavior of all other households is therefore lost after averaging across households by income quintile.

Together, the observation that most households do not own liquid financial assets, that interest rates on common liabilities adjust slowly to Federal Funds Rate changes and the clear division of households into three groups by the average checking account balance to income ratio justifies the extension of the canonical New Keynesian model with limited participation in asset markets and labor market risk for a subset of agents.

3 The Model

I extend a standard medium scale New Keynesian model similar to Smets and Wouters (2007) and Christiano, Eichenbaum and Evans (2005) in two ways to capture the low ownership of liquid investments and strong countercyclical savings behavior in the data. First, I add an exogenously sticky wage equation that sets the wage above the market clearing level and creates involuntary unemployment. Second, on the demand side, there are two types of households, traders, fraction \( n^p \) of all households, and non-traders, \( 1 - n^p \).
The setup for traders is similar to the standard representative agent in NK models. They invest into bonds and capital, are subject to investment adjustment cost and choose capital utilization. All traders belong to a large family and are insured against the loss of income in case of unemployment. Non-traders, on the other hand, only participate in the bond market and are subject to irreversible unemployment risk. Unemployment implies zero income until death. These agents’ only option is to consume their savings when unemployed. The risk of unemployment thus creates a precautionary saving motive. These limited agents are modeled following Carroll and Toche (2011).

On the supply side, a competitive final goods firms aggregates intermedi- ate goods into one final good. Intermediate goods are produced by a continuum of monopolistic firms that employ labor and capital services. Prices are set according to Calvo (1983) and indexed to inflation in periods in which an intermediate firm cannot adjust its prices. A monetary policy rule and passive government expenditure equation close the model.

All derivations are detailed in Appendix A.3.

3.1 Final Goods Firm

The final goods firm aggregates the intermediate products into one final good. Each intermediate good is sold at price $P_t(j)$ and the final good has price $P_t$. The production function is

$$y_t = \left( \int y_t(j)^{\frac{\varepsilon - 1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon - 1}}$$

Cost minimization gives the demand function for intermediate good $y_t(j)$ and an expression for the aggregate price level $P_t$, where $\varepsilon$ is price elasticity of demand for good $j$:

$$y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\varepsilon} y_t$$  \hspace{1cm} (1)
\[ P_t = \left( \int P_t(j)^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}} \]  

\section*{3.2 Intermediate Good Firms}

A continuum of identical intermediate good firms use utilized capital \( \hat{K}_t(j) \) and effective labor \( \hat{N}_t(j) \) (both defined further below) as well as technology \( A_t \) to produce output \( y_t(j) \). They minimize cost and maximize profit by choosing the optimal inputs and price \( P_t(j) \) given the demand function (1) and factor prices. Price setting follows Calvo (1983). Firms can reset their prices with probability \( 1 - \phi \) every period. Prices that are not reset are indexed to the inflation rate. The production function is:

\[ y_t(j) = A_t \hat{K}_t(j)^\alpha \hat{N}_t(j)^{1-\alpha} \]

where \( \alpha \) is the share of capital in production.

Cost minimization given factor prices and subject to the demand function of the final goods producer leads to the following first order conditions:

\[ w_t = mc_t(j)(1 - \alpha)y_t(j)\hat{N}_t^{-1}(j) \]
\[ R_t = mc_t(j)\alpha y_t(j)\hat{K}_t^{-1}(j) \]

\[ w_t \text{ and } R_t \text{ are the real factor prices and } mc_t \text{ is the real marginal cost of producing one extra unit of output.} \]

Under Calvo pricing, firms can reset their price \( P_t(j) \) with probability \( (1 - \phi) \) every period. Conversely, with probability \( \phi \), prices are indexed to lagged inflation \( \pi_t^{\xi_p} \) where \( \xi_p \in [0, 1] \). Intermediate firms are owned by traders and future profit is discounted by their stochastic discount factor \( \beta^s \frac{\lambda_{t+\xi_p}}{\lambda_t} \), defined in equation 14 below. Intermediate firms maximize expected future profit given the demand function by choosing the optimal price \( P_t(j) \):
\[
\max_{P_t(j)} \mathbb{E}_t \sum_{s=0}^{\infty} (\beta \phi)^s \frac{\lambda_{t+s}}{\lambda_t} \left( \frac{\pi_{t-1,t+s-1}^\# P_t(j)}{P_{t+s}} y_{t+s}(j) - mc_{t+s}(j) y_{t+s}(j) \right)
\]

s.t. \( y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon} y_t \)

The optimal price, written in terms of optimal price inflation is:

\[
\pi_t^\# = \frac{\epsilon}{\epsilon - 1} \hat{A}_t \pi_t
\]  

(4)

where

\[
\hat{A}_t = \lambda_t y_t + \beta \phi \pi_{t+1}^\# \pi_t^{\epsilon-1} \mathbb{E}_t \hat{A}_{t+1}
\]

\[
\hat{B}_t = \lambda_t m c_t y_t + \beta \phi \pi_{t+1}^\# \pi_t^{\epsilon-\epsilon} \mathbb{E}_t \hat{B}_{t+1}
\]

As usual, without the Calvo friction, \( \phi = 0 \) implies that firms choose prices above nominal marginal cost inflation.

Since all intermediate firms are homogeneous, we can integrate across all intermediaries to get aggregate output \( y_t \)

\[
y_t = \frac{A_t \hat{K}^{\alpha} \hat{N}^{1-\alpha}}{v_t^p}
\]  

(5)

where \( v_t \) is a measure of price dispersion:

\[
v_t^p = \pi_t^{\epsilon} \left( (1 - \phi)(\pi_t^\#)^{-\epsilon} + \pi_t^{-\epsilon} \phi v_{t-1}^p \right)
\]

3.3 Labor market

The link between precautionary saving and the labor market is created via an exogenous sticky wage equation and involuntary unemployment. There is no leisure-labor decision. Both types of households work full time when employed, and do not work any hours when unemployed. Traders, however,
are more efficient than non-traders and provide $\eta > 1$ labor per agent. $\eta$ both reflects the fact that traders earn higher income in the data and hold more wealth. Firms cannot discriminate between workers and, by the law of large numbers, employ both types in the same proportion. Traders earn a wage of $\eta w_t$, and limited agents $w_t$.\(^{13}\)

The exogenous wage rule is adaptive, as proposed by Hall (2005) and an average between last period’s wage and a fixed wage parameter $\bar{w}$. The latter is multiplied by an adjustment factor which varies with the tightness of the labor market and is subject to a wage shock $\epsilon^{z_w}$. In the literature, this wage parameter is usually taken as a short-cut around wage bargaining and equal to the outcome of some bargaining process (see for example Blanchard and Gali (2010)). Here, absent a labor-leisure decision, this wage parameter is calibrated to yield the targeted unemployment rate in steady state by setting the wage above the marginal cost. In real terms, the wage rule is as follows:

$$w_t = \left( \frac{w_{t-1}}{\pi_t} \right)^{\gamma_w} \left( \bar{w} e^{z_w} \left[ \frac{n_t}{n_{ss}} \right] \phi_w \right)^{(1-\gamma_w)} \tag{6}$$

where the parameter $\phi_w \geq 0$ determines by how much the wage adjusts to labor market conditions, and $\gamma_w \in [0, 1]$ the inertia in nominal wages. If $\gamma_w = 1$, nominal wages are perfectly sticky.\(^{14}\) The larger $\phi_w$, the more does the wage react to labor market tightness.

The wage shock follows:

$$z_w^t = \rho_w z_w^{t-1} + \epsilon^w_t \tag{7}$$

Given the assumption of non-discriminatory hiring by firms above, unemployment is equally split between both traders and limited households. The fraction of limited, unemployed households is $n_t^u = urate_t (1 - n^p)$ and given

\(^{13}\)The setup of the labor market is very close to Challe and Ragot (2016) but I abstract from hiring frictions.

\(^{14}\)I explore the importance of rigid nominal wages in an extension at the end of the paper.
recursively by

\[ n_t^u = (1 - \sigma^d)n_{t-1}^u + \sigma_t^u n_{t-1}^e \]  

(8)

where \( \sigma^d \) is the fixed death probability of unemployed agents\(^{15}\), \( \sigma_t^u \) the probability of unemployment at the beginning of period \( t \) and \( n_{t-1}^e \) the share of limited, employed workers at the end of period \( t - 1 \). The firm FOCs yield labor demand and thus unemployment. Given unemployment, equation (8) determines \( \sigma_t^u \). Traders are insured against unemployment in large families and the share of unemployed traders is given by: \( urate_t n_p \). Together, limited, employed households and traders provide aggregate, effective labor \( \hat{N}_t \):

\[ \hat{N}_t = n_t^e + n_p \eta (1 - urate_t) \]  

(9)

and the total labor supply is

\[ N_t = n_t^e + n_p (1 - urate_t) \]  

(10)

### 3.4 Traders

Traders are similar to the standard representative agent setup in NK models as in Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007). I assume that these agents belong to one large family, fraction \( n_p \) of all agents, and within that family are fully insured against unemployment. The family head maximizes utility by choosing consumption, investment, capital utilization, next period’s capital stock and saving in real bonds. When employed, an agent provides labor of 1 and 0 otherwise.

Investment is subject to quadratic investment cost \( S_t(I_t, I_{t-1}) \) when \( \tau > 0 \) which is paid in consumption units for changes in the level of investment per unit of investment, and includes an investment adjustment cost shock \( e^{z_t} \).

These cost are modeled as in Christiano, Eichenbaum and Evans (2005).

\(^{15}\)\( \sigma^d \) can be thought of as the hiring probability.
Existing capital depreciates at rate $\delta^K$. Investment per period is thus given by:

$$K_{t+1} - (1 - \delta^K)K_t = e^{z^I_t} \left( 1 - \frac{\tau}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right) I_t \tag{11}$$

Cost shock $z^I_t$ evolves according to the following process:

$$z^I_t = \rho_I z^I_{t-1} + \epsilon^I_t \tag{12}$$

Investment cost play a key role in representative agent NK models and as I will discuss later, one implication of adding households with a precautionary saving motive is that the importance of these costs decreases substantially.

Capital $K_t$ is utilized at rate $u_t$ to yield effective capital $\hat{K}_t = u_t K_t$. Utilization cost $\eta^K(u_t)$ are paid in consumption units and are calibrated such that in steady state the utilization rate is $u_{ss} = 1$ and cost are $\eta^K(u_{ss}) = 0$. In addition, $\eta^K'(u_{ss}) > 0$ and $\eta^K''(u_{ss}) > 0$. The functional form follows Christiano, Trabandt and Walentin (2010), chapter 7.

$$\eta^K(u_t) = \chi_1 (1 - u_t) + \frac{\chi_2}{2} (u_t - 1)^2 \tag{13}$$

Traders maximize utility given prices, bond holding from last period, the capital stock, the previous level of investment, the previous level of consumption as well as the level of employment (in real terms):
\[ V_t = \max_{c_t, I_t, u_t, b_{t+1}, K_{t+1}} \left( \ln(c_t - bc_{t-1}) + \beta \mathbb{E}_t V_{t+1} \right) \]

subject to

\[ c_t + I_t + b_{t+1} \leq R_t u_t K_t + \omega \omega N_t - \eta(u_t) K_t + (1 + r_t^i) \frac{1}{\pi_t} b_t + \frac{1}{w_{t+1}} \frac{\Pi_t}{\pi_t} + T_t^G \]

\[ K_{t+1} - (1 - \delta^K) K_t = e^{\tau t} \left( 1 - \frac{\tau}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right) I_t \]

\[ N_t = (1 - urate_t) \]

where, \( b \) is the consumption habit parameter, \( \Pi_t \) is the profit of the intermediaries, \((1 + r_t^i)\) is the nominal interest rate, \( T_t^G \) are real lump sum taxes and \( b_t \) are real bonds.

Let \( \lambda_t \) and \( \mu_t \) be the multipliers on the budget constraint and capital accumulation equation. The optimality conditions are:

\[ \lambda_t = \frac{1}{c_t - bc_{t-1}} - \beta \mathbb{E}_t \left[ \frac{1}{c_{t+1} - bc_t} \right] \]  \hspace{1cm} (14)

\[ R_t = \chi_1 + \chi_2(u_t - 1) \]  \hspace{1cm} (15)

\[ \lambda_t = (1 + i_t) \beta \mathbb{E}_t \left[ \lambda_{t+1} \frac{1}{\pi_{t+1}} \right] \]  \hspace{1cm} (16)

\[ \lambda_t = \mu_t e^{\tau t} \left( 1 - \frac{\tau}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right) - \tau \frac{I_t}{I_{t-1}} \left( \frac{I_t}{I_{t-1}} - 1 \right) \]

\[ - \beta \mathbb{E}_t \left[ \mu_{t+1} e^{\tau t} \left( \frac{I_{t+1}}{I_t} - 1 \right) \frac{I_{t+1}^2}{I_t^2} \right] \]  \hspace{1cm} (17)

\[ \mu_t = \beta \mathbb{E}_t \left[ \lambda_{t+1} (R_{t+1} u_{t+1} - \eta(u_{t+1})) + \mu_{t+1} (1 - \delta^K) \right] \]  \hspace{1cm} (18)

Equation 14 is the stochastic discount factor. Since traders are also firm
owners, this SDF was used in the optimization problem of intermediaries. Equation 15 shows the role of capital utilization. Utilization rate adjustments alleviate the effect of interest rate shocks on the return on capital services. Equation 16 is the Euler Equation. And the last two equations are the optimality conditions for investment and capital utilization.

3.5 Employed, limited agents

Limited households are a fraction \((1 - n_p)\) of all households and split into employed and unemployed agents. Their two defining features are that they can only save in riskless bonds and face uninsurable unemployment risk. Employed agents are assigned to large families\(^{16}\) when they are born and share income and assets. A family head maximizes the utility of all agents and assigns each member a level of consumption and saving. If an agent becomes unemployed, however, it must take its savings and leave the family. Once unemployed, labor income is zero and agents consume their savings until they die with probability \(\sigma^d\). This risk of unemployment and zero income gives rise to the precautionary saving motive.

The employed agent problem has a family level wealth state variable \(X_t^e\), after the labor market transitions have occurred. Since there are infinitely many employed and newborn agents, the rates of loss of agents to unemployment and the allocation of newborn workers to families are the same across all families by the law of large numbers. While \(n_u^t\) and \(n_e^t\) can vary over time, the total fraction of limited agents is constant at \(1 - n_p\).

The choice variables for the family head are per-member (lowercase) wealth, \(x_t^e\), and consumption, \(c_t^e\). Employed workers supply one unit of labor inelastically. The law of motion for real pooled resources takes into account the wealth inflows and outflows in the labor market transition stage at the

\(^{16}\text{This assumption solves the aggregation problem with different cohorts of limited, employed households.}\)
beginning of the period:

\[ X_t^e = (1 - \sigma_t^u) n_t^e - \frac{1}{\pi_t} (1 + i_{t-1}) x_{t-1}^e + D_t^u \]  

where the first term on the right determines the outflow of resources that unemployed workers take with them, and \( D_t^u \) is the transfer of wealth from newly deceased unemployed agents to newborn, employed workers.

The family head then solves the following problem (in real terms):

\[ V_t^e = \max_{c_t^e, x_t^e} n_t^e \ln(c_t^e) + \beta E_t \left[ V_{t+1}^e + \sigma_{t+1}^u n_{t+1}^e V_{t+1}^u \right] \]

subject to

\[ n_t^e (c_t^e + x_t^e - T_t^G) \leq X_t^e + n_t^e w_t \]  

\[ X_{t+1}^e = (1 - \sigma_{t+1}^u) n_{t+1}^e \frac{1}{\pi_{t+1}} (1 + i_t) x_{t+1}^e + D_{t+1}^u \]

\[ x_t^e \geq 0 \]

where \( T_t^G \) is a lumpsum tax. Assuming that the BC constraint holds with equality, that \( x_t^e > 0 \) and substituting for \( X_t^e \), the first order conditions and envelope conditions are:

\[ \frac{\partial V_t^e}{\partial c_t^e} = n_t^e \frac{1}{c_t^e} - n_t^e \lambda_t = 0 \]

\[ \frac{\partial V_t^e}{\partial x_t^e} = -n_t^e \lambda_t + \beta E_t \left[ \frac{\partial V_{t+1}^e}{\partial x_{t+1}^e} + \sigma_{t+1}^u n_{t+1}^e \frac{\partial V_{t+1}^u}{\partial x_{t+1}^e} \right] = 0 \]

\[ \frac{\partial V_t^e}{x_{t-1}^e} = \lambda_t (1 - \sigma_t^u) n_{t-1}^e \frac{1}{\pi_t} (1 + i_{t-1}) \]

\[ \frac{\partial V_t^u}{\partial x_t^e} = \frac{1}{c_t^u \pi_t} (1 + i_{t-1}) \]

Where the last envelope condition is given by equation (55) in the appendix of the problem for unemployed agents. Iterating forward the envelope conditions and substituting we get:
\[ \lambda_t = \frac{1}{c^e_t} \]
\[ n^e_t \lambda_t = \beta_e (1 + i_t) \mathbb{E}_t \left[ \lambda_{t+1} (1 - \sigma^u_{t+1}) n^e_t \frac{1}{\pi_{t+1}} + \sigma^u_{t+1} n^e_t \frac{1}{c^u_{t+1} \pi_{t+1}} \right] \]

which together give the Euler Equation for employed, limited households:

\[ \frac{1}{c^e_t} = \beta_e (1 + i_t) \mathbb{E}_t \left[ (1 - \sigma^u_{t+1}) \frac{1}{c^e_{t+1} \pi_{t+1}} + \sigma^u_{t+1} \frac{1}{c^u_{t+1} \pi_{t+1}} \right] \]  

(21)

As long as \( \mathbb{E}_t \beta_e \frac{1}{\pi_{t+1}} < 1 \), absent the unemployment probability, impatience implies consumption up to the borrowing limit of \( x^e_t = 0 \). The probability of unemployment next period, however, creates a savings motive to avoid a potential consumption level of zero. This target savings level depends on the unemployment probability, expected length of unemployment, as well as income, the discount factor and expected inflation.

### 3.6 Unemployed, limited Households

Unemployed households face fixed death probability \( \sigma^d \), do not receive any income and only consume their savings. These unemployed households solve the following problem:

\[ V^u_t = \max_{c^u_t, x^u_t} \ln(c^u_t) + \beta_e (1 - \sigma^d) \mathbb{E} V^u_{t+1} \]

subject to

\[ c^u_t + x^u_t \leq \frac{1}{\pi_t} (1 + r^u_{t-1}) x^u_{t-1} \]
\[ x^u_t \geq 0 \]

where \( x^u_t \) are real savings in bonds in period \( t \) and there is a no-borrowing limit. In the initial period of unemployment \( x^u_{t-1} = x^e_{t-1} \). Taking FOCs\(^{17}\)

\(^{17}\)A binding no-borrowing constraint implies zero consumption.
yields the Euler Equation:

\[ \frac{1}{c_t^u} = \beta_e (1 - \sigma^d) (1 + r_t^i) \mathbb{E}_t \left[ \frac{1}{c_{t+1}^u} \frac{1}{\pi_{t+1}} \right] \]  \hspace{1cm} (23)

From the Euler Equation and the lifetime budget constraint, the perfect foresight solution becomes:

\[ c_t^u = (1 - \beta_e (1 - \sigma^d)) \frac{1}{\pi_t} (1 + r_{t-1}^i) x_{t-1}^u = k_u \frac{1}{\pi_t} (1 + i_{t-1}) x_{t-1}^u \]  \hspace{1cm} (24)

Thus, unemployed households consume a constant fraction of their income.

### 3.7 Aggregate unemployment variables

Aggregate dynamics are described by three variables: consumption, \( \bar{C}_t^u \), saving, \( \bar{S}_t^u \), and transfers from the deceased households, \( \bar{D}_t^u \).

Consumption consists of the newly unemployed agents’ consumption (no death probability in first period) and consumption of the surviving, previously unemployed agents.

\[ \bar{C}_t^u = \sigma_t^u n_{t-1}^e c_t^u + (1 - \sigma^d) k_u \frac{1}{\pi_t} (1 + r_{t-1}^i) \bar{S}_{t-1}^u \]  \hspace{1cm} (25)

Aggregate saving combines the saving of the newly unemployed with the saving of the surviving unemployed:

\[ \bar{S}_t^u = \frac{1}{\pi_t} (1 + r_{t-1}^i) (1 - k_u) (x_{t-1}^e n_{t-1}^e \sigma_t^u + (1 - \sigma^d) \bar{S}_{t-1}^u) \]  \hspace{1cm} (26)

And, lastly, transfers are the real savings of the deceased agents, out of the group of previously unemployed households:

\[ \bar{D}_t^u = \sigma^d \frac{1}{\pi_t} (1 + r_{t-1}^i) \bar{S}_{t-1}^u \]  \hspace{1cm} (27)
3.8 Monetary Policy and Government

Monetary policy follows a Taylor rule that reacts to deviations of inflation from the target of steady state inflation and to economic growth (see e.g. Guerrieri and Iacoviello (2017)).

\[(1 + r_t^i) = (1 + r_{t-1}^i)^{r_R} \left( \frac{\pi_t}{\pi_{SS}} \right)^{(1-r_R) \rho_y} \left( \frac{y_t}{y_{t-1}} \right)^{(1-r_R) \rho_y} (1 + r_{SS}^i)^{1-r_R} \epsilon_{r,t} \] (28)

where \( \epsilon_{r,t} \) is the monetary policy shock.

Government spending is an exogenous fraction of output:

\[ G_t = \omega_t y_t \]

where \( \omega_t \) is follows an AR(1) process:

\[ \omega_t = \rho_g \omega_{t-1} + (1 - \rho_g) \bar{\omega} + \epsilon^G_t \]

The government levies lumpsum taxes on traders and employed, limited households according to a balanced budget rule:

\[ G_t = n_t^e T_t^G + n_p T_t^g \] (29)

3.9 Aggregation and Equilibrium

In equilibrium, effective labor and capital markets clear:

\[ \hat{N}_t = n_t^e + (1 - urate_t) \eta m_p \]

\[ \hat{K}_t = n_p u_t K_t \]

The bond market clears:

\[ n_t^e x_t^e + S_t^u = n_p b_{t+1} \]
The aggregate resource constraint is:

\[ y_t = G_t + n_t^e c_t^e + C_t^u + n_t(p_t + I_t + \eta^K(u_t)K_t) \]

The law of motion for capital is

\[ K_{t+1} - (1 - \delta^K)K_t = e^{\varepsilon_t} \left( 1 - \frac{\tau}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right) \right)^2 I_t \]

The law of motion for the fraction of limited, unemployed agents is:

\[ n_t^u = (1 - \sigma^d)n_{t-1}^u + \sigma^u n_{t-1}^e \]

Inflation is given by

\[ \pi_t^{1-\epsilon} = (1 - \phi)\pi_t^{#(1-\epsilon)} + \phi\pi_{t-1}^{#(1-\epsilon)} \]

where \( \pi_t^{#} \) is defined in equation (4).

The real wage is determined by the wage equation:

\[ w_t = \left( \frac{w_{t-1}}{\pi_t} \right)^{\gamma_w} \left( \bar{\omega}e^{\varepsilon_t} \frac{n_t}{n_{ss}} \right)^{\phi_w} (1-\gamma_w) \] (30)

The shock processes are:

\[ z_t^I = \rho_I z_{t-1}^I + \varepsilon_t^I \]
\[ z_t^w = \rho_w z_{t-1}^w + \varepsilon_t^w \]
\[ \omega_t = \rho_G \omega_{t-1} + (1 - \rho_G) \bar{\omega} + \varepsilon_t^G \]
\[ A_t = \rho_A A_{t-1} + (1 - \rho_A) \bar{A} + \varepsilon_t^A \] (31)

Therefore, an equilibrium in this economy is a set of value and policy functions, a set of prices and a set government policies, such that given prices and state variables, (1) the policy functions solve the household problems of traders, limited and employed as well as limited and unemployed households,
(2) firms maximize profits, (3) the bond, labor and capital markets clear, (4) wages are set according to the wage rule, and (5) government policy is given by the balanced budget equation and monetary policy rule.

4 Estimation and Results

In this section, I estimate the benchmark model and answer the questions listed in the introduction: How well does a precautionary saving model match the observed saving pattern in the data? How does the inclusion of households that engage in precautionary saving change the transmission of monetary policy? What are the implications for monetary policy? And, can a model of precautionary saving explain the sluggish inflation rate and seemingly low effectiveness of monetary policy since the financial crisis?

4.1 Calibration and Estimation

Table 2: Calibrated parameters

<table>
<thead>
<tr>
<th>parameter</th>
<th>description</th>
<th>value</th>
<th>target</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\pi_{ss})</td>
<td>steady state inflation</td>
<td>0</td>
<td>mean unemployment duration</td>
</tr>
<tr>
<td>(\sigma^d)</td>
<td>death probability</td>
<td>0.6011</td>
<td>mean unemployment duration</td>
</tr>
<tr>
<td>(\beta)</td>
<td>discount factor traders</td>
<td>0.9904</td>
<td>quarterly interest rate</td>
</tr>
<tr>
<td>(\beta_e)</td>
<td>discount factor limited</td>
<td>0.96</td>
<td>closed match of SCF chkg-inc ratio</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>share of capital in production</td>
<td>1/3</td>
<td></td>
</tr>
<tr>
<td>(\chi_1)</td>
<td>capital utilization parameter</td>
<td>0.0297</td>
<td>(n_{ss} = 1)</td>
</tr>
<tr>
<td>(\chi_2)</td>
<td>capital utilization parameter</td>
<td>0.01</td>
<td>Christiano et.al. 2005</td>
</tr>
<tr>
<td>(\delta^K)</td>
<td>capital depreciation rate</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>(\epsilon_p)</td>
<td>price elasticity of demand</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>(n_p)</td>
<td>share of traders</td>
<td>0.4</td>
<td>SCF data, see section 2</td>
</tr>
<tr>
<td>(\bar{w})</td>
<td>log wage parameter</td>
<td>0.6456</td>
<td>average unemployment</td>
</tr>
<tr>
<td>(\omega)</td>
<td>share of government</td>
<td>0.2</td>
<td>Christiano et.al. 2005</td>
</tr>
<tr>
<td>(\eta)</td>
<td>wage premium</td>
<td>1.5</td>
<td>Challe and Ragot (2016)</td>
</tr>
</tbody>
</table>
The model is estimated using five quarterly time series from the end of the Volker period in 1982Q2 until 2017Q1. To examine how well this model with uninsurable labor market risk and precautinary saving can match the data I will compare the model implied checking account to quarterly income ratio of limited, employed households to the data points in the SCF for the middle 30% of households by income, as outlined in the data section.

The five time series used in the estimation procedure are the change in real investment per capita, the change in real output per capita, the inflation rate, the secondary market 3-month T-Bill rate\(^\text{18}\) and the unemployment rate. The time series are described in detail in section A.7.

The parameters of this model are grouped into two categories: calibrated parameters and estimated parameters. Table 2 lists the calibrated parameters and the targets used. The model is linearized around a zero inflation steady state. \(\alpha\) is set to 1/3, the quarterly depreciation rate is 2% and the price elasticity of demand equal to 10. These four values are commonly used in the NK literature. The death probability of unemployed agents, \(\sigma^d\) is chosen to match the average unemployment duration in quarters. The real interest rate pins down the discount factor of traders, \(\beta\). The capital utilization parameters are chosen to set capital utilization to 1 in steady state. This calibration implies \(\chi_1 = 0.0297\). Since \(\chi_2\) is problematic to estimate and not the focus of this paper, I take the value of 0.01 from Christiano, Eichenbaum and Evans (2005). The wage parameter \(\bar{w}\) is implied by the steady state unemployment rate. Given the steady state unemployment rate and death probability, the law of motion for limited, unemployed agents, equation (8), implies an unemployment probability around 4%. Both the calibrated death probability and implied unemployment probability are in line with Shimer (2005).

\(^{18}\)A drawback of this estimation method is that the ZLB is matched by a series of unexpected shocks to the Taylor Rule and households do not expect the interest rate to stay at that level. This approach has been used by other papers as well, see Challe and Ragot (2016).
The fraction of limited agents is 60% given the relative proportions of the middle and high income groups in the data. The wage premium is set to 1.5 following Challe and Ragot (2016). Lastly, the discount factor of limited, employed agents, \( \beta_e \), is difficult to pin down since SCF data on saving is only available every three years, starting in 1989. Matching the average checking account balance to income ratio for that period leads to a discount factor of 0.945. Following the estimation, however, the model implied path of this ratio is quantitatively too low. I thus choose a discount factor of 0.96 which leads to a close match between the model ratio and SCF data. This adjustment does not alter the qualitative result but increases the amount of saving at every point in time by roughly the same amount, as shown in figure A.6 in the Appendix.

Table 3: Estimated parameters

<table>
<thead>
<tr>
<th>Model Description</th>
<th>Parameters</th>
<th>Prior [bound]</th>
<th>Mode</th>
<th>Posterior</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>10%</td>
<td>Median</td>
<td>90%</td>
</tr>
<tr>
<td>( \gamma_w ) wage inertia</td>
<td>beta [0.5,0.1]</td>
<td>0.7708</td>
<td>0.7339</td>
<td>0.7712</td>
</tr>
<tr>
<td>( \phi_w ) wage adjustment</td>
<td>gamma [1,0.2]</td>
<td>1.2614</td>
<td>1.1053</td>
<td>1.2930</td>
</tr>
<tr>
<td>b habit persistence</td>
<td>beta [0.7,0.1]</td>
<td>0.5389</td>
<td>0.3973</td>
<td>0.5144</td>
</tr>
<tr>
<td>( \tau ) investment cost</td>
<td>normal [1,0.5]</td>
<td>1.0047</td>
<td>0.7909</td>
<td>1.0996</td>
</tr>
<tr>
<td>( \phi_p ) calvo probability</td>
<td>beta [0.5,0.1]</td>
<td>0.6746</td>
<td>0.6365</td>
<td>0.6659</td>
</tr>
<tr>
<td>( \zeta_p ) inflation index</td>
<td>beta [0.5,0.2]</td>
<td>0.1386</td>
<td>0.0804</td>
<td>0.1739</td>
</tr>
<tr>
<td>( \phi_y ) output Taylor</td>
<td>normal [0.125,0.05]</td>
<td>0.2094</td>
<td>0.1558</td>
<td>0.2133</td>
</tr>
<tr>
<td>( \phi_t ) inflation Taylor</td>
<td>normal [1.5,0.1]</td>
<td>1.8635</td>
<td>1.7821</td>
<td>1.8746</td>
</tr>
<tr>
<td>( \rho_i ) inertia Taylor</td>
<td>beta [0.9,0.05]</td>
<td>0.7803</td>
<td>0.7531</td>
<td>0.7786</td>
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<tr>
<td>( \rho_w ) AR(1) wage shock</td>
<td>beta [0.9,0.05]</td>
<td>0.9242</td>
<td>0.8455</td>
<td>0.9158</td>
</tr>
<tr>
<td>( \rho_a ) AR(1) TFP shock</td>
<td>beta [0.9,0.05]</td>
<td>0.9535</td>
<td>0.9291</td>
<td>0.9532</td>
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<tr>
<td>( \rho_g ) AR(1) gov shock</td>
<td>beta [0.9,0.05]</td>
<td>0.9744</td>
<td>0.9637</td>
<td>0.9731</td>
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<tr>
<td>( \rho_z ) AR(1) invest cost shock</td>
<td>beta [0.9,0.05]</td>
<td>0.9129</td>
<td>0.8856</td>
<td>0.9086</td>
</tr>
<tr>
<td>( \sigma_w ) sd wage shock</td>
<td>inv gamma [0.01,0.002]</td>
<td>0.0520</td>
<td>0.0460</td>
<td>0.0529</td>
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<tr>
<td>( \sigma_t ) sd Taylor shock</td>
<td>inv gamma [0.002,0.002]</td>
<td>0.0015</td>
<td>0.0014</td>
<td>0.0015</td>
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<tr>
<td>( \sigma_a ) sd TFP shock</td>
<td>inv gamma [0.01,0.002]</td>
<td>0.0052</td>
<td>0.0048</td>
<td>0.0052</td>
</tr>
<tr>
<td>( \sigma_g ) sd gov shock</td>
<td>inv gamma [0.01,0.002]</td>
<td>0.0071</td>
<td>0.0065</td>
<td>0.0072</td>
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<tr>
<td>( \sigma_z ) sd invest cost shock</td>
<td>inv gamma [0.01,0.002]</td>
<td>0.0137</td>
<td>0.0124</td>
<td>0.0143</td>
</tr>
</tbody>
</table>
The estimated parameters are shown in table 3. The third column details the prior distribution and parameters, while the last four columns show the estimation results. The estimated parameters align well with the literature. There are two prominent exceptions, however, which are the key difference between the benchmark model and canonical NK models: the capital adjustment cost parameter and the habit parameter. Both are markedly lower in the benchmark model. Furthermore, the benchmark model allows for more price and wage flexibility. These results indicate that the transmission mechanism of monetary policy does not solely work through the intertemporal substitution channel here. I will return to this issue after first evaluating how well the benchmark model can match the observed data on precautionary saving.

4.2 Model implied saving

The benchmark model fits the data on precautionary saving quantitatively well until the mid 2000s but overestimates the savings ratio in the run up to and time since the Great Recession. Across all available years, however, the same trend in savings is clearly visible in the model implied savings ratio and the SCF data.

Following the discussion in part 2, I compare the model implied average checking account balance to quarterly income ratio of the limited, employed households to the values derived from the Survey of Consumer Finances for the middle income group of households. For the model outcome, I use end-of-period savings per agent $x_t^e$ divided by 2 over wage income $w_t$ as target variable:

$$chkinc_t = \frac{x_t^e}{2w_t}$$

I retrieve the time series for this savings ratio by feeding the shocks that were extracted at the estimation stage back into the model. From the SCF I
take the values calculated in the data section which are available every three years for the period 1989 - 2013.

The resulting comparison is shown in figure 3. The model matches the long-run business cycle variation of savings quite well. The drop in savings during the 1990s and eventual increase around the dotcom bust and following short recession as well as the decrease during the housing boom and sharp increase during and after the Great Recession are clearly visible.

In addition, during the nineties and up until around 2004, the implied savings ratio is quite close to the SCF data points. For the last 10 years of the sample, however, the model implies a much larger magnitude of precautionary saving, starting just before the Great Recession.

Two recent papers that estimate medium scale NK models with a precautionary saving motive are Challe and Ragot (2016) and Ravn and Sterk
Both papers include hiring cost which amplify the labor market movements but in the case of Challe and Ragot (2016) the authors use the ratio of consumption of the bottom 60% of households to the top 40% as a target in their estimation while Ravn and Sterk (2017) compare the implied vacancy rate to the data. The benchmark model, abstracting from any amplification mechanism already overestimates the amount of household saving. One implication of my result is that the powerful feedback loop between labor market outcomes and aggregate demand found by these two papers might be overestimated.

Further evidence that this model fits the data better than a standard representative agent model is the movement of the real interest rate. In Appendix A.10 I show that this model is able to reproduce the result of Canzoneri, Cumby and Diba (2007). The estimated real interest rate in this model moves counter to the implied aggregate interest rate.

4.3 Transmission of Monetary Policy

The precautionary saving motive links consumption of limited, employed agents to the labor market. This link breaks the dominance of the direct effect of monetary policy via the intertemporal substitution channel in standard NK models and strengthens the importance of the impact monetary policy has on the labor market.

A strong indication of the strength of the indirect effect can be seen in the estimated parameters of the benchmark model compared to a representative agent version\(^{19}\) in which the share of limited agents is set to 0. Standard NK models rely on strong real and nominal frictions to inhibit the consumption smoothing motive of the representative agent. Capital adjustment costs and consumption habits increase the cost of using capital to save and dissave, see Rupert and Sustek (2016) for a detailed discussion. Furthermore, sticky prices and wages prohibit the immediate response of prices to changes in

\(^{19}\)See Appendix A.5 for a description of the model and the estimation result.
monetary policy.

All of these frictions are lower and therefore less important in the estimated benchmark model. The capital adjustment cost and habit parameters are 1 and 0.54, respectively, compared to 1.6 and 0.78 in the representative agent economy. Prices are also more flexible in the benchmark model. The estimated calvo and inflation indexation parameters are 0.67 and 0.14, lower than 0.7 and 0.07 in the representative agent model. The same is true for wages. Nominal wage inertia is estimated at 0.77 and the labor market tightness adjustment parameter is 1.26 in the benchmark economy, whereas these are estimated at 0.8 and 1 in the representative agent model. Since both models were estimated with the same data these results indicate that the benchmark model relies less on frictions and more on an indirect transmission mechanism for monetary policy.

I highlight the relative importance of the indirect effect in two steps: First, the direct effect of monetary policy on limited, employed households is given in figure 4. The IRFs depict the response to a monetary policy shock for the benchmark model and an augmented benchmark economy in which the non-interest income for limited, employed households is held constant. Second, in figure 5 I compare the IRFs for a monetary policy shock in the benchmark model and a cash version of the benchmark model in which limited, employed households do not earn interest income.

The separation of the direct from the total effect of monetary policy for limited, employed agents is clearly visible when all real non-interest income is held constant. Specifically, government expenses and thus taxes, the unemployment rate and real wages are fixed at their respective steady state values. Figure 4 shows that the response of limited, employed agents is an order of magnitude larger than the response of traders in this model and the response of traders is very similar in both models, despite different paths of output and

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20 The result of Werning (2015) holds and the total effect of a monetary policy shock is the same as in the representative agent model, see figure A.11 in the Appendix.
inflation, highlighting the strengths of the direct effect of monetary policy on these agents.

The reaction of limited, employed agents shows that absent a change in labor market risk, they increase their savings initially but following strong deflation dissave their additional real savings. In the benchmark model that includes labor market risk, the economy takes longer to return to the steady state and the additional saving of limited agents decreases the initial drop of inflation and associated drop in output (since real costs increase less for firms that cannot adjust prices). The indirect effect softens the impact of the direct effect and extends the transition back to the steady state.

To investigate how strongly the precautionary saving channel dominates
the intertemporal substitution channel for limited, employed agents I compare the benchmark model in which all agents earn interest on their savings to a "cash" model\textsuperscript{21} in which limited, employed agents save in cash which does not earn interest\textsuperscript{22}. Importantly, the interest rate is not part of the Euler Equation for limited agents in the cash economy and they only react to the path of inflation and the labor market. Figure 5 shows the IRFs following a positive shock to the interest rate for both models. These paths are

\textsuperscript{21}Described in Appendix A.4.

\textsuperscript{22}Since cash is dominated by bonds, traders do not invest into cash, a condition I verify ex-post using the multiplier on the non-negativity constraint for cash savings. This constraint is binding for traders but not binding for limited, employed households.
virtually the same\textsuperscript{23}.

The implication for monetary policy in a model with a strong indirect transmission channel is that, following aggregate shocks, interest rate changes need to overcome movements of demand into the opposite direction. For example, if the economy is pushed into a recession through an aggregate demand shock, an application I will look at in the next section, the response of monetary policy has to be more forceful. The initial shock is amplified when households increase savings in response to greater labor market uncertainty. At first, only households that directly react to the interest rate will adjust consumption and this adjustment has to be strong enough to cancel the amplified drop in demand.

\subsection*{4.4 Missing Inflation since the Great Recession}

Inflation since the Great Recession has been persistently below the 2\% target of the Federal Reserve Bank despite strong output growth and decreasing unemployment. Evidence that the saving mechanism might play an important role in answering this question can be seen in the data. Both middle and low income households strongly increased their savings during and after the Great Recession. Especially the data of the 2013 Survey of Consumer Finances shows that even several years after the recession, household savings were at a record high.

I simulate the Great Recession through a shock to government demand. This shock both lowers output and inflation, and, intuitively, is a shift of the aggregate demand curve to the left. In a model of precautionary saving, this shift is amplified when households reduce consumption to save, in response to an increase in the unemployment probability.

The optimal response of monetary policy is to lower interest rates to stimulate household consumption and increase demand. The dominating indirect effect of monetary policy on limited households’ savings behavior implies,\textsuperscript{23} Saving in the benchmark economy is higher in the steady state.
Figure 6: Comparison of IRFs in benchmark and representative agent model after a government spending shock which is both deflationary and reduces output. Both models use the same process for government spending and Taylor Rule. The reduction in output causes a rise in unemployment and in turn increases savings. Despite similar paths for output and unemployment, in the benchmark economy inflation drops more despite a lower interest rate. The benchmark interest rate stays below the representative agent model rate until after output and unemployment return to the steady state.

However, that traders have to adjust consumption even more than in a representative agent NK model to overcome the additional drop in aggregate demand due to the precautionary savings motive. The lower the fraction of traders in the economy, the more the target rate has to be adjusted. Figure 6 illustrates this effect. I compare IRFs of the estimated benchmark economy to the estimated representative agent model\textsuperscript{24}.

Following the same shock to government spending, the initial impact is the same in both models. Output and unemployment follow similar transition paths, but limited agents increase their savings in the benchmark economy. The initial impact on inflation is therefore stronger despite a lower interest rate.

\textsuperscript{24}The parameters governing the AR process for government spending and the Taylor Rule are the same and as estimated in the benchmark model to ensure that the comparison is valid.
rate. The optimal policy rate in the benchmark economy stays below the representative agent model rate until after output and unemployment return to the steady state.

The IRFs show that the precautionary saving channel is a promising candidate to explain the consistently low inflation rate since the Great Recession as it aligns well with observed household saving behavior in the data and implies both a lower inflation rate and lower interest rates. Other factors such as the zero lower bound constraint on the ability to lower the target rate as well as the strong increase in unemployment duration will amplify this channel even more. Models with hiring frictions, however, also imply much stronger precautionary saving during and after the financial crisis, and may overestimate the saving response to the Great Recession. I therefore explore a different extension of the benchmark economy, downward rigid nominal wages.

4.5 Downward Rigid Nominal Wages

The benchmark model abstracted from any mechanism that would intensify or weaken the precautionary saving channel. This abstraction allowed me to analyze how the saving channel itself changes the workings of standard New Keynesian models. I now extend the model with downward rigid nominal wages, a defining feature of the data during the Great Recession in many countries.25 This fact is important in the context of precautionary saving because the strength of saving motive increases with wage rigidity since more rigid nominal wages lead to a larger variation in unemployment. The previous discussion has shown that the effectiveness of a given monetary policy intervention weakens the more households respond to labor market movements and save. Downward rigid nominal wages therefore imply an asymmetric response of the labor market and thus higher saving in downturns compared to the benchmark model. In turn, one would expect monetary policy to be

25Schmitt-Grohé and Uribe (2013) provide a good overview.
Figure 7: Comparison of IRFs in benchmark model and extended model with downward rigid wages. The red line is identical to the IRFs of the benchmark model shown in figure 6. Nominal wages fall for 6 periods, and the economy follows the augmented model. Note that the wage graph displays real wages. The black line, shows that the impact of the constraint continues even after the constraint does not bind anymore. While the initial impact on inflation is lower in the beginning, higher unemployment leads to more saving and a longer and lower transition path of inflation during the recovery.

less effective during recessions. Empirical evidence in Tenreyro and Thwaites (2016) supports this hypothesis.

To capture this asymmetry, I augment the benchmark economy with a second wage equation that has a higher $\gamma^A = 0.95$ parameter, compared to the estimated $\gamma^w = 0.7708$. The higher this value, the more nominally rigid is the wage. The augmented wage equation takes effect when nominal wages are falling in the model, while the economy behaves as in the benchmark model when nominal wages are rising. Apart from this change in the wage parameter, the two economies are identical.

To solve the model under two different wage setting regimes, I use the
method of endogenously binding constraints, developed by Guerrieri and Iacoviello (2015). An advantage of the piecewise linear solution method is that it can accommodate a large state space. I exploit this feature to solve the medium scale NK model under varying wage equations. The algorithm switches between the two models based on whether the nominal wage is increasing or decreasing. As shown in their paper, the solution of this algorithm approximates closely the solution found using fully nonlinear models.

Figure 7 shows the response of the extended and benchmark models to the same government demand shock as in the previous section. The red, dotted line is the same as shown in figure 6 for the benchmark model. The black line depicts the response of the augmented model. Following a government demand shock, nominal wages fall for 6 periods and the rigid wage equation takes effect. The slower wage adjustment leads to higher unemployment and saving, but the path of output is virtually identical.

The largest difference between the benchmark model and the augmented model is the path of inflation. While the initial drop is not as large as in the benchmark economy, it now takes longer to return to the steady state and at a lower level during the transition.

A second difference between the benchmark economy and the extended model is the path of investment. Even though interest rates are slightly

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26 One criticism of the piecewise linear solution method is that households do not expect the change between models. This drawback might be problematic in the case of binding borrowing constraints that affect household decisions directly, but is unlikely to change the outcome in this model. Households never reach the borrowing constraint and households do not take the effect of their decision making on wages into account directly. In addition, Guerrieri and Iacoviello (2017) use this method to solve a model with an occasionally binding leverage constraint and show that the piecewise linear solution approximates the global solution well.

27 See the bottom right graph in figure A.13.

28 Since government spending is a share of output, the slightly different path of output explain why the response of government spending is not exactly the same in both models.

29 Interestingly, this result might be a compromise between economists that argue inflation has not been low enough and economists who wonder why inflation is not higher, see Hall (2011).
higher, investment is actually lower. Monetary policy is less potent in stimulating investment during the simulated recession, in line with recent evidence of asymmetric effects of monetary policy by Tenreyro and Thwaites (2016).

5 Conclusion

Standard New Keynesian models rely almost exclusively on the intertemporal substitution channel which is responsible for 95% of the total effect of monetary policy. The fact that most households do not own financial assets that are directly exposed to the target interest rate and exhibit a counter-cyclical saving behavior indicate that models of precautionary saving and incomplete markets might resolve this issue.

I define as measure of precautionary saving the \textit{average monthly checking account balance to income} ratio and find that an estimated medium scale New Keynesian model in which a large fraction of households are not insured against permanent income loss through unemployment matches this statistic well. I further show that in such a model the direct effect of monetary policy is dominated by the state of the labor market for these agents. In a simulation of the Great Recession, the benchmark model both implies a lower inflation rate and longer transition back to the steady state compared to an estimated representative agent version of the model. This result is strengthened when the model is extended with downward rigid nominal wages.

Precautionary saving, therefore, is a promising extension of the canonical NK model. The measure of precautionary saving presented in this paper can be a valuable guide to evaluate how well other extensions can improve the match between the model and data.

For example, one possible explanation of the low quantitative match between the model implied savings ratio and the data the last 10 years is unemployment duration. As shown in figure A.7, average unemployment duration is quite volatile, but fixed in the benchmark model. The model both
implies a lower savings ratio in the mid-90s when unemployment duration was above average and a higher savings ratio in the mid-2000s when unemployment duration was low. In addition, the benchmark model presented in this paper assumes that firms cannot discriminate between high and low productive workers. Unemployment, however, increases much more sharply for less productive workers and it takes longer for these jobs to reappear after a recession.

Furthermore, Carroll, Sommer and Slacalek (2012) find that fluctuations in net worth and access to credit are important determinants of the aggregate savings rate, in addition to labor market risk. Few households in the data hold liquid assets but house ownership and mortgage debt increased greatly before the Great Recession. The house price boom and expanded subprime mortgage sector in the mid 2000s would imply a much lower savings ratio in that period as households bought real estate, while binding borrowing constraints and leverage shocks can explain why this ratio did not increase to the degree that the model suggests.

Another factor is the role of automatic stabilizers. During the Great Recession, the Obama administration extended unemployment benefits from 26 weeks (depending on the State) to 99 weeks, see Rothstein (2011). A combination of varying unemployment duration and unemployment benefits might well explain why despite an increase in average unemployment duration, savings did not increase as much as the benchmark model would indicate. As Galí, Vallés and López-Salido (2007) show, the presence of non-Ricardian households strongly increases the effect of government spending on consumption.

Beyond that, the extension of the benchmark model with downward rigid nominal wages also fits the evidence on asymmetric effects of monetary policy found by Tenreyro and Thwaites (2016).

Lastly, models with precautionary saving can potentially answer the issue raised in King and Watson (2012), namely that traditional NK models have
difficulty matching the inflation rate starting in the early 2000s and heavily rely on cost shocks to do so. According to the Survey of Consumer Finances, saving behavior became very volatile around that time compared to the 90s and will therefore have impacted the inflation rate in a way that cannot be captured with a representative agent model.
References


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A Appendix

A.1 Total liquid investments by income quintile

The following graphs show the ratio of total liquid investment, average checking account balance in the month prior to the survey date, as well as the amount invested into stocks, bonds and money market mutual funds for the same income groups as in the data analysis section. As figure A.1 shows, the few households that own financial investments dominate after aggregation and hide the business cycle variation in saving of all other households.

Figure A.1: Ratio of liquid investment to quarterly income. While few households hold any such investments, their amount dwarfs the savings of all other households after aggregation into income groups, and hides the business cycle variation of savings of households that do not own any financial assets.

The results presented in section 2 are robust to the exclusion of households that hold financial assets. The pattern of saving changes a bit, but not fundamentally, as shown in figure A.2. The main difference is a stronger
increase of saving for middle income households in the mid 90s, a period of high unemployment duration.

Figure A.2: Average monthly checking account balance to quarterly income ratio excluding households with financial assets.
A.2 Interest Rates on illiquid debt

The following graphs show the distribution of interest rates on credit card debt, car loans and mortgages paid by all households in the SCF for the period of 2007 - 2013. For most of 2007, the Federal Funds target rate was 5.25% and it was lowered to 0-0.25% in December 2008.

The rates shown here were calculated based on SCF data. In the SCF, for each household up to three mortgages, four credit cards and four car loans are given, with the interest rate on each. I use the total amount outstanding and the current annual rate at the time of the interview to generate a weighted interest payment for all loans of the same type per household. To give a simple example: two loans ($100,$20) with i-rate (10%,5%) and thus annual payments ($10,$1). Then, total debt = $120, total payment = $11, weighted i-rate = $11 / $120 * 100% = 9.17%.

What these graphs show is that despite a very low Federal Funds Rate, rates paid by households only decreased slowly over a period of 6 years. While investments that are directly exposed to the Federal Funds Rate might be adjusted in response FFR changes, this kind of debt that usually constitutes a large part of a household’s financial portfolio does not exhibit a strong short-term reaction to the FFR. Household consumption therefore can be assumed to respond less to FFR changes if they mainly hold this kind of debt, in line with Vissing-Jørgensen (2002).
Figure A.3: Distribution of Interest Rates on Credit Card Debt 2007 - 2013 in SCF data
Figure A.4: Distribution of Interest Rates on car loans 2007 - 2013 in SCF data
Figure A.5: Distribution of Interest Rates on Mortgage Debt 2007 - 2013 in SCF data
A.3 The Benchmark Model

A.3.1 Final Good Firms

The final goods firm aggregates the intermediate products into one final good. Each intermediate good is sold at price $P_t(j)$ and the final good has price $P_t$.

\[
y_t = \left( \int y_t(j)^{\frac{1}{\epsilon}} dj \right)^{\frac{1}{1-\epsilon}}
\]

Cost minimization gives the demand function for intermediate good $y_t(j)$:

\[
\min_{y_t(j)} \left( \int P_t(j) y_t(j) dj \right) - P_t \left( y_t - \left( \int y_t(j)^{\frac{1}{\epsilon}} dj \right)^{\frac{1}{1-\epsilon}} \right)
\]

(32)

\[
\frac{\partial}{\partial y_t(j)} = P_t(j) - P_t \left( y_t(j)^{\frac{1}{\epsilon} - 1} \left( \int y_t(j)^{\frac{1}{\epsilon}} dj \right)^{\frac{1}{1-\epsilon} - 1} \right) = 0
\]

(33)

Plugging back in the expression for $y_t$ gives:

\[
P_t(j) = P_t y_t(j)^{-\frac{1}{\epsilon}} y_t^{\frac{1}{\epsilon}}
\]

(34)

Solving for $y_t(j)$ yields the demand function:

\[
y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon} y_t
\]

(35)

To find an expression for the price level $P_t$, substitute the demand function (35) back into the resource constraint:

\[
y_t = \left( \int \left( \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon} y_t^{\frac{1}{\epsilon}} \right) \frac{1}{\epsilon} dj \right)^{\frac{1}{1-\epsilon}}
\]

(36)

\[
= \left( \left( \int (P_t(j)^{-\epsilon} dj)^{\frac{1}{\epsilon} - 1} P_t^{-\epsilon} y_t^{\frac{1}{\epsilon}} \right)^{\frac{1}{1-\epsilon}}
\]

and solve for $P_t$
\[ y_t = \left( \int (P_t(j)^{-\epsilon}) \frac{d j}{1 - \epsilon} \right) P^t y_t \]

\[ 0 = \left( \int (P_t(j)^{-\epsilon}) \frac{d j}{1 - \epsilon} \right) P^t \]

\[ P_t = \left( \int P_t(j)^{1-\epsilon} d j \right)^{\frac{1}{1-\epsilon}} \]

which yields

\[ P_t = \left( \int P_t(j)^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}} \quad (37) \]

### A.3.2 Intermediate Good Firms

A continuum of identical intermediate good firms use utilized capital \( \tilde{K}_t(j) \) and effective labor \( \tilde{N}_t(j) \) (both defined further below) as well as technology \( A_t \) to produce output \( y_t(j) \). They minimize cost and maximize profit by choosing the optimal inputs and price \( P_t(j) \) given the demand function (1) and factor prices. Price setting follows Calvo (1983). Firms can reset their prices with probability \( 1 - \phi \) every period. Those who cannot optimize price \( P_t(j) \) index their prices to the inflation rate. The production function is:

\[ y_t(j) = A_t \tilde{K}_t(j)^{\alpha} \tilde{N}_t(j)^{1-\alpha} \]

where \( \alpha \) is the share of capital in production.

Cost minimization given factor prices and subject to the demand function of the final goods producer leads to the following first order conditions:

\[
\min_{\tilde{N}_t(j), \tilde{K}_t(j)} \quad P_t w_t \tilde{N}_t(j) + P_t R_t \tilde{K}_t(j) \\
\text{s.t.} \quad A_t \tilde{K}_t(j)^{\alpha} \tilde{N}_t(j)^{1-\alpha} \geq \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon} y_t \quad (38)
\]

Forming the Lagrangian and maximizing the negative of the cost function:
\[
\max_{N_t(j), K_t(j)} L_t(j) = -P_t w_t \hat{N}_t(j) - P_t R_t \hat{K}_t(j) + P_t mc_t(j) \left( A_t \hat{K}_t(j) \right)^\alpha \hat{N}_t(j)^{1-\alpha} - \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon} y_t
\]

where \(P_t mc_t(j)\) is the Lagrangian multiplier, i.e. the marginal cost of producing one extra unit of output. \(w_t\) and \(R_t\) are the real factor cost for each input good.

The FOCs are, in real terms (dividing by \(P_t\)):
\[
\frac{\partial}{\partial \hat{N}_t(j)} = -w_t + mc_t(j)(1 - \alpha) y_t \hat{N}_t^{-1}(j) = 0
\]
\[
\frac{\partial}{\partial \hat{K}_t(j)} = -R_t + mc_t(j)\alpha y_t \hat{K}_t^{-1}(j) = 0
\] (39)

The real marginal cost is thus the ratio of the real wage to the marginal product of labor. Under competitive markets, this cost is equal to 1.

Total cost of production:
\[
\hat{N}_t(j) w_t + \hat{K}_t(j) R_t = mc_t y_t(j)
\]

Profit for each intermediary thus given by
\[
\Pi_t(j) = \frac{P_t(j) y_t(j)}{P_t} - mc_t(j) y_t(j)
\] (40)

Under Calvo pricing, firms can reset their price \(P_t(j)\) with probability \((1 - \phi)\) every period. With probability \(\phi\), prices can only be adjusted by an inflation index. This indexation to lagged inflation \((\pi_{t-1})\)is governed by parameter \(\xi_p \in [0, 1]\). Intermediate firms are owned by traders and future profit is discounted by their SDF \(\beta \lambda_t\), defined in the household problem below. Intermediate firms maximize expected future profit given the demand
function:

\[
\max_{P_t(j)} \mathbb{E}_t \sum_{s=0}^{\infty} (\beta \phi)^s \frac{\lambda_{t+s}}{\lambda_t} \left( \frac{\pi_{t-1,t+s-1}^{\xi_p} P_t(j)}{P_{t+s}} \right) y_{t+s}(j) - mc_{t+s}(j) y_{t+s}(j) \quad (41)
\]

\[
\text{s.t. } y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon} y_t
\]

Substituting for \( y_{t+s}(j) \), the problem becomes (and splitting it into two terms to make it easier to read):

\[
\max_{P_t(j)} \mathbb{E}_t \sum_{s=0}^{\infty} (\beta \phi)^s \frac{\lambda_{t+s}}{\lambda_t} \left( \frac{\pi_{t-1,t+s-1}^{\xi_p} P_t(j)}{P_{t+s}} \right) \left( \frac{\pi_{t-1,t+s-1}^{\xi_p} P_t(j)}{P_{t+s}} \right)^{-\epsilon} y_{t+s}
\]

\[
- \mathbb{E}_t \sum_{s=0}^{\infty} (\beta \phi)^s \frac{\lambda_{t+s}}{\lambda_t} \left( mc_{t+s}(j) \left( \frac{\pi_{t-1,t+s-1}^{\xi_p} P_t(j)}{P_{t+s}} \right)^{-\epsilon} y_{t+s} \right)
\]

Simplifying:

\[
\max_{P_t(j)} \mathbb{E}_t \sum_{s=0}^{\infty} (\beta \phi)^s \frac{\lambda_{t+s}}{\lambda_t} \left( \pi_{t-1,t+s-1}^{(1-\epsilon)\xi_p} P_{t-1}(j) P_{t+s}^{-\epsilon} y_{t+s} \right)
\]

\[
- \mathbb{E}_t \sum_{s=0}^{\infty} (\beta \phi)^s \frac{\lambda_{t+s}}{\lambda_t} \left( mc_{t+s}(j) \pi_{t-1,t+s-1}^{\epsilon} P_{t-1}(j) P_{t+s}^{\epsilon} y_{t+s} \right)
\]
Taking the FOC with respect to $P_t(j)$ results in:

$$\frac{∂}{∂P_t(j)} = (1 - \epsilon)P_t^{-\epsilon}(j)E_t \sum_{s=0}^{∞} (\beta \phi)^s \frac{λ_{t+s}}{λ_t} \left( \pi_{t-1,s+1}^{-1}y_{t+s}P_{t+s}^{-1}\right)$$

$$+ \epsilon P_t^{-\epsilon-1}(j)E_t \sum_{s=0}^{∞} (\beta \phi)^s \frac{λ_{t+s}}{λ_t} \left( mc_{t+s}(j)\pi_{t-1,s+1}^{-1}y_{t+s}P_{t+s}^\epsilon \right)$$

$$= 0$$

Next, multiply by $λ_t$ and divide by $P_t^{-\epsilon}(j)$:

$$(1 - \epsilon)A_t + \epsilon P_t^{-1}(j)B_t(j) = 0$$

where

$$A_t = E_t \sum_{s=0}^{∞} (\beta \phi)^s \lambda_{t+s} \left( \pi_{t-1,s+1}^{(1-\epsilon)}y_{t+s}P_{t+s}^{-(1-\epsilon)} \right)$$

$$B_t(j) = E_t \sum_{s=0}^{∞} (\beta \phi)^s \lambda_{t+s} \left( mc_{t+s}(j)\pi_{t-1,s+1}^{-\epsilon}y_{t+s}P_{t+s}^{\epsilon} \right)$$

And finally solve for $P_t(j)$

$$\frac{ε}{ε - 1} \cdot \frac{B(j)}{A} = P_t^\#(j)$$

Since the optimal choice for $P_t^\#(j)$ does not depend on individual firm characteristics $j$ ($mc_t(j)$ is the homogeneous across firms, as can be seen from firm FOCs in equations (39)), it is the same for all intermediaries: $P_t^\#$.

Further, $A_t$ and $B_t$ can be written in recursive form (dropping $j$):

$$A_t = \lambda_t y_t P_t^{\epsilon-1} + \beta \phi π_t^{\epsilon (1-\epsilon)}E_t A_{t+1}$$

$$B_t = \lambda_t mc_t y_t + \beta \phi π_t^{-\epsilon}E_t B_{t+1}$$
Since the price level is not stationary, define \( \hat{A}_t = \frac{A_t}{P_t} \) and \( \hat{B}_t = \frac{B_t}{P_t} \).

We get:

\[
\hat{A}_t = \lambda_t y_t + \beta \phi \pi^{(e-1)}_t \pi^{(e-1)}_t E_t \hat{A}_{t+1}
\]

\[
\hat{B}_t = \lambda_t mc_t y_t + \beta \phi \pi^{\epsilon}_t \pi^{\epsilon}_t E_t \hat{B}_{t+1}
\]

Equation (43) thus becomes:

\[
P_t^\# = \frac{\epsilon}{\epsilon - 1} \frac{\hat{B}_t}{\hat{A}_t} \frac{P^\epsilon_t}{P^{(\epsilon-1)}_t}
\]

And optimal price inflation \( \pi_t^\# = \frac{p^\#_t}{P_t^{(\epsilon-1)}} \) is:

\[
\pi_t^\# = \frac{\epsilon}{\epsilon - 1} \frac{\hat{B}_t}{\hat{A}_t} \pi_t
\] (44)

If firms could adjust prices every period, i.e. \( \phi = 0 \), then the optimality condition reduces to \( P_t^\# = \frac{\epsilon}{\epsilon - 1} mc_t P_t \). The optimal price is a markup over nominal marginal cost \( mc_t P_t \).

Since all intermediate firms are homogeneous, we can integrate across all intermediaries to get aggregate output \( y_t \). As a reminder, individual firm production is given by:

\[
A_t \hat{K}_t^\alpha(j) \hat{N}_t^{1-\alpha}(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon} y_t
\] (45)

All intermediaries hire capital and labor in the same ratio, and this ratio has to hold in the aggregate. Plugging in:

\[
A_t \hat{K}_t^\alpha(j) \hat{N}_t(j) = A_t \hat{K}_t^\alpha \hat{N}_t = \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon} y_t
\]

Next, integrate over all firms:

\[
A_t \hat{K}_t^\alpha \int \hat{N}_t(j) dj = y_t \int \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon} dj
\]
By labor market clearing, \( \int \hat{N}_t(j) \, dj = \hat{N}_t \) and define a measure of price dispersion \( v_t^p = \int \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon} \, dj \) to get an expression for aggregate output:

\[
y_t = \frac{A_t \hat{K}_t^\alpha \hat{N}_t^{1-\alpha}}{v_t^p}
\]  

(46)

Next, we need to solve for \( v_t^p \). A fraction of \((1 - \phi)\) intermediary firms choose optimal price \( P_{t^#}(j) \), whereas all other firms index their price to inflation with parameter \( \xi_p \). Thus, we can split the integral in \( v_t^p \) into two parts:

\[
v_t^p = \int_0^{1-\phi} \left( \frac{P_{t^#}(j)}{P_t} \right)^{-\epsilon} \, dj + \int_{1-\phi}^1 \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon} \, dj
\]

\( P_t(j) \) is indexed to lagged inflation by \( \pi_{t-1}^{\xi_p} P_{t-1}(j) \). Plugging in and solving the first term:

\[
v_t^p = (1 - \phi) \left( \frac{P_{t^#}}{P_t} \right)^{-\epsilon} P_t^\epsilon + \int_{1-\phi}^1 \pi_{t-1}^{\xi_p} P_{t-1}^{\epsilon-\epsilon} P_t^\epsilon \, dj
\]

Adding \( P_{t-1}^{\epsilon-\epsilon} P_t^\epsilon \) as factors to the integral simplifies the problem:

\[
v_t^p = (1 - \phi) \left( \frac{P_{t^#}}{P_t} \right)^{-\epsilon} P_t^\epsilon + \pi_{t-1}^{\xi_p} \int_{1-\phi}^1 P_{t-1}^{\epsilon-\epsilon} P_t^\epsilon \, dj
\]

\[
= (1 - \phi) \left( \frac{P_{t^#}}{P_t} \right)^{-\epsilon} P_t^\epsilon + \pi_{t-1}^{\xi_p} \pi_t \phi v_{t-1}^p
\]

Last, this equation can be written recursively in terms of inflation (since the price levels are not stationary):

\[
v_t^p = \pi_t^\epsilon \left( (1 - \phi) \left( \frac{P_{t^#}}{P_t} \right)^{-\epsilon} + \pi_{t-1}^{\xi_p} \phi v_{t-1}^p \right)
\]  

(47)
A.3.3 Traders

Traders are similar to the standard representative agent setup in NK models as in Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007). I assume that these agents belong to one large family, fraction $n_p$ of all agents, and within that family are fully insured against unemployment. The family head maximizes utility by choosing consumption, investment, capital utilization, next period’s capital stock and saving in real bonds subject to investment adjustment cost and utilization cost for each agent. When employed, an agent provides labor of 1 and 0 otherwise.

Investment is subject to quadratic investment cost $S_t(I_t, I_{t-1})$ when $\tau > 0$ which is paid in consumption units for changes in the level of investment per unit of investment, and includes an investment adjustment cost shock $e^{zt}$. These cost follow the form in Christiano, Eichenbaum and Evans (2005). Existing capital depreciates at rate $\delta K$. Investment per period is thus given by:

$$K_{t+1} - (1 - \delta K)K_t = e^{zt} \left(1 - \frac{\tau}{2} \left(\frac{I_t}{I_{t-1}} - 1\right)^2\right)I_t$$

(48)

Investment cost play a key role in representative agent NK models and as I will discuss later, one implication of adding households with a precautionary saving motive is that the importance of these costs decreases substantially.

Capital $K_t$ is utilized at rate $u_t$ to yield effective capital $\hat{K}_t = u_tK_t$. Utilization cost $\eta^K(u_t)$ are paid in consumption units and are calibrated that in steady state $u_{ss} = 1$ and $\eta^K(u_{ss}) = 0$. In addition, $\eta^K'(u_{ss}) > 0$ and $\eta^K''(u_{ss}) > 0$. These are modeled as in Christiano, Trabandt and Walentin (2010), chapter 7.

$$\eta^K(u_t) = \chi_1(1 - u) + \frac{\chi_2}{2} (u_t - 1)^2$$

(49)

Traders maximize utility given prices, saving in bonds from last period, the capital stock, the previous level of investment, the previous level of con-
sumption as well as the level of employment:

\[ V_t = \max_{c_t, I_t, u_t, b_{t+1}, K_{t+1}} \ln(c_t - bc_{t-1}) + \beta \mathbb{E}_t V_{t+1} \]

subject to

\[ c_t + I_t + b_{t+1} \leq R_t u_t K_t + w_t \omega N_t - \eta(u_t) K_t + (1 + r^i_t) \frac{1}{\pi_t} b_t + \frac{1}{n_p} \Pi_t + T^G_t \]

\[ K_{t+1} - (1 - \delta^K) K_t = e^{\lambda_t} \left( 1 - \frac{\tau}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right) I_t \]

\[ N_t = (1 - urate_t) \]

where, \( \Pi_t \) is the profit of the intermediaries, \( (1 + r^i_t) \) is the nominal interest rate and \( T^G_t \) are lump sum taxes.

Let \( \lambda_t \) and \( \mu_t \) be the multipliers on the budget constraint and capital accumulation equation. The Lagrangian is:

\[ \mathcal{L}_t = \ln(c_t - bc_{t-1}) \]

\[ + \lambda_t \left( R_t u_t K_t + w_t \omega N_t - \eta(u_t) K_t + (1 + r^i_t) \frac{1}{\pi_t} b_t + \frac{1}{n_p} \Pi_t + T^G_t - c_t - I_t - b_{t+1} \right) \]

\[ + \mu_t \left( e^{\lambda_t} \left( 1 - \frac{\tau}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right) I_t - K_{t+1} + (1 - \delta^K) K_t \right) \]

\[ + \beta \mathbb{E}_t V_{t+1} \]

The FOCs are:
\[
\begin{align*}
\frac{\partial V_t}{\partial c_t} &= \frac{1}{c_t - bc_t-1} - \lambda_t + \beta E_t \left[ \frac{\partial V_{t+1}}{\partial c_t} \right] = 0 \\
\frac{\partial V_t}{\partial K_{t+1}} &= -\mu_t + \beta E_{t+1} \left[ \frac{\partial V_{t+1}}{\partial K_{t+1}} \right] = 0 \\
\frac{\partial V_t}{\partial b_{t+1}} &= -\lambda_t + \beta E_t \left[ \frac{\partial V_{t+1}}{\partial b_{t+1}} \right] = 0 \\
\frac{\partial V_t}{\partial I_t} &= -\lambda_t + \mu_t e^{z_t} \left( \left( 1 - \frac{\tau}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right) - \tau \frac{I_t}{I_{t-1}} \left( \frac{I_t}{I_{t-1}} - 1 \right) \right) + \beta E_t \left[ \frac{\partial V_{t+1}}{\partial I_t} \right] = 0 \\
\frac{\partial V_t}{\partial u_t} &= \lambda_t (R_t K_t - \eta'(u_t) K_t) = 0
\end{align*}
\]

Where \( \eta'(u_t) = \chi_1 + \chi_2 (u_t - 1) \). The envelope conditions are:

\[
\frac{\partial V_t}{\partial c_{t-1}} = -b \frac{1}{c_t - bc_{t-1}}
\]

\[
\frac{\partial V_t}{\partial K_t} = \lambda_t (R_t u_t - \eta(u_t)) + \mu_t (1 - \delta^K)
\]

\[
\frac{\partial V_t}{\partial b_t} = \lambda_t (1 + i_t) \frac{1}{\pi_t}
\]

\[
\frac{\partial V_t}{\partial I_{t-1}} = -\mu_t e^{z_t} \tau \frac{I_t^2}{I_{t-1}^2} \left( \frac{I_t}{I_{t-1}} - 1 \right)
\]

Iterating the envelope conditions forward and plugging in yields the following optimality conditions:

\[
\lambda_t = \frac{1}{c_t - bc_t} - \beta E_t \left[ \frac{1}{c_{t+1} - bc_{t+1}} \right] \tag{50}
\]

\[
R_t = \chi_1 + \chi_2 (u_t - 1) \tag{51}
\]
\[ \lambda_t = (1 + i_t) \beta \mathbb{E}_t \left[ \frac{\lambda_{t+1}}{\pi_{t+1}} \right] \]  
\[ \lambda_t = \mu_t e^{\gamma t} \left( \left( 1 - \frac{\tau}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right) \right)^2 - \tau \frac{I_t}{I_{t-1}} \left( \frac{I_t}{I_{t-1}} - 1 \right) \right) \]  
\[ \lambda_t = \beta \mathbb{E}_t \left[ \mu_{t+1} e^{\gamma (t+1)} \left( \frac{I_{t+1}}{I_t} - 1 \right) \frac{I_{t+1}^2}{I_t^2} \right] \]  
\[ \mu_t = \beta \mathbb{E}_t \left[ \lambda_{t+1} \left( R_{t+1} u_{t+1} - \eta(u_{t+1}) \right) + \mu_{t+1} (1 - \delta^K) \right] \]

Equation 14 is the stochastic discount factor. Since traders are also firm owners, the SDF was used in the optimization problem of intermediaries. Equation 15 shows the role of capital utilization. By adjusting the utilization rate, interest rate shocks are not transmitted directly to the return on capital services. Equation 16 is the Euler Equation. And the last two equations are the optimality conditions for investment and capital utilization.

### A.3.4 Employed, limited Households

The family head then solves the following problem (in real terms):

\[ V^e_t = \max_{c^e_t, x^e_t} n^e_t ln(c^e_t) + \beta \mathbb{E}_t \left[ V^e_{t+1} + \sigma^u_{t+1} n^e_{t+1} V^u_{t+1} \right] \]

subject to

\[ n^e_t (c^e_t + x^e_t - T^G_t) \leq X^e_t + n^e_t w_t \]

\[ X^e_{t+1} = (1 - \sigma^u_{t+1}) n^e_t \frac{1}{\pi_{t+1}} (1 + i_t) x^e_t + D^u_{t+1} \]

\[ x^e_t \geq 0 \]

where \( x^e_t \) represents savings during the consumption period, and \( T^G_t \) is a lumpsum tax. Assuming that the BC constraint holds with equality, that \( x^e_t > 0 \) and substituting for \( X^e_t \) and in the BC, the first order conditions and envelope conditions are:
\[ \frac{\partial V_t^e}{\partial c_t^e} = n_t^e \frac{1}{c_t^e} - n_t^e \lambda_t = 0 \]

\[ \frac{\partial V_t^e}{\partial x_t^e} = -n_t^e \lambda_t + \beta_t E_t \left[ \frac{\partial V_{t+1}^e}{\partial x_t^e} + \sigma_t^u n_t^e \frac{\partial V_{t+1}^u}{\partial x_t^e} \right] = 0 \]

\[ \frac{\partial V_t^e}{x_{t-1}^e} = \lambda_t (1 - \sigma_t^u) n_{t-1}^e \frac{1}{\pi_t} (1 + i_{t-1}) \]

\[ \frac{\partial V_t^u}{\partial x_t^e} = \frac{1}{c_t^u} \frac{1}{\pi_t} (1 + i_{t-1}) \]

Where the last envelope condition is given by equation (55) of the problem for unemployed agents. Iterating forward the envelope conditions and substituting we get:

\[ \lambda_t = \frac{1}{c_t^e} \]

\[ n_t^e \lambda_t = \beta_t (1 + i_t) E_t \left[ \lambda_{t+1} (1 - \sigma_{t+1}^u) n_{t+1}^e \frac{1}{\pi_{t+1}} + \sigma_{t+1}^u n_{t+1}^e \frac{1}{c_{t+1}^u} \frac{1}{\pi_{t+1}} \right] \]

which together give the Euler Equation for employed, limited households:

\[ \frac{1}{c_t^e} = \beta_t (1 + i_t) E_t \left[ (1 - \sigma_{t+1}^u) \frac{1}{c_{t+1}^e} \frac{1}{\pi_{t+1}} + \sigma_{t+1}^u \frac{1}{c_{t+1}^u} \frac{1}{c_{t+1}^u} \frac{1}{\pi_{t+1}} \right] \]

### A.3.5 Unemployed, limited Households

Limited households are of two types: employed and unemployed. Unemployed households face fixed death probability \( \sigma^d \) and do not receive any income. These unemployed households solve the following problem:
\[ V_t^u = \max_{c_t^u, x_t^u} \ln(c_t^u) + \beta_e (1 - \sigma^d) \mathbb{E} V_{t+1}^u \]

subject to
\[ c_t^u + x_t^u \leq \frac{1}{\pi_t} (1 + r_{t-1}^i) x_{t-1}^u \]
\[ x_t^u \geq 0 \]

where \( x_t^u \) are real savings in bonds in period \( t \) and there is a no-borrowing limit. Bonds pay nominal interest rate \( r_t^i \). The FOCs are (with \( \lambda \) on the budget constraint\(^{30}\)):
\[
\frac{\partial V_t^u}{\partial c_t^u} = \frac{1}{c_t^u} - \lambda_t = 0 \\
\frac{\partial V_t^u}{\partial x_t^u} = -\lambda_t + \beta_e (1 - \sigma^d) \mathbb{E}_t \left[ \frac{\partial V_{t+1}^u}{\partial x_t^u} \right] = 0
\]

The envelope condition is:
\[
\frac{\partial V_t^u}{\partial x_{t-1}^u} = \lambda_t (1 + r_{t-1}^i) \frac{1}{\pi_t}
\]  \hspace{1cm} (55)

Iterating forward the envelope condition:
\[
\lambda_t = \frac{1}{c_t^u} \\
\lambda_t = \beta_e (1 - \sigma^d) \mathbb{E}_t \left[ \lambda_{t+1} \frac{1}{\pi_{t+1}} (1 + r_t^i) \right]
\]

Combining both results in the Euler Equation:
\[
\frac{1}{c_t^u} = \beta_e (1 - \sigma^d) (1 + r_t^i) \mathbb{E}_t \left[ \frac{1}{c_{t+1}^u} \frac{1}{\pi_{t+1}} \right]
\]

From the Euler Equation and the lifetime budget constraint, the perfect foresight solution becomes:

\(^{30}\)A binding no-borrowing constraint implies zero consumption, and therefore it will never bind
Thus, unemployed households consume a constant fraction of their income. This result allows me to aggregate all unemployed households.

A.3.6 Aggregate unemployment variables

Aggregate dynamics are described by three variables: consumption, $\bar{C}^u_t$, saving, $\bar{S}^u_t$, and transfers from the deceased households, $\bar{D}^u_t$.

Consumption consists of the newly unemployed agents’ consumption (no death probability in first period) and consumption of the surviving, previously unemployed agents.

$$c^u_t = (1 - \beta_c(1 - \sigma^d))(1 + r^i_{t-1})x^u_{t-1} = k^u_{\pi_t}(1 + i^u_{t-1})x^u_{t-1}$$

Aggregate saving combines the saving of the newly unemployed with the saving of the surviving unemployed:

$$\bar{S}^u_t = \frac{1}{\pi_t}(1 + r^i_{t-1})(1 - k^u_{\pi_t})(x^e_{t-1}n^e_{t-1}\sigma^u_t + (1 - \sigma^d)\bar{S}^u_{t-1})$$

And, lastly, transfers are the real savings of the deceased agents, out of the group of previously unemployed households:

$$\bar{D}^u_t = \sigma^d\frac{1}{\pi_t}(1 + r^i_{t-1})\bar{S}^u_{t-1}$$

A.3.7 Monetary Policy and Government

Monetary policy follows a Taylor rule that reacts to deviations of inflation from the target of steady state inflation and to economic growth (see e.g. Guerrieri and Iacoviello (2017)).
\[(1 + r_t^i) = (1 + r_{t-1}^i)R_t \left( \frac{\pi_t}{\pi_{SS}} \right)^{(1-r_R)\pi_t} \left( \frac{y_t}{y_{t-1}} \right)^{(1-r_R)\gamma} (1 + r_{SS}^i)^{(1-r_R)\epsilon_{r,t}} \]

where \( \epsilon_{r,t} \) is the monetary policy shock.

Government spending is an exogenous fraction of output:

\[G_t = \omega_t y_t\]

where \( \omega_t \) is follows an AR(1) process:

\[\omega_t = \rho_g \omega_{t-1} + (1 - \rho_g) \bar{\omega} + \epsilon^G_t\]

The government levies lumpsum taxes on traders and employed, limited households according to a balanced budget rule:

\[G_t = n_t^e T_t^G + n_p T_t^g\]

**A.3.8 Aggregation and Equilibrium**

In equilibrium, effective labor and capital markets clear:

\[
\hat{N}_t = n_t^e + (1 - urate_t)\eta_m_p \\
\hat{K}_t = n_p u_t K_t
\]

The bond market clears:

\[n_t^e x_t^e + S_t^u = n_p b_{t+1}\]

The aggregate resource constraint is:

\[y_t = G_t + n_t^e c_t^e + C_t^u + n_p (c_t + I_t + \eta^K (u_t) K_t)\]

The law of motion for capital is
\[ K_{t+1} - (1 - \delta^K)K_t = e^{z_t} \left( 1 - \frac{\tau}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right) I_t \]

The law of motion for limited, unemployed agents is:

\[ n_t^u = (1 - \sigma^d)n_{t-1}^u + \sigma_t n_{t-1}^e \]

The price index is given by equation (37):

\[ P_t = \left( \int P_t(j)^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}} \]

Since all intermediate firms are identical, we can rewrite this equation using the optimal price derived from Calvo pricing:

\[
\begin{align*}
P_t^{1-\epsilon} &= \int P_t(j)^{1-\epsilon} \\
&= \int_0^{1-\phi} P_t^{#(1-\epsilon)} dj + \int_{1-\phi}^1 \pi_t^{\xi_p(1-\epsilon)} P_t-1^{(j)^{1-\epsilon}} dj \\
&= (1 - \phi) P_t^{#(1-\epsilon)} + \phi \pi_t^{\xi_p(1-\epsilon)} \int_0^1 P_t-1^{(j)^{1-\epsilon}} dj \\
&= (1 - \phi) P_t^{#(1-\epsilon)} + \phi \pi_t^{\xi_p(1-\epsilon)} P_t-1^{1-\epsilon}
\end{align*}
\]

In the last step, I substituted in the term for the aggregate price level last period. Dividing both sides by \( P_t^{1-\epsilon} \) yields an equation for inflation:

\[
\pi_t^{1-\epsilon} = (1 - \phi) \pi_t^{#(1-\epsilon)} + \phi \pi_t^{\xi_p(1-\epsilon)}
\]

(56)

where \( \pi_t^{#} \) is defined in equation (44).

The real wage is determined by the wage equation:

\[
w_t = \left( \frac{w_{t-1}}{\pi_t} \right)^{\gamma_w} \left( \tilde{w} e^{z_t} \left[ \frac{n_t}{n_{ss}} \right] \gamma_w \right)^{(1-\gamma_w)}
\]

The shock processes are:
\[
\begin{align*}
\dot{z}_t^I &= \rho_I z_{t-1}^I + \epsilon_t^I \\
\dot{z}_t^w &= \rho_w z_{t-1}^w + \epsilon_t^w \\
\omega_t &= \rho_G \omega_{t-1} + (1 - \rho_G) \bar{\omega} + \epsilon_t^G \\
A_t &= \rho_A A_{t-1} + (1 - \rho_A) \bar{A} + \epsilon_t^A
\end{align*}
\]

Therefore, an equilibrium in this economy is a set of value and policy functions, a set of prices and a set government policies, such that given prices and state variables, (1) the policy functions solve the household problems of traders, limited and employed as well as limited and unemployed households, (2) firms maximize profits, (3) the bond, labor and capital markets clear and (4) government policy is given by the balanced budget equation and monetary policy rule.

A.4 The Cash Model

The firm side of this model is the same as in the benchmark economy, only some household equations and the market clearing conditions change.

A.4.1 Traders

Traders can also choose to hold cash, with an associated no-borrowing constraint. In addition, they are the sole recipients of transfers from the monetary authority $T_t^M$. 

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\[ V_t = \max_{c_t,I_t,u_t,bt+1,K_{t+1}} ln(c_t - bc_{t-1}) + \beta E_t V_{t+1} \]

subject to

\[ c_t + I_t + bt+1 + m_t \leq R_t u_t K_t + wt\omega N_t - \eta(u_t)K_t + (1 + r^i_t)\frac{1}{\pi_t}b_t + \frac{1}{\pi_t}m_{t-1} + \frac{1}{n_p} \frac{\Pi_t}{P_t} + T^G_t + \frac{1}{n_p} T^M_t \]

\[ K_{t+1} - (1 - \delta^K_t)K_t = e^{zt} \left( 1 - \frac{\tau}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right) \right) I_t \]

\[ N_t = (1 - urate_t) \]

\[ m_t \geq 0 \]

The additional FOC is:

\[ \frac{\partial V_t}{\partial m_t} = -\lambda_t + \mu^M_t + \beta E_t \left[ \frac{\partial V_{t+1}}{\partial m_t} \right] = 0 \]

and the envelope condition is:

\[ \frac{\partial V_t}{\partial m_{t-1}} = \lambda_t \frac{1}{\pi_t} \]

Together with the other FOCs there is an additional cash Euler Equation:

\[ \lambda_t = \beta E_t \frac{1}{\pi_t+1} \lambda_{t+1} + \mu^M_t \]

(57)

Since cash is dominated by bonds as long as the interest rate is non-zero, \( m_t = 0 \). When I solve the cash model I verify that this condition holds true.

### A.4.2 Employed, limited Households

The problem of the household head for employed but limited agents changes to:

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70
\[ V_t^e = \max_{c_t^e, x_t^e} n_t^e \ln(c_t^e) + \beta e \mathbb{E} \left[ V_{t+1}^e + \sigma_{t+1}^u n_t^e V_{t+1}^u \right] \]

subject to

\[ n_t^e (c_t^e + x_t^e - T_t) \leq X_t^e + n_t^e w_t \]

\[ X_{t+1} = (1 - \sigma_{t+1}) n_t^e \frac{1}{\pi_{t+1}} x_t^e + D_{t+1}^u \]

\[ x_t^e \geq 0 \]

where \( x_t^e \) represents cash savings during the consumption period and \( T_t \) are lumpsum taxes. Assuming that the BC constraint holds with equality, that and that \( x_t^e > 0 \), the first order conditions and envelope conditions are:

\[ \frac{\partial V_t^e}{\partial c_t^e} = n_t^e \frac{1}{c_t^e} - n_t^e \lambda_t = 0 \]

\[ \frac{\partial V_t^e}{\partial x_t^e} = -n_t^e \lambda_t + \beta e \mathbb{E}_t \left[ \frac{\partial V_{t+1}^e}{\partial x_t^e} + \sigma_{t+1}^u n_t^e \frac{\partial V_{t+1}^u}{\partial x_t^e} \right] = 0 \]

\[ \frac{\partial V_t^e}{x_{t-1}^e} = \lambda_t (1 - \sigma_t^u) n_{t-1}^e \frac{1}{\pi_t} \]

\[ \frac{\partial V_t^u}{\partial x_t^e} = \frac{1}{c_t^u} \frac{1}{\pi_t} \]

Where the last envelope condition is given by equation (58). Iterating forward the envelope conditions and substituting we get:

\[ \lambda_t = \frac{1}{c_t^e} \]

\[ n_t^e \lambda_t = \beta e \mathbb{E}_t \left[ \lambda_{t+1} (1 - \sigma_{t+1}^u) n_{t+1}^e \frac{1}{\pi_{t+1}} + \sigma_{t+1}^u n_t^e \frac{1}{c_{t+1}^t} \frac{1}{\pi_{t+1}} \right] \]

which together give the Euler Equation for employed, limited households:

\[ \frac{1}{c_t^e} = \beta e \mathbb{E}_t \left[ (1 - \sigma_{t+1}^u) \frac{1}{c_{t+1}^t} \frac{1}{\pi_{t+1}} + \sigma_{t+1}^u \frac{1}{c_{t+1}^u} \frac{1}{\pi_{t+1}} \right] \]

This Euler Equation does not feature the interest rate.
A.4.3 Unemployed, limited Households

As with limited, employed households, the problem of unemployed agents does not contain the interest rate.

\[
V_t^u(x_{t-1}^u, c_t^u, x_t^u) = \max_{c_t^u, x_t^u} \ln(c_t^u) + \beta_e(1 - \sigma^d)\mathbb{E}V_{t+1}^u(x_{t}^u)
\]

subject to

\[
c_t^u + x_t^u \leq \frac{1}{\pi_t}x_{t-1}^u
\]

\[
x_t^u \geq 0
\]

where \( x_t^u \) are real savings in cash in period \( t \) and there is a no-borrowing limit. The FOCs are (with \( \lambda \) on the budget constraint\(^{31} \)):

\[
\frac{\partial V_t^u}{\partial c_t^u} = \frac{1}{c_t^u} - \lambda_t = 0
\]

\[
\frac{\partial V_t^u}{\partial x_t^u} = -\lambda_t + \beta_e(1 - \sigma^d)\mathbb{E}_t \left[ \frac{\partial V_{t+1}^u}{\partial x_t^u} \right] = 0
\]

The envelope condition is:

\[
\frac{\partial V_t^u}{\partial x_{t-1}^u} = \lambda_t \frac{1}{\pi_t} \tag{58}
\]

Iterating forward the envelope condition:

\[
\lambda_{t+1} \frac{1}{\pi_{t+1}} = \beta_e(1 - \sigma^d)\mathbb{E}_t \left[ \lambda_{t+1} \frac{1}{\pi_{t+1}} \right]
\]

Combining both results in the Euler Equation:

\[
\frac{1}{c_t^u} = \beta_e(1 - \sigma^d)\mathbb{E}_t \left[ \frac{1}{c_{t+1}^u} \frac{1}{\pi_{t+1}} \right]
\]

\(^{31}\)A binding no-borrowing constraint implies zero consumption, and therefore it will never bind.
From the Euler Equation and the lifetime budget constraint, the perfect foresight solution becomes:

\[ c_t^u = (1 - \beta_t(1 - \sigma^d)) \frac{1}{\pi_t} x_{t-1}^u = k_u \frac{1}{\pi_t} x_{t-1}^u \]

Thus, unemployed households consume a constant fraction of their income. This result allows me to aggregate all unemployed households.

A.4.4 Aggregate unemployment variables

The aggregate laws of motion for unemployed agents become:

\[
\bar{C}_t^u = \sigma^u_t n_{t-1}^e c_{t-1}^u + (1 - \sigma^d) k_u \frac{1}{\pi_t} \bar{S}_{t-1}^u
\]

\[
\bar{S}_t^u = \frac{1}{\pi_t} \left( x_{t-1}^e n_{t-1}^e \sigma_t^u (1 - k_u) + (1 - \sigma^d)(1 - k_u) \bar{S}_{t-1}^u \right)
\]

\[
D_t^u = \sigma^d \frac{1}{\pi_t} \bar{S}_{t-1}^u
\]

A.4.5 Equilibrium

In addition to the equilibrium conditions of the benchmark model, the following changes apply:

Bond market clearing

\[
\int_0^{n_p} b_i^t di = 0
\]

Money market clearing

\[
n_t^e x_t^e + S_t^u + n_p m_t = M_t
\]

where \( M_t \) is the aggregate real money supply which evolves according to:
\[ M_t = \frac{1}{\pi_i} M_{t-1} + T^M_t \]

with \( T^M_t \) is the transfer from the monetary authority.
A.5 Representative Agent Model

The representative agent version of the benchmark model essentially sets the fraction of limited agents to 0 and therefore $n_p = 1$. The representative agent solves the same problem as traders in the benchmark model. Substituting $n^e_i = n^u_i = 0$ into the benchmark model yields the representative agent model.

A.5.1 Estimation

The representative agent model is calibrated as the benchmark model and estimated with the same priors. The result is shown in table 4:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Prior distribution [bounds]</th>
<th>Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_w$</td>
<td>wage inertia</td>
<td>beta [0.5,0.1]</td>
<td>0.8058</td>
</tr>
<tr>
<td>$\phi_w$</td>
<td>wage adjustment</td>
<td>gamma [1.0,2]</td>
<td>1.0053</td>
</tr>
<tr>
<td>$b$</td>
<td>habit persistence</td>
<td>beta [0.7,0.1]</td>
<td>0.7761</td>
</tr>
<tr>
<td>$\tau$</td>
<td>investment cost</td>
<td>normal [1,0.5]</td>
<td>1.6071</td>
</tr>
<tr>
<td>$\phi_p$</td>
<td>calvo probability</td>
<td>beta [0.5,0.1]</td>
<td>0.7071</td>
</tr>
<tr>
<td>$\zeta_p$</td>
<td>inflation index</td>
<td>beta [0.5,0.2]</td>
<td>0.0697</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>output Taylor</td>
<td>normal [0.125,0.05]</td>
<td>0.1797</td>
</tr>
<tr>
<td>$\phi_x$</td>
<td>inflation Taylor</td>
<td>normal [1.5,0.1]</td>
<td>1.6905</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>inertia Taylor</td>
<td>beta [0.9,0.05]</td>
<td>0.8280</td>
</tr>
<tr>
<td>$\rho_w$</td>
<td>AR(1) wage shock</td>
<td>beta [0.9,0.05]</td>
<td>0.9243</td>
</tr>
<tr>
<td>$\rho_u$</td>
<td>AR(1) TFP shock</td>
<td>beta [0.9,0.05]</td>
<td>0.9588</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>AR(1) gov shock</td>
<td>beta [0.9,0.05]</td>
<td>0.9790</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>AR(1) investment cost shock</td>
<td>beta [0.9,0.05]</td>
<td>0.9163</td>
</tr>
<tr>
<td>$\sigma_w$</td>
<td>sd wage shock</td>
<td>inv gamma [0.01,0.002]</td>
<td>0.0590</td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>sd Taylor shock</td>
<td>inv gamma [0.002,0.002]</td>
<td>0.0014</td>
</tr>
<tr>
<td>$\sigma_u$</td>
<td>sd TFP shock</td>
<td>inv gamma [0.01,0.002]</td>
<td>0.0052</td>
</tr>
<tr>
<td>$\sigma_g$</td>
<td>sd gov shock</td>
<td>inv gamma [0.01,0.002]</td>
<td>0.0046</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>sd investment cost shock</td>
<td>inv gamma [0.01,0.002]</td>
<td>0.0147</td>
</tr>
</tbody>
</table>
A.6 Steady State

The steady state wage is derived as follows: Prices are constant in steady state and real marginal cost are given by equation (43) as \( mc = \frac{\epsilon - 1}{\epsilon} \), where I set \( \epsilon = 10 \). As discussed in the data section, the share of traders is set to \( n^p = 40\% \). Next, the equilibrium unemployment rate of 6.28\% gives \( n^e = (1 - n^p)N_{agg}^{SS} \) and the effective labor supply as \( \hat{N} = n^e + n^p \omega N_{agg}^{SS} \). I follow Challe and Ragot (2016) and set \( \omega = 1.5 \).

From here, the steady state wage is:

\[
w_{ss} = (\chi_1 (mc \alpha \hat{N}^{1-\alpha})^{-1})^{1/(-1)}
\]

with \( \alpha = 1/3 \).

Figure A.6: Comparison of model implied checking account balance to income ratio with beta of 0.96 and 0.945. The higher beta helps to quantitatively match the SCF data, but does not change the qualitative result.

The limited, employed household discount factor is set to \( \beta_e = 0.96 \) to match the average checking to income ratio in the data to more closely align the implied time series of this ratio with the data. Figure A.6 shows that the
higher value of beta mainly reduces the magnitude of this ratio but not the trend.
A.7 Time Series

All time series were extracted from the St. Louis Fed FRED data base fred.stlouisfed.org for the time 1982Q1 to 2017Q1.

Table 5: Time Series

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLF16OV</td>
<td>Civilian Labor Force, Thousands of Persons, Quarterly, Seasonally Adjusted</td>
</tr>
<tr>
<td>GDPC1</td>
<td>Real Gross Domestic Product, Billions of Chained 2009 Dollars, Quarterly, Seasonally Adjusted Annual Rate</td>
</tr>
<tr>
<td>GDPDEF</td>
<td>Gross Domestic Product: Implicit Price Deflator, Index 2009=100, Quarterly, Seasonally Adjusted</td>
</tr>
<tr>
<td>UEMPMEAN</td>
<td>Average (Mean) Duration of Unemployment, Weeks, Quarterly, Seasonally Adjusted</td>
</tr>
<tr>
<td>UNRATE</td>
<td>Civilian Unemployment Rate, Percent, Quarterly, Seasonally Adjusted</td>
</tr>
<tr>
<td>TB3MS</td>
<td>3-Month Treasury Bill: Secondary Market Rate, Percent, Quarterly, Not Seasonally Adjusted</td>
</tr>
<tr>
<td>PCDG</td>
<td>Personal Consumption Expenditures: Durable Goods, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate</td>
</tr>
<tr>
<td>DDURRD3Q086SBEA</td>
<td>Personal consumption expenditures: Durable goods (implicit price deflator), Index 2009=100, Quarterly, Seasonally Adjusted</td>
</tr>
<tr>
<td>PNFI</td>
<td>Private Nonresidential Fixed Investment, Billions of Dollars, Quarterly, Seasonably Adjusted Annual Rate</td>
</tr>
<tr>
<td>A008RD3Q086SBEA</td>
<td>Gross private domestic investment: Fixed investment: Nonresidential (implicit price deflator), Index 2009=100, Quarterly, Seasonally Adjusted</td>
</tr>
</tbody>
</table>

The following time series were used in the estimation of the model:

1. Change in log real GDP per capita: GDPC1 divided by CLF16OV and then demeaned. Model variable: $\Delta y_t$.

2. Inflation: GDPDEF demeaned and divided by 400. Model variable: $\pi_t - 1$.


4. Interest Rate: TB3MS divided by 400 and minus average inflation. Model variable: $r_t^i - 1$.
5. Change in log real investment per capita: Weighted average of PCDG and PNFI. Both time series were converted to real terms with DDURRD3Q086SBEA and A008RD3Q086SBEA respectively and to per capita terms with CLF16OV. The weights are the one period lagged share of nominal PCDG (D) and PNFI (FI):

\[ \ln \Delta I_t = \ln D_t \left( \frac{D_{t-1}^N}{D_{t-1}^N + F_{t-1}^N} \right) + \ln F_t \left( \frac{F_{t-1}^N}{D_{t-1}^N + F_{t-1}^N} \right) \]

where the superscript N indicates nominal variable. All variables are in per capita terms and demeaned. Model variable: \( \Delta I_t \).

To calibrate the death probability of limited, unemployed agents I convert UEMPMEAN to quarters and take the average. Figure A.7 shows the 5 time series used as well as unemployment duration.

![Figure A.7](image-url)

Figure A.7: Final time series used in estimation and calibration of the model, period 1982Q2 to 2017Q1.
A.8 Estimation Plots

The following figures show the posterior distributions and estimated shocks of the benchmark model.

Figure A.8: Posterior Distributions 1
Figure A.9: Posterior Distributions 2

Figure A.10: Smoothed Shocks
A.9 IRF Monetary Policy Shock

The result of Werning (2015) holds in the benchmark economy and the response of inflation to a policy shock is very similar in the estimated representative agent model compared to the benchmark model.

Figure A.11: Comparison of IRFs after a monetary policy shock in the benchmark model and representative agent version.
A.10 Comparison of Aggregate and Actual Real Interest Rate

Canzoneri, Cumby and Diba (2007) show that for representative agent models, the model implied real interest rate is negatively correlated with the real interest rate in the data. The benchmark model is able to partially solve this puzzle. While the correlation of real interest rates is not negative, the actual interest rate in the model moves in opposite direction from the interest rate that is implied by aggregate consumption in this model especially at the beginning and end of recessions.

After estimating the benchmark model, I feed the shocks back into the model and calculate the implied aggregate interest rate following Canzoneri, Cumby and Diba (2007) by feeding aggregate consumption growth as well as inflation into a representative agent version of the Euler Equation: \[
\frac{1}{C_{t+1}^{agg}} = \beta (1 + r_{t}^{i}) \frac{1}{\pi_{t+1}} \frac{1}{\pi_{t}^{agg}}.
\]
Figure A.12 plots the resulting \((1 + r_{t}^{i}) \frac{1}{\pi_{t+1}}\) together with the actual real interest rate of the benchmark model. The graph looks similar to figure 1 in their paper (which covers the period 1966 to 2004).

A.11 Downward Rigid Wages

To simulate downward rigid wages, I change the wage parameters \(\gamma_{w}\) to 0.95. The algorithm switches between both models based on whether the nominal wage is increasing or decreasing. The more rigid wage equation takes effect when nominal wages are decreasing. The two criteria are as follows. When the economy is in the benchmark model, \(w_{t} < \frac{1}{\pi_{t}} w_{t-1}\) and under the more rigid regime: \(w_{t}^{flex} > \frac{1}{\pi_{t}} w_{t-1}^{flex}\), where \(w_{t}\) and \(w_{t}^{flex}\) are determined by the more flexible wage equation. When the economy is in the more rigid state, however, \(w_{t}\) is determined by the rigid wage equation.
Figure A.12: Comparison of the Aggregate (dashed) and Actual Real Interest Rate (dotted).
Figure A.13: Comparison of IRFs in benchmark model and extended model with downward rigid wages. The bottom right graphs shows that the constraint binds for the first six periods.