Physics 157 Final Exam Review Package - Solutions

UBC Engineering Undergraduate Society

Attempt questions to the best of your ability. This review package consists of 28 pages, including 1 cover page and 18 questions. Problems are ranked in difficulty as (*) for easy, (**) for medium, and (***) for difficult. Difficulty is subjective, so don’t be discouraged if you are stuck on a (*) problem.

Solutions posted at: https://ubcengineers.ca/tutoring/

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Want a warm up? These are the easier problems
[1,2,4,5]

Short on study time? These cover most of the material
[1,8,10,11,17,18]

Want a challenge? These are some tougher questions
[13,14,15,16,17,18]

Some of the problems in this package were not created by the EUS. Those problems originated from one of the following sources:

• Fundamentals of Physics / David Halliday, Robert Resnick, Jearl Walker. – 9th ed.
• Exercises for the Feynman Lectures on Physics / Matthew Sands, Richard Feynman, Robert Leighton.

All solutions prepared by the EUS.

EUS Health and Wellness Study Tips

• Eat Healthy—Your body needs fuel to get through all of your long hours studying. You should eat a variety of food (not just a variety of ramen) and get all of your food groups in.

• Take Breaks—Your brain needs a chance to rest: take a fifteen minute study break every couple of hours. Staring at the same physics problem until your eyes go numb won’t help you understand the material.

• Sleep—We have all been told we need 8 hours of sleep a night, university shouldn’t change this. Get to know how much sleep you need and set up a regular sleep schedule.

Good Luck!
1. A 103 kg non-uniform log hangs by two steel wires, $A$ and $B$, both of radius $1.20 \text{ mm}$. Initially, wire $A$ was 2.5 m long, and wire $B$ 2 mm longer than that. The log is now horizontal. If the Young’s modulus for steel is $Y = 200 \text{ GPa}$, what are the magnitudes of the forces on the log from,

(a) wire $A$?
(b) wire $B$?
(c) what is the ratio $d_A/d_B$?

Solution:

- Let $A = 0.0012^2 \pi$ be the cross sectional area of the wires.
- Let $l_A = 2.5 \text{ m}$, $l_B = 2.502 \text{ m}$, and $l'_A = l'_B$ be the lengths of the wires after stretching.

From the stress/strain relations, we have

$$\frac{F_A}{A} = \frac{Y l'_A - l_A}{l_A} \quad (1.1)$$

$$\frac{F_B}{A} = \frac{Y l'_B - l_B}{l_B} \quad (1.2)$$

From the force equilibrium, we have

$$F_A + F_B = 103 \cdot g \quad (1.3)$$

Multiplying both (1.1) and (1.2) through by $A$, then substituting into (1.3), we then have

$$F_A + F_B = AY \frac{l'_A - l_A}{l_A} + AY \frac{l'_B - l_B}{l_B} = 103 \cdot g \quad (1.4)$$

Substituting in some known numerical values, as well as $l'_A = l'_B$,

$$\frac{l'_A - 2.5}{2.5} + \frac{l'_A - 2.502}{2.502} = \frac{103 g}{AY} = 0.00111 \quad (1.5)$$

(a) From this, we obtain

$$l'_A = l'_B = 2.502395 \text{ m}$$

Substituting this back in to (1.1) and (1.2), we find that

$$F_A = 866 \text{ N}$$
(b) \[ F_B = 143 \text{ N} \]

(c) From the torque equilibrium, we have

\[ d_A F_A = d_B F_B \]  \hspace{1cm} (1.6)

so

\[ \frac{d_A}{d_B} = \frac{F_B}{F_A} = 0.165 \]  \hspace{1cm} (1.7)
2. Three equal length straight rods, of aluminum, Invar, and steel, all at 20.0 °C, form an equilateral triangle with hinge pins at the vertices. At what temperature will the angle opposite the Invar be 59.95? The coefficients of thermal expansion for aluminum, Invar, and steel, are $23 \cdot 10^{-6} / ^\circ C$, $0.7 \cdot 10^{-6} / ^\circ C$, and $11 \cdot 10^{-6} / ^\circ C$ respectively.

Solution:

- Let the length of each bar at 20°C be $L$.
- Let $L'_A$ be the length of the aluminum bar after changing temperature
- Let $L'_I$ be the length of the Invar bar after changing temperature
- Let $L'_S$ be the length of the steel bar after changing temperature.

Then, using the thermal expansion formula,

$$L'_A = L(1 + 23 \cdot 10^{-6} \Delta T) \quad (2.1)$$
$$L'_I = L(1 + 0.7 \cdot 10^{-6} \Delta T) \quad (2.2)$$
$$L'_S = L(1 + 11 \cdot 10^{-6} \Delta T) \quad (2.3)$$

Plugging these expanded lengths (all of which depend on $\Delta T$) into the cosine law,

$$L'_I^2 = L'_A^2 + L'_S^2 - 2L'_A L'_S \cos(59.95^\circ) \quad (2.4)$$

Any second order terms will be ignored because thermal expansion coefficients are already small, and their squares are even smaller. Thus, plugging in (2.1), (2.2), (2.3) into (2.4), we solve for $\Delta T$.

The result is

$$\Delta T = 48^\circ C$$

so then we conclude that the temperature needs to be raised to 68 °C.
3. (∗) Given the following stress vs strain curve for a material under tension, find the following values/intervals:

(a) Proportional limit
(b) region of elastic behaviour
(c) region of plastic deformation
(d) Young’s modulus of the material
(e) Fracture point

Solution: This question only tests one’s knowledge of the definitions of the terms described in the textbook about stress-strain curves.

(a) Proportional limit is at strain = 0.004
(b) region of elastic behaviour is for strain between 0 and 0.004
(c) region of plastic deformation is for strain between 0.004 and 0.1 (this is where the material would not return to regular length if released from tension)
(d) Young’s modulus is the slope of the curve in the elastic behaviour region, so

\[ Y = \frac{225 \text{ MPa}}{0.004} = 56.3 \text{ GPa} \]

(e) Fracture point is at strain = 0.1 (this is where the material breaks)
4. A rigid beam ABC of length $2L$ is hinged to a fixed rigid wall at end A and horizontally supported using a vertical hanger rod hinged at the other end C. The hanger rod DC is uniform with cross sectional area $A = 100\text{mm}^2$, and length $l = 3\text{ m}$. The end D of the hanger rod is hinged to a fixed rigid ceiling as shown. A vertical downward load $P = 70\text{ kN}$ is applied at the mid-point B of the beam ($AB = L = BC$). The material of the hanger rod has obeys the stress-strain curve shown. Determine the following values:

(a) Normal (tensile) stress in the hanger rod
(b) Downward movement at the point of load application (downward movement at point B)

![Diagram of beam and hanger rod](image)

**Solution:**

(a) Let $A_y$ be the upward force on the horizontal rod at A, and $C_y$ be the upward force on the horizontal rod at C. Then

$$A_y + C_y = P$$

and by symmetry, $A_y = C_y = P/2$.

The stress in the hanger rod will be $35 \cdot 10^6 / 100 \cdot 10^{-6} = 350\text{ MPa}$.

(b) Note that the value of 350 MPa falls outside the elastic region. Thus we have to use the graph value to find the strain at this value of stress. We can see that we have a strain of 0.002, so then

$$\delta = 0.002l = 0.002(3) = 0.006\text{ m}$$

Thus point C moves down by 6 mm. This means that point B moves down by 3 mm, since it is halfway along the rod. (Later, in Solid Mechanics, you will learn that this isn’t actually the case, but for now, this approximation will do)
5. In the figure, string 1 has a linear density of 3.00 g/m, and string 2 has a linear density of 5.00 g/m. They are under tension due to the hanging block of mass $M = 500$ g. Calculate the wave speed on

(a) string 1
(b) string 2
(c) Find $M_1, M_2$ such that the wave speeds in the two strings are equal.

**Solution:** Both strings will have tension $T = 0.25g$ in them.

(a) 
\[ v_1 = \sqrt{\frac{T}{\mu_1}} = \sqrt{\frac{0.25g}{0.003}} = 28.6 \text{ m/s} \]

(b) 
\[ v_2 = \sqrt{\frac{T}{\mu_2}} = \sqrt{\frac{0.25g}{0.005}} = 22.1 \text{ m/s} \]

(c) We set the two speeds equal, so $v_1 = v_2$. Thus

\[ \sqrt{\frac{T_1}{\mu_1}} = \sqrt{\frac{T_2}{\mu_2}} \]

Since the block is divided, we have a system of equations:

\[ \begin{cases} M_1 + M_2 = 0.5 \\ T_1 = M_1 g \\ T_2 = M_2 g \end{cases} \]

Solving the system of equations,

\[ M_1 = 188 \text{ grams} \]
\[ M_2 = 312 \text{ grams} \]
6. A heat engine is proposed that operates with heat reservoirs at temperatures $T_1$ and $T_2$, using a monatomic ideal gas as the working fluid. The engine cycle consists of the following three steps:

(i) Expansion at constant pressure, $P$, from $V_1$ and $T_1$ to $V_2$ and $T_2$.
(ii) Cooling at constant volume from $T_2$ to $T_1$.
(iii) Reversible isothermal compression at $T_1$ from $V_2$ to $V_1$.

(a) Draw the cycle on a $P$–$V$ diagram.
(b) Calculate the heat and work for each step in the cycle.
(c) Derive an expression for the efficiency of the engine in terms of $T_2$ and $T_1$.
(d) For $T_2 = 600$ K and $T_1 = 300$ K, what is the efficiency of this engine? How does this compare with the efficiency of a Carnot cycle engine operating between reservoirs at the same two temperatures?

**Solution:**

(a)

(b) (i) This process is isobaric, so the equations for $W$ and $Q$ will be:

$$W = P(V_2 - V_1) \quad (6.1)$$

$$Q = U + W = nC_P(T_2 - T_1) \quad (6.2)$$

(ii) This process is isochoric, so there is no work done.

$$W = 0 \quad (6.3)$$

$$Q = nC_V(T_1 - T_2) \quad (6.4)$$

(iii) This process is isothermal, so then there is no change in internal energy.

$$W = nRT_1 \ln \left( \frac{V_1}{V_2} \right) \quad (6.5)$$

$$Q = nRT_1 \ln \left( \frac{V_1}{V_2} \right) \quad (6.6)$$
(c) Only some of these $Q$’s are positive. Only the ones that are positive contribute to the *heat added*. The positive one is the one from Step (i).

$$ e = \frac{\sum W}{\sum Q_H} = \frac{P(V_2 - V_1) + nRT_1 \ln(V_1/V_2)}{nC_P(T_2 - T_1)} \quad (6.7) $$

Now we need to perform some substitutions so that the answer is only in terms of $T_1$ and $T_2$. Since $C_P = 5R/2$ (monatomic gas), and we have the ideal gas law

$$ PV_1 = nRT_1 $$
$$ PV_2 = nRT_2 $$

Which means that we have the ratio

$$ \frac{V_1}{V_2} = \frac{T_1}{T_2} $$

we can do some substitutions and cancellations to find:

$$ e = \frac{T_2 - T_1 + T_1 \ln(T_1/T_2)}{(5/2)(T_2 - T_1)} \quad (6.8) $$

(d) Plugging in those values, we obtain

$$ e = \frac{2(1 + \ln(0.5))}{5} = 12.3\% $$

The Carnot efficiency is 50%, which is significantly higher than what was obtained for this particular engine.
7. A sinusoidal wave of angular frequency 1200 rad/s and amplitude 3.00 mm is sent along a cord with linear mass density 2.00 g/m and tension 1200 N.

(a) What is the average rate at which energy is transported by the wave to the opposite end of the cord?

(b) If, simultaneously, an identical wave travels along a separate, identical cord, what is the total average rate at which energy is transported to the opposite ends of the two cords by the waves?

(c) If, instead, those two waves are sent along the same cord simultaneously, what is the total average rate at which they transport energy when their phase difference is

(i) 0
(ii) 0.4\pi\text{ rad}
(iii) \pi\text{ rad}

Solution:

(a) We use the formula for power transfer along a string, so

\[ P = \frac{1}{2} \mu \omega^2 A^2 v = 10 \text{ W} \]

(b) The total power transferred will just be double of the previous one, so 20 W. This is because there is no superposition involved, since the waves travel on distinct strings.

(c) (i) The amplitudes will add together, so we will have double the amplitude and thus four times the power of part (a). So it becomes 40 W.

(ii) If we add two waves, we will have to use the trigonometric identity

\[ \sin \theta_1 + \sin \theta_2 = 2 \cos \left( \frac{\theta_1 - \theta_2}{2} \right) \sin \left( \frac{\theta_1 + \theta_2}{2} \right) \]

Since \( \theta_1 = \omega t \), and \( \theta_2 = \omega t + 0.4\pi \), we have 2A cos(0.2\pi) as the new amplitude, so plugging this amplitude into the formula yields

\[ P = 26.3 \text{ W} \]

(iii) There will be destructive interference all along the wire, so then no power is transferred and

\[ P = 0 \text{ W} \]
8. Consider a planet in a binary star system. There are two stars, $A$ and $B$. The planet is $r_A$ away from star $A$, which emits total power $P_A$. The planet is $r_B$ away from star $B$, which emits total power $P_B$. The planet has albedo $\alpha$ and emissivity $e$. What is the temperature $T$ of the planet? Express your answer in terms of the given variables.

**Solution:** The intensities of radiation of each star at the planet are

$$ I_A = \frac{P_A}{4\pi r_A^2} \quad (8.1) $$

$$ I_B = \frac{P_B}{4\pi r_B^2} \quad (8.2) $$

Let $S$ be the total surface area of the planet. The power absorbed by the planet is given by

$$ P_{in} = \frac{S}{4} (1 - \alpha)(I_A + I_B) \quad (8.3) $$

$$ = \frac{S(1 - \alpha)}{16\pi} \left[ \frac{P_A}{r_A^2} + \frac{P_B}{r_B^2} \right] \quad (8.4) $$

Note that we use $S/4$ because the cross sectional area of a sphere is a quarter of the sphere’s total surface area. The power emitted by the planet is given by

$$ P_{out} = e\sigma ST^4 \quad (8.5) $$

Because the planet is in thermal steady state, we equate these two quantities.

$$ P_{in} = P_{out} \quad (8.6) $$

$$ \frac{S(1 - \alpha)}{16\pi} \left[ \frac{P_A}{r_A^2} + \frac{P_B}{r_B^2} \right] = e\sigma ST^4 \quad (8.7) $$

Solving for $T$ yields

$$ T = \sqrt[4]{\frac{(1 - \alpha)}{16\pi e\sigma} \left[ \frac{P_A}{r_A^2} + \frac{P_B}{r_B^2} \right]} \quad (8.8) $$
9. A girl is sitting near the open window of a train that is moving at a velocity of 10.00 m/s to the East. The girl’s uncle stands near the tracks and watches the train move away. The locomotive whistle emits sound at frequency 500.0 Hz. The air is still.

(a) What frequency does the uncle hear?
(b) What frequency does the girl hear?

A wind begins to blow from the east at 10.00 m/s.
(c) What frequency does the uncle now hear?
(d) What frequency does the girl now hear?

Solution: The shifted frequency due to Doppler shift is given by

\[ f' = f \cdot \left( \frac{v \pm v_D}{v \mp v_S} \right) \]  

(9.1)

where \( f \) is the unshifted frequency, \( v \) is the speed of the wave (speed of sound), \( v_D \) is the speed of the detector/observer, and \( v_S \) is the speed of the source. We will take velocities with respect to the uncle/ground.

(a) The uncle (detector/observer) is at rest with respect to the ground, so \( v_D = 0 \). The speed of the source is \( v_S = 10 \) m/s. The train is moving away, choose the plus sign for the denominator. Thus

\[ f' = 500 \left( \frac{343}{343 + 10} \right) = 485.8 \text{ Hz} \]

(b) The girl is now the detector/observer. She is in the same reference frame as the locomotive’s whistle, and thus has no relative velocity to the whistle. Thus \( (v + v_D) = (v + v_S) \) and

\[ f' = f = 500.0 \text{ Hz} \]

(c) For parts (c) and (d) we will have to take velocities relative to the air. This is because the wave is moving with its wave speed 343 m/s with respect to the air, and we can then use the familiar formula. Relative to the air the locomotive is moving at \( v_S = 20.00 \) m/s away from the uncle. Use the plus sign in the denominator. Relative to the air the uncle is moving at \( v_D = 10.00 \) m/s toward the locomotive. Use the plus sign in the numerator. Thus

\[ f' = f \cdot \left( \frac{v \pm v_D}{v \mp v_S} \right) = 500 \left( \frac{343 + 10}{343 + 20} \right) = 486.2 \text{ Hz} \]

(d) Relative to the air the locomotive is moving at \( v_S = 20.00 \) m/s away from the girl and the girl is moving at \( v_D = 20.00 \) m/s toward the locomotive. Use the plus signs in both the numerator and the denominator. Thus \( (v + v_D) = (v + v_S) \) and

\[ f' = f = 500.0 \text{ Hz} \]
10. In the figure, an aluminum wire, of length $L_1 = 60.0\text{cm}$, cross-sectional area $1.00 \cdot 10^{-2}\text{cm}^2$, and density $2.60\text{ g/cm}^3$, is joined to a steel wire, of density $7.80\text{ g/cm}^3$ and the same cross-sectional area. The compound wire, loaded with a block of mass $m = 10.0\text{ kg}$, is arranged so that the distance $L_2$ from the joint to the supporting pulley is $86.6\text{ cm}$. Transverse waves are set up on the wire by an external source of variable frequency; a node is located at the pulley.

(a) Find the lowest frequency that generates a standing wave having the joint as one of the nodes.

(b) How many nodes are observed at this frequency? (including the nodes at the endpoints)

![Diagram](image)

| Solution: Note: In this solution, $m$ is not the mass of the block featured in the figure, it is an integer indexing the frequencies of standing modes of the string. |

(a)  
- The frequency of the wave is the same for both sections of the wire
- The wave speed and wavelength can be different in the different sections
- We have the equation $f_m = mv/2L$ for the frequency of the $m$th standard mode, and we want it to be satisfied in both sections, where $m$ is a positive integer.

Since we want these frequencies to be the same in both sections (otherwise the wire would be discontinuous!), we have

$$\frac{mv_1}{2L_1} = \frac{nv_2}{2L_2} \tag{10.1}$$

where $m$ is the integer corresponding to wire $L_1$ and $n$ is the integer corresponding to wire $L_2$. In order to compute the velocity of the wave in each medium, we must first compute the linear mass densities $\mu$ in each medium. We have:

$$\mu_1 = A\rho_1 = 0.0026\text{ kg/m}$$

$$\mu_2 = A\rho_2 = 0.0078\text{ kg/m}$$

Now we have (for a tension $T = mg$)

$$v_1 = \sqrt{\frac{T}{\mu_1}} = 194.1\text{ m/s}$$

$$v_2 = \sqrt{\frac{T}{\mu_2}} = 112.1\text{ m/s}$$

Substituting these in to (7.1), we have $m = 0.4n$, or

$$5m = 2n$$

The smallest integer solution to this equation is $m = 2$, $n = 5$. Thus the frequency will be

$$f = \frac{mv_1}{2L_1} = 324\text{ Hz}$$

(b) Since $m = 2$ corresponds to 3 nodes, and $n = 5$ corresponds to 6 nodes, but there is one node shared by the two sections (where $L_1$ meets $L_2$), there are 8 nodes in total.
11. Two bulbs of volumes 200 cm$^3$ and 100 cm$^3$, as shown, are connected by a short tube containing an insulating porous plug that permits equalization of pressure but not of temperature between the bulbs. The system is sealed at 27$^\circ$C when it contains oxygen under atmospheric pressure. The small bulb is immersed in an ice bath at 0$^\circ$C and the large bulb is placed in a steam bath at 100$^\circ$C. What is the final pressure $P$ inside the system? Neglect thermal expansion of the bulbs.

**Solution:** We will call the smaller volume (1), and the larger volume (2). Then we can write the ideal gas law equations:

\begin{align*}
    P_1 V_1 &= n_1 R T_1 \quad (11.1) \\
    P_2 V_2 &= n_2 R T_2 \quad (11.2)
\end{align*}

Some known variables are

- $P_1 = P_2 = 101.3$ kPa
- $T_1 = T_2 = 300$ K

Let the primed variables be the values at equilibrium. Then we can write the ideal gas law for each bulb at equilibrium:

\begin{align*}
    P V_1 &= n'_1 R T'_1 \quad (11.3) \\
    P V_2 &= n'_2 R T'_2 \quad (11.4)
\end{align*}

Since the temperatures at equilibrium will be the same as the baths, we know that $T'_1 = 273$ K, and $T'_2 = 373$ K. Let $n = n_1 + n_2 = n'_1 + n'_2$ be the total number of moles in the system. Using the ideal gas law, we can calculate the number of moles initially in each bulb. To do that, plug

- $V_1 = 0.0001$ m$^3$
- $V_2 = 0.0002$ m$^3$
- $P_1 = P_2 = 101.3$ kPa
- $T_1 = T_2 = 300$ K

$n_1 = 0.00406$ mol, and $n_2 = 0.00813$ mol, so $n = 0.0122$ mol. Thus we three have equations in three unknowns,

\begin{align*}
    n'_1 + n'_2 &= 0.0122 \\
    P V_1 &= n'_1 R T'_1 \\
    P V_2 &= n'_2 R T'_2
\end{align*}

Solving the system, we obtain $P = 112.2$ kPa
**Leidenfrost effect:** A water drop that is slung onto a skillet with a temperature between 100\(^\circ\)C and about 200\(^\circ\)C will last about 1 s. However, if the skillet is much hotter, the drop can last several minutes, an effect named after an early investigator. The longer lifetime is due to the support of a thin layer of air and water vapour that separates the drop from the metal (by distance \(L\) as shown). Let \(L = 0.100\) mm, and assume that the drop is flat with height \(h = 1.50\) mm and bottom face area \(A = 4.00 \cdot 10^{-6}\) m\(^2\). Also assume that the skillet has a constant temperature \(T_s = 300\)\(^\circ\)C and the drop has a temperature of 100\(^\circ\)C. Water has density \(\rho = 1000\) kg/m\(^3\), latent heat of vaporization \(L_v = 2265\) J/g, and the supporting layer has thermal conductivity \(k = 0.026\) W/m \(
abla\) K.

(a) At what rate is energy conducted from the skillet to the drop through the drop’s bottom surface?

(b) If conduction is the primary way energy moves from the skillet to the drop, how long will the drop last?

\[ \text{Water drop} \]
\[ \text{Skillet} \]

**Solution:**

(a) The rate of energy flow through the surface is given by
\[ H = \frac{kA\Delta T}{L} \]  

Plugging in the given numbers,
\[ H = \frac{(0.026)(4.00 \cdot 10^{-6})(300 - 100)}{0.0001} = 0.21\ \text{W} \]

(b) First we find mass of the droplet, which is given by:
\[ m = \rho Ah = 6 \cdot 10^{-6}\ \text{kg} \]

Then we observe that the heat transferred can be related to the latent heat of vaporization
\[ Q = Ht = L_v m = (6 \cdot 10^{-6})(2265 \cdot 10^3) \]

so then
\[ t = 65\ \text{s} \]
13. (* *) A massless rod with mass $M$ on the end of it, length $L$ swings as a pendulum with two horizontal springs of negligible mass and constants $k_1$ and $k_2$ acting at the bottom end, as shown. Both springs are relaxed when the rod is vertical. What is the period $P$ of small oscillations? Express your answer in terms of $M$, $L$, $k_1$, and $k_2$.

**Solution:** There will be a tension $T$ in the rod, and thus the gravitational force on the mass $Mg$ is balanced by $T \cos \theta$, where $\theta$ is the angle between the rod and the vertical. Since $\theta$ is small, $T \approx Mg$. Thus the horizontal force on the mass due to the rod will be $T \sin \theta \approx Mg \theta \approx Mg/L$. Thus the total force on the mass will be

$$F = -[Mgx/L + (k_1 + k_2)x]$$  \hspace{1cm} (13.1)

Setting up Newton’s second law $F = ma = m \frac{d^2x}{dt^2}$,

$$M \frac{d^2x}{dt^2} + \frac{Mgx}{L} + (k_1 + k_2)x = 0$$  \hspace{1cm} (13.2)

$$\frac{d^2x}{dt^2} + \frac{gx}{L} + \frac{(k_1 + k_2)x}{M} = 0$$  \hspace{1cm} (13.3)

$$\frac{d^2x}{dt^2} + x \left( \frac{g}{L} + \frac{k_1 + k_2}{M} \right) = 0$$  \hspace{1cm} (13.4)

Thus

$$\omega = \sqrt{\frac{g}{L} + \frac{k_1 + k_2}{M}} = \sqrt{\frac{Mg + L(k_1 + k_2)}{ML}}$$  \hspace{1cm} (13.5)

By the definition of angular frequency,

$$P = \frac{2\pi}{\omega}$$  \hspace{1cm} (13.6)

this means that

$$P = 2\pi \sqrt{\frac{ML}{Mg + L(k_1 + k_2)}}$$  \hspace{1cm} (13.7)
14. Suppose we expand 1.00 mol of a monatomic gas initially at 5.00 kPa and 600 K from initial volume \( V_i = 1.00 \text{ m}^3 \) to final volume \( V_f = 2.00 \text{ m}^3 \). Suppose that at any instant during the expansion, the pressure \( P \) and volume \( V \) of the gas are related by \( P = 5e^{(V_i-V)/a} \), with \( P \) in kilopascals, \( V_i \) and \( V \) in cubic metres, and \( a = 1.00 \text{ m}^3 \). What are

(a) The final pressure
(b) Final temperature
(c) Find the work done by the expansion
(d) What is \( \Delta S \) for the expansion?

**Solution:** Let \( n = 1 \), \( P_i = 5 \), \( T_i = 600 \), and both \( P_f, T_f \) unknown.

(a) The final pressure is
\[
P(2) = 5e^{1-2} = \frac{5}{e} \text{ kPa} = 1839.4 \text{ Pa}
\]

(b) To find the temperature, plug the final pressure from part (a) into the ideal gas law.
\[
P_fV_f = nRT_f
\]
This produces
\[
T_f = \frac{10000}{eR} = 441 \text{ K}
\]

(c) We use the knowledge of antiderivatives learned in Math 100.
The work is given by
\[
W = \int PdV
\]
\[
= 5 \int e^{V_i-V} dV
\]
\[
= -5e^{V_i-V} + C
\]
To solve for the constant \( C \), we note that no work has been done before the gas has been expanded, so \( W(V_i) = 0 \). This means that \( C = 5 \). The work is then
\[
W = -5e^{V_i-V} + 5
\]
We evaluate at \( V_f \), so
\[
W(V_f) = \frac{-5}{e} + 5 \text{ kJ}
\]
Then we have
\[
W = 5000(1 - 1/e) \text{ J} = 3160.6 \text{ J}
\]

(d) To find \( \Delta S \), we will have to recall how the work formula for a isothermal expansion is derived, and use that information to find the formula for \( \Delta S \) in this case.
**Remark 1.** Recall the process of finding the work for an isothermal process (in the textbook). For isothermal process,

\[
W = \int PdV \tag{14.6}
\]

\[
= \int \frac{nRTdV}{V} \tag{14.7}
\]

\[
= nRT \ln \left( \frac{V_f}{V_i} \right) \tag{14.8}
\]

We can use a similar procedure to find the formula for part (d). From the first law of thermodynamics,

\[
dQ = dW + dU \tag{14.9}
\]

the second law of thermodynamics

\[
dS = \frac{dQ}{T} \tag{14.10}
\]

We can combine (3.9) and (3.10) to make

\[
dS = \frac{dW + dU}{T} \tag{14.11}
\]

Thus

\[
\Delta S = \int \frac{dW}{T} + \frac{dU}{T} \tag{14.12}
\]

\[
= \int \frac{PdV}{T} + \frac{nC_VdT}{T} \tag{14.13}
\]

\[
= \int \frac{nRdT}{V} + \frac{nC_VdT}{T} \tag{14.14}
\]

\[
= nR \ln \left( \frac{V_f}{V_i} \right) + nC_V \ln \left( \frac{T_f}{T_i} \right) \tag{14.15}
\]

Note that this is the same form as the work formula that we’ve been given, so we can logically argue that a result of the same form is obtained here.

Therefore

\[
\Delta S = R \ln \left( \frac{2}{1} \right) + \frac{3R}{2} \ln \left( \frac{10000}{600eR} \right) = 1.94 \text{ J/K}
\]
15. A crude musical instrument is constructed by stretching a wire of negligible mass under tension $T$ between two points and firmly attaching a mass $m$ to the wire at a distance $x$ from one end, as shown in figure. The mass is displaced from equilibrium by a small distance $A$ (i.e. $A \ll x, A \gg l - x$), and then allowed to vibrate. Neglect the force of gravity.

(a) Find the frequency $\nu$ of the sound.

(b) Write the equation for the displacement of the mass from equilibrium as a function of time $y(t)$.

(c) As $x$ is varied, what are the minimum and maximum frequencies available? 

Express your answers in terms of $T$, $m$, $l$, $x$, and $A$.

![Diagram of the instrument with a mass attached to a wire and an amplitude A from the equilibrium position.](image)

**Solution:**

(a) Let $\theta_1$ be the angle that the string on the left makes with the horizontal, and $\theta_2$ be the angle that the string on the right makes with the horizontal. Since the tension $T$ is uniform throughout the wire, the force $F_y$ on the mass $m$ is given by

\[
F_y = -(T \sin \theta_1 + T \sin \theta_2) = -T \left( \frac{y}{x} + \frac{y}{l-x} \right) = -y \frac{Tl}{x(l-x)} \tag{15.1}
\]

Setting up Newton’s second law, $F_y = ma = m\frac{d^2y}{dt^2}$

\[
m \frac{d^2y}{dt^2} + y \frac{Tl}{x(l-x)} = 0 \tag{15.4}
\]

As discussed in class, the coefficient of $y$ is $\omega^2$. Thus we find $\omega$ to be:

\[
\omega = \sqrt{\frac{Tl}{mx(l-x)}} = 2\pi\nu \tag{15.6}
\]

Therefore

\[
\nu = \frac{1}{2\pi} \sqrt{\frac{Tl}{mx(l-x)}} \tag{15.7}
\]
(b) The general equation for oscillation will then be:

\[ y(t) = A \cos(\omega t) \]  \hspace{1cm} (15.8)

\[ = A \cos \left( t \sqrt{\frac{Tl}{mx(l-x)}} \right) \]  \hspace{1cm} (15.9)

Note that you could have used sine instead of cosine and it would also be completely correct.

(c) When \( x \) is taken close to 0 or \( l \), the frequency becomes unbounded and races off towards infinity, so

\[ \nu_{\text{max}} \to \infty \]

When \( x \) is \( l/2 \) (mass is in the middle) we have

\[ \nu_{\text{min}} = \frac{1}{\pi} \sqrt{\frac{T}{ml}} \]
A cylinder, filled with argon, is equipped with a spring loaded piston of mass \(m\) and area \(A\). At equilibrium the argon is at total pressure \(P_0\), the piston is distance \(L_0\) from either end of the system as shown, and the spring (constant \(K\)) is compressed by \(x_0\) (its free length being \(L_0 + x_0\)). Find the angular frequency \(\omega\) of small oscillations \((x_0 \ll L_0)\) of the piston if the gas compresses isothermally. Hint: You may use \(1/(1 + x) \approx 1\) for \(x \ll 1\). Express your answer in terms of \(K\), \(P_0\), \(A\), \(L_0\), and \(m\).

**Solution:** At equilibrium, the forces on the piston are balanced, so then \(P_0 A = Kx_0\). For convenience, set \(x = 0\) at the equilibrium point. Since the process is isothermal, we know that (for the pressure and volume inside the argon filled cylinder)

\[
P_0 V_0 = P(x) \cdot V(x) \quad (16.1)
\]
\[
P_0 A L_0 = P(x) A \cdot (L_0 + x) \quad (16.2)
\]

To find the contribution to the force that this pressure (caused by a small displacement \(x\)) will exert on the piston, we take

\[
P_0 - P(x) = P_0 - P_0 L_0 \frac{x_0}{L_0 + x} \quad (16.3)
\]
\[
= P_0 \frac{x_0}{L_0 + x} \quad (16.4)
\]

Thus the total force on the piston will be the sum of the spring force \(Kx\) and the force due to the pressure change (11.5)

\[
Kx + A \frac{P_0 x}{L_0 + x} = x \left( K + \frac{P_0 A}{L_0 + x} \right) \quad (16.5)
\]

Since we can approximate \(1/(1 + x) \approx 1\) for \(x \ll 1\), we can perform the following manipulation:

\[
F = x \left( K + \frac{P_0 A}{L_0(1 + x/L_0)} \right) \quad (16.6)
\]
\[
\approx x \left( K + \frac{P_0 A}{L_0} \right) \quad (16.7)
\]

Then using Newton’s second law, we have \(F = ma = m \frac{d^2x}{dt^2}\), which, for this physical scenario, is

\[
m \frac{d^2x}{dt^2} + x \left( K + \frac{P_0 A}{L_0} \right) = 0 \quad (16.8)
\]
\[
\frac{d^2x}{dt^2} + x \left( \frac{K + \frac{P_0 A}{L_0}}{m} \right) = 0 \quad (16.9)
\]
The coefficient of the $x$ will be $\omega^2$, so we have

$$\omega^2 = \frac{\left(K + \frac{P_0A}{L_0}\right)}{m} \tag{16.10}$$

and the final answer becomes

$$\omega = \sqrt{\frac{\left(K + \frac{P_0A}{L_0}\right)}{m}} \tag{16.11}$$
An object of mass 5.0 kg is found to oscillate with negligible damping when suspended from a spring which causes it to perform 10 complete cycles in 10.0 seconds. Thereafter, a certain small magnetic damping is applied, proportional to the velocity of the motion and the amplitude decreases from an initial 0.2 m to 0.1 m in 10 cycles.

(a) Write the differential equation associated with this motion, with the coefficients of $d^2x/dt^2$, $dx/dt$ and $x$ represented by numbers in SI units.

(b) What is now the period $T'$ of the motion? Find $T'$ to six decimal places.

(c) In how many cycles, $N$ (starting from the 0.2 m amplitude) will the amplitude reach
   (i) 0.05 m?
   (ii) 0.02 m?

(d) What is average rate of energy dissipation $P$ by damping during the first cycle?

Solution:

(a) The general form of the differential equation for the motion is

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0 \quad (17.1)$$

It is clear that the mass $m = 5$. Since $T = 1$ because the oscillation does 10 cycles in 10 seconds, we have that since

$$\omega = \frac{2\pi}{T} \quad (17.2)$$

plugging in $T = 1$ for the undamped motion, we find

$$\omega_0 = 2\pi \quad (17.3)$$

which is the angular frequency for the undamped motion. Since

$$\omega_0^2 = \frac{k}{m} \quad (17.4)$$

we have

$$k = 20\pi^2$$

To find $b$ is more complicated. When the damping force is applied, the period will change, so 10 cycles no longer means 10 seconds. For damped motion we will have a new period $T'$ with corresponding angular frequency $\omega$. This new $\omega$ is given by

$$\omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2} \quad (17.5)$$

$$\omega = \sqrt{(2\pi)^2 - \left(\frac{b}{10}\right)^2} \quad (17.6)$$

Since (12.2) still holds, but this time for different period and angular frequency, we have

$$T' = \frac{2\pi}{\omega} \quad (17.7)$$

Plugging (12.6) into (12.7), we have

$$T' = \frac{2\pi}{\sqrt{(2\pi)^2 - \left(\frac{b}{10}\right)^2}} \quad (17.8)$$
The amplitude of the motion as a function of time is

\[ A(t) = A_0 e^{-bt/2m} \]  

and since the amplitude of the motion drops from 0.2 m to 0.1 m after 10 cycles (10 periods), we have so we will have

\[ 0.1 = 0.2e^{-b10T'/10} \]  

Rearranging,

\[ \ln \left( \frac{1}{2} \right) = -bT' \]  

Plugging (12.8) into this, we have

\[ \ln(2) = \frac{b(20\pi)}{10\sqrt{(2\pi)^2 - \left( \frac{b}{m} \right)^2}} \]  

Solving for \( b \), we find that

\[ b = 0.693 \text{ kg/s} \]

Thus the differential equation is

\[ 5\frac{d^2x}{dt^2} + 0.693 \frac{dx}{dt} + 20\pi^2 x = 0 \]  

(b) Now that we have \( b \), we can plug it back in to the (12.6) to find \( \omega \).

\[ \omega = \sqrt{(2\pi)^2 - \left( \frac{0.693}{10} \right)^2} = 6.282803 \text{ rad/s} \]

and thus, by (12.7),

\[ T' = 1.000061 \text{ s} \]

(c) (i) Setting the amplitude to 0.05, we obtain

\[ 0.05/0.2 = e^{-bNT'/10} \]

and since we already know \( b \) and \( T' \), we solve for \( N \) to find

\[ N = 20 \text{ cycles} \]

(ii) A similar calculation to (i) will yield

\[ N = 33 \text{ cycles} \]

(d) We must find the initial energy \( E_0 \). This can be calculated with the formula for spring potential energy.

\[ E_0 = \frac{kA_0^2}{2} = \frac{(20\pi^2)(0.2)^2}{2} = \frac{2\pi^2}{5} \]

Since

\[ E(t) = E_0 e^{-bt/m} \]

we take the difference quotient

\[ \frac{E(T') - E_0}{T'} = -0.2644 \text{ J/s} \]

Thus the average rate of energy dissipation over the first period will be 0.2644 J/s.
18. Energy can be removed from water as heat at and even below the normal freezing point (0.0 °C at atmospheric pressure) without causing the water to freeze; the water is then said to be supercooled. Suppose a 1.00 g water drop is supercooled until its temperature is that of the surrounding air, which is at −5.00 °C. The drop then suddenly and irreversibly freezes, transferring energy to the air as heat. What is the entropy change for the drop? The specific heat of ice is 2220 J/K·kg, the specific heat of water is 4186 J/K·kg, and the latent heat of fusion of ice is 334 kJ/kg.

**Solution:** Since entropy is a state function, this means that the change in entropy is independent of the path taken between the initial state and the final state. For this problem, we consider a three step process:

(i) In the first process, consider the water drop heating up to 0 °C
(ii) In the second process, consider the water drop freezing at constant temperature of 0°C
(iii) In the third process, consider the water drop (now as a solid ice) cooling down to its original temperature

First, we need to find out how to compute the change in entropy for a non-constant temperature process. This can be done in the following way: Since

\[
dS = \frac{dQ}{T}
\]  
(18.1)

and

\[
dQ = mc\,dT
\]  
(18.2)

we can plug (17.2) into (17.1):

\[
dS = \frac{mc\,dT}{T}
\]  
(18.3)

Taking the antiderivative of both sides, we have

\[
\Delta S = mc\ln\left(\frac{T_f}{T_i}\right)
\]  
(18.4)

Now we are ready to calculate the appropriate entropies:

(i) 

\[
\Delta S_1 = mc\ln\left(\frac{T_f}{T_i}\right)
\]  

\[
= (0.001)(4168)\ln\left(\frac{273}{268}\right)
\]  

\[
= 0.077 \text{ J/K}
\]

(ii) 

\[
\Delta S_2 = -\frac{mL}{T}
\]  

\[
= -0.001(334000)/273
\]  

\[
= -1.22 \text{ J/K}
\]
(iii)

\[ \Delta S_3 = mc \ln \left( \frac{T_f}{T_i} \right) \]

\[ = (0.001)(2220) \ln \left( \frac{268}{273} \right) \]

\[ = -0.041 \text{ J/K} \]

Summing these together,

\[ \Delta S = \Delta S_1 + \Delta S_2 + \Delta S_3 \]

\[ = 0.077 - 1.22 - 0.041 \]

\[ = -1.18 \text{ J/K} \]
Useful Constants and Conversion Ratios:
- R = Ideal Gas constant = 8.31451 J/molK, 1 atm = 1.013 \times 10^5 Pa, 1 atm \cdot litre = 101.3 J
- \sigma = Stefan-Boltzmann constant = 5.6704 \times 10^{-8} \text{W/m}^2\text{K}^4, \gamma_{\text{air}} = 1.4, C_{V_{\text{air}}} = 20.8 J/molK
- \rho_{\text{water}} = \text{Density of water} = 1 \text{gram/cm}^3 = 1000 \text{kg/m}^3

Mechanics:
- Linear Motion: \( x = x_0 + \frac{1}{2} (v_0 + v)t \), \( x = x_0 + v_0t + \frac{1}{2}at^2 \), \( v = v_0 + at \), \( v^2 = v_0^2 + 2a(x - x_0) \)
- Circular Motion: \( a_c = \frac{v^2}{r} \)
- Forces: \( \mathbf{F} = ma = \frac{d}{dt} \mathbf{p} \), Friction: \( |\mathbf{F}| = \mu |\mathbf{N}| \), Spring: \( \mathbf{F} = -kx \), Damping: \( \mathbf{F} = -bv \)
- Buoyant Force: \( \mathbf{F}_b = \rho \mathbf{V}_g \)
- Work: \( W = \mathbf{F} \cdot \mathbf{dr} = \mathbf{F} \cdot \Delta \mathbf{r} \), \( K = \frac{1}{2}mv^2 \), \( \Delta U_{\text{gravity}} = mg\Delta h \), \( \Delta U_{\text{spring}} = \frac{1}{2}kx^2 \)
- Power: \( P = \frac{dW}{dt} = \mathbf{F} \cdot \mathbf{v} \)

Thermodynamics:
- Thermal Expansion: \( \Delta L = \alpha L_0 \Delta T \), Stress and Strain: \( \frac{\mathbf{F}}{\mathbf{A}} = \gamma \frac{\Delta L}{L} \), Ideal Gas Law: \( PV = nRT \)
- Thermal Conductivity: \( I = \frac{\Delta Q}{\Delta t} = kA \frac{\Delta T}{\Delta x} \)
- Black Body Radiation: \( P = e\sigma AT^4 \), \( \lambda_{\text{max}}T = 2.8977685 \times 10^{-3} m \cdot K \)
- Internal Energy: \( U = nC_V T \)
- First Law of Thermodynamics: \( dQ = dU + dW \) For an ideal gas, \( dW = PdV \)
- Work for an isothermal process: \( W = nRT \ln(V_f/V_i) \)
- Work for an adiabatic expansion: \( TV^{\gamma-1} = \text{constant} \), if the number of moles is constant \( PV^\gamma = C \) where \( C \) is a constant and \( \gamma = C_p/C_V \)
- Work for adiabatic process: \( W = \int_{V_i}^{V_f} PdV = C \int_{V_i}^{V_f} \frac{dV}{V^\gamma} = \frac{C}{1-\gamma} (V_2^{1-\gamma} - V_1^{1-\gamma}) \)

Heat Transfer: \( Q = mC\Delta T \), \( Q = mL, C_p = C_V + R, C_V = \frac{f}{2}R \), where \( f \) is degrees of freedom.
- \( f = 3 \) for monatomic and \( f = 5 \) for diatomic.
- \( dS = \frac{dQ}{T} \)
- \( \epsilon = W/Q_H \), \( \text{COP}_{\text{Cooling}} = \frac{|Q_C|}{|W|} \), \( \text{COP}_{\text{Heating}} = \frac{|Q_H|}{|W|} \), \( \epsilon_{\text{Carnot}} = 1 - \frac{T_C}{T_H} \)

Integrals:
- \( \int x^n dx = x^{n+1}/n + 1, C, n \neq 1 \)
- \( \int x^{-1} dx = \ln x + C \)

Trigonometry:
- \( \sin \theta_1 + \sin \theta_2 = 2 \cos \left( \frac{\theta_1 - \theta_2}{2} \right) \sin \left( \frac{\theta_1 + \theta_2}{2} \right) \)

Area and Volume:
- Surface Area of a sphere: \( A = 4\pi r^2 \). Lateral surface area of a cylinder: \( A = 2\pi rl \).
- Area of a circle: \( A = \pi r^2 \). Volume of a cylinder: \( V = \pi r^2 \). Volume of a sphere: \( V = \frac{4}{3} \pi r^3 \)

Oscillations:
- \( \omega = 2\pi f, T = \frac{1}{f}, x = A \cos(\omega t + \phi), \omega^2 = \frac{k}{m} \)
- Damped Oscillations: \( x = A_0 e^{-\frac{bt}{2m}} \cos(\omega t + \phi) \), where \( \omega = \sqrt{w_0^2 - \left( \frac{b}{2m} \right)^2} \), \( Q = 2\pi \frac{E}{\Delta E} \)
- Energy for damped \( E = E_0 e^{-\frac{bt}{2m}} \)
Waves:
\[ v = \sqrt{\frac{T}{\mu}}, \quad k = \frac{2\pi}{\lambda}, \quad P = \frac{1}{2} \mu \omega^2 A^2 v, \quad p_o = \rho \omega s_0 \]
\[ v = \sqrt{\frac{\gamma RT}{M}}, \quad I = \frac{P_{ac}}{4\pi r^2}, \quad \beta = 10dB \log_{10} \left( \frac{l}{l_0} \right), \quad \text{Doppler Effect } f' = f_0 \left( \frac{v \pm v_L}{v \mp v_S} \right) \]

Beats: \[ \Delta f = f_2 - f_1, \quad y = A \cos(2\pi \omega t + \phi) \]

Interference: \[ kAx + \Delta \phi = 2\pi n \quad \text{or} \quad \pi(2n + 1), \quad n = 0, \pm 1, \pm 2, \pm 3, \pm 4, \ldots \]

Standing Waves \[ f_m = \frac{mv}{2L}, \quad m = 1, 2, 3, \ldots, \quad f_m = \frac{mv}{4L}, \quad m = 1, 3, 5, \ldots \]

Constants:
\[ k = \frac{1}{4\pi \epsilon_0} \approx 9 \times 10^9 \text{Nm}^2/\text{C}^2, \quad \epsilon_0 = 8.84 \times 10^{-12} \text{C}^2/\text{Nm}^2, \quad e = 1.6 \times 10^{-19} \text{C} \]
\[ \mu_0 = 4\pi \times 10^{-7} \text{Tm/Av}, \quad c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 299,792,458 \text{m/s} \]

Point Charge:
\[ |F| = \frac{k|q_1 q_2|}{r^2}, \quad |E| = \frac{k|q|}{r}, \quad V = \frac{kq}{r} + \text{Constant} \]

Electric potential and potential energy \[ \Delta V = V_a - V_b = \int_a^b E \cdot dl = -\int_b^a E \cdot dl \]
\[ E_x = -\frac{dV}{dx}, \quad E = -\nabla V, \quad \Delta U = U_a - U_b = q(V_a - V_b) \]

Maxwell’s Equations:
\[ \int_S E \cdot dA = \frac{Q_{enc}}{\epsilon_0} = 4\pi kQ_{enc} \]
\[ \int_C B \cdot dl = \mu_0 (I_{enclosed}) + \epsilon_0 \mu_0 \frac{d\Phi_E}{dt} \]
\[ \int_C E \cdot dl = -\frac{d\Phi_B}{dt} \]

Where \( S \) is a closed surface and \( C \) is a closed curve, \( \Phi_E = \int E \cdot dA \) and \( \Phi_B = \int B \cdot dA \)

Energy Density:
\[ u_E = \frac{1}{2} \epsilon_0 E^2 \] and \[ u_B = \frac{1}{2} \mu_0 B^2 \] (energy per volume)

Forces:
\[ \mathbf{F} = q \mathbf{E} + q \mathbf{v} \times \mathbf{B}, \quad \mathbf{F} = IL \times \mathbf{B} \]

Capacitors:
\[ q = CV, \quad U_C = \frac{q^2}{2C}, \quad \text{For parallel plate capacitor with vacuum (air): } C = \frac{\epsilon_0 A}{d}, \quad C_{\text{dielectric}} = KC_{\text{vacuum}} \]

Inductors:
\[ \mathcal{E}_L = -L \frac{dI}{dt}, \quad U_L = \frac{1}{2} LI^2, \quad \text{where } L = N\Phi_B/I \text{ and } N \text{ is the number of turns.} \]

For a solenoid \( B = \mu_0 n I \) where \( n \) is the number of turns per unit length.

**DC Circuits:** \( V_R = IR, \quad P = VI, \quad P = I^2 R \)

(For RC circuits) \( q = ae^{-t/\tau} + b, \quad \tau = RC, \quad a \text{ and } b \text{ are constants} \)

(For RL circuits) \( I = ae^{-t/\tau} + b, \quad \tau = L/R, \quad a \text{ and } b \text{ are constants} \)

**AC circuits:** \( X_L = \omega L, \quad X_C = 1/(\omega C), \quad V_C = X_C I, \quad V_L = X_L I \)

\[ V = ZI, \quad Z = \sqrt{(X_L - X_C)^2 + R^2}, \quad P_{\text{average}} = I_{\text{rms}}^2 R, \quad I_{\text{rms}} = \frac{I_{\text{max}}}{\sqrt{2}} \]

If \( V = V_0 \cos(\omega t) \), then \( I = I_{\text{max}} \cos(\omega t - \phi) \), where \( \tan \phi = \frac{X_L - X_C}{R} \)

\[ P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos \phi \]

**Additional Equations:** \[ dB = \frac{\mu_0}{4\pi} \cdot \frac{dI}{r^3} \]

LRC Oscillations: \[ q = A_0 e^{-\frac{\mu_0}{4\pi} \cos(\omega t + \phi)} \], where \( \omega = \sqrt{\omega_0^2 - \left( \frac{R}{2L} \right)^2} \) and \( \omega_0 = \frac{1}{LC} \)