Mathematics 152 Midterm 1 Review Package

UBC Engineering Undergraduate Society

Attempt questions to the best of your ability. This review package consists of 14 pages, including 1 cover page and 19 questions. Problems are ranked in difficulty as (•) for easy, (••) for medium, and (•••) for difficult.

Solutions posted at: [http://ubcengineers.ca/tutoring/](http://ubcengineers.ca/tutoring/)

If you believe that there is an error in these solutions, or have any questions, comments, or suggestions regarding EUS Tutoring sessions, please e-mail us at: [tutoring@ubcengineers.ca](mailto:tutoring@ubcengineers.ca). If you are interested in helping with EUS tutoring sessions in the future or other academic events run by the EUS, please e-mail [vpacademic@ubcengineers.ca](mailto:vpacademic@ubcengineers.ca).

Some of the problems in this package were not created by the EUS. Those problems originated from one of the following sources (All solutions prepared by the EUS.):

- Schuam’s Outline of Matrix Operations; Richard Bronson
- Calculus 7th ed; James Stewart
- Linear Algebra; Sterling K. Berberian
- Linear Algebra and Its Applications 3rd ed; Gilbert Strang
- Linear Algebra and Matrix Theory; Robert Stoll

Want a warm up? These are the easier problems

1, 2, 4, 5

Short on study time? These cover most of the material

6, 7, 8, 9, 11, 14

Want a challenge? These are some tougher questions

17, 18

EUS Health and Wellness Study Tips

- **Eat Healthy**—Your body needs fuel to get through all of your long hours studying. You should eat a variety of food (not just a variety of ramen) and get all of your food groups in.

- **Take Breaks**—Your brain needs a chance to rest: take a fifteen minute study break every couple of hours. Staring at the same physics problem until your eyes go numb won’t help you understand the material.

- **Sleep**—We have all been told we need 8 hours of sleep a night, university shouldn’t change this. Get to know how much sleep you need and set up a regular sleep schedule.
1. Let \( \mathbf{A} = (3, 0, 2), \mathbf{B} = (-4, 1, 6), \mathbf{C} = (10, 9, 0), \mathbf{D} = (7, 3, 5) \). Compute the following:

(a) \( 2\mathbf{A} + 4\mathbf{D} \)
(b) \( \|\mathbf{D}\| \)
(c) \( \|\mathbf{B} - \mathbf{C}\| \)
(d) Compute the angle between \( \mathbf{A} \) and \( \mathbf{B} \)
2. Consider the augmented matrix \(
\begin{pmatrix}
1 & 2 & 6 \\
3 & 6 & 7 \\
\end{pmatrix}
\). Determine if its associated linear system has one solution, no solutions, or infinitely many solutions.

3. Consider the augmented matrix \(
\begin{pmatrix}
2 & 8 & 10 & 4 \\
1 & 7 & 7 & 5 \\
2 & 3 & 3 & 3 \\
\end{pmatrix}
\). Determine whether the linear system associated with this matrix has one solution, no solutions, or infinitely many solutions.
4. Consider the following lines of MATLAB code:
   
   ```matlab
   A = [1 0 0; 3 5 2; 2 3 4];
   A = A + [3 2 1; 0 0 0; 1 2 1];
   
   What will be the output if \( A(2,1) + A(1,2) \) is called?
   ```

5. Find the projection of \((3,5)\) onto the line \(3x + 2y = 7\).
6. Let \( \mathbf{A} = (2, 1, 5) \), \( \mathbf{B} = (-1, 5, -2) \), and \( \mathbf{C} = (k, -3, 12) \).

(a) For what value(s) of \( k \) will \( \mathbf{A} \), \( \mathbf{B} \), \( \mathbf{C} \) form a linearly dependent set?

(b) Find the area of the triangle spanned by \( \mathbf{A} \) and \( \mathbf{B} \)

(c) Now redefine \( \mathbf{C} = (1, -3, 4) \). Find the volume of the parallelepiped spanned by \( \mathbf{A} \), \( \mathbf{B} \), and \( \mathbf{C} \).
7. Consider the linear system
\[
\begin{align*}
 x + 2y + z &= 1 \\
 -x + 3z &= 1 \\
 x - y - 3z &= 0
\end{align*}
\]
(a) Write this system as an augmented matrix.
(b) Write the system to row echelon form
8. The line \( L \) passes through the points \((9, 0, 1)\) and \((7, 2, 3)\).

(a) i. Find a parametric equation for \( L \).
    ii. Find an equation form of the line \( L \).

(b) The plane \( P \) has the equation \(-x + y + z = 5\). Is the line \( L \) parallel to \( P \), perpendicular to \( P \), or neither?

(c) The plane \( Q \) has the equation \(2x - 2z = 1\). Is the line \( L \) parallel to \( Q \), perpendicular to \( Q \), or neither?

(d) Find an equation for the plane that is perpendicular to \( L \) and passes through the point \((6, 2, 4)\)
(* *) 9. Find the plane that passes through the points \((0, -2, 5)\) and \((-1, 3, 1)\) and is perpendicular to the plane \(2z = 5x + 4y\).
10. Solve the vector equation $\mathbf{a} = \mathbf{a} \times (1, 2, 3) + (13, 5, -6)$.

11. Does \{$(2, 3), (-1, 2), (4, -7)$\} form a linearly independent set?
12. Find a 2 by 3 system whose general solution is \( \mathbf{x}(w) = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + w \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \)

13. Find the angles which the vector \( \mathbf{A} = 3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k} \) makes with each of the coordinate axes. You may leave your answer in calculator ready form.
14. (*** Find the minimum distance between the point $(9, 0, -2)$ and the plane $z = 3x - 2y + 6$
15. Write the general solution to the linear system associated with the following augmented matrix.

\[
\begin{pmatrix}
1 & 2 & 2 & | & 1 \\
1 & 4 & 5 & | & 4
\end{pmatrix}
\]

16. (a) Find an equation for the plane perpendicular to the vector \( \mathbf{A} = 2i + 3j + 6k \) and passing through the terminal point of the vector \( \mathbf{B} = i + 5j + 3k \).

(b) Find the distance from the origin to the plane.
17. Find the minimum distance between the point \((3, 2, 6)\) and the line \(r(t) = (3t - 2)i + tj - (2t + 5)k\). You may leave your answer in calculator ready form.
18. Find the point of intersection between the line $\mathbf{r}(t) = (3t - 2)i + tj - (2t + 5)k$ and the plane $z = 3x - 2y + 6$.

19. Consider the following lines of Matlab code:
   
   ```matlab
   A = ones(5);
   A(:,3) = [1 2 3 4 5]';
   ```
   
   (a) What is $A$?
   
   (b) What will the output be if we call `det(A)`?