Mathematics 152 Midterm 2 Review Package

UBC Engineering Undergraduate Society

Attempt questions to the best of your ability. Problems are ranked in difficulty as (•) for easy, (••) for medium, and (•••) for difficult.

Solutions posted at: https://ubcengineers.ca/tutoring

If you believe that there is an error in these solutions, or have any questions, comments, or suggestions regarding EUS Tutoring sessions, please e-mail us at: tutoring@ubcengineers.ca. If you are interested in helping with EUS tutoring sessions in the future or other academic events run by the EUS, please e-mail vpacademic@ubcengineers.ca.

Some of the problems in this package were not created by the EUS. Those problems originated from one of the following sources (All solutions prepared by the EUS.):

- Schuam’s Outline of Matrix Operations; Richard Bronson
- Calculus 7th ed; James Stewart
- Linear Algebra; Sterling K. Berberian
- Linear Algebra and Its Applications 3rd ed; Gilbert Strang
- Linear Algebra and Matrix Theory; Robert Stoll

Want a warm up?  These are the easier problems 2, 5, 6
Short on study time? These cover most of the material 6, 7, 9
Want a challenge? These are some tougher questions 9, 11, 12

EUS Health and Wellness Study Tips

- **Eat Healthy**—Your body needs fuel to get through all of your long hours studying. You should eat a variety of food (not just a variety of ramen) and get all of your food groups in.

- **Take Breaks**—Your brain needs a chance to rest: take a fifteen minute study break every couple of hours. Staring at the same physics problem until your eyes go numb won’t help you understand the material.

- **Sleep**—We have all been told we need 8 hours of sleep a night, university shouldn’t change this. Get to know how much sleep you need and set up a regular sleep schedule.
1. Consider the linear system
\[
\begin{cases}
x + 2y + z = 1 \\
-x + 3z = 1 \\
x - y - 3z = 0
\end{cases}
\]
(a) Write this system as an augmented matrix.
(b) Write the system to row echelon form
(c) Write the system in reduced row echelon form
(d) Find the solution to the system
2. Compute the rank of \( A = \begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 2 & 4 & 0 & 2 \end{pmatrix} \)

3. (a) Find the work done in moving an object along a vector \( \mathbf{r} = 3\mathbf{i} + 2\mathbf{j} - 5\mathbf{k} \) if the applied force is \( \mathbf{F} = 2\mathbf{i} - \mathbf{j} - \mathbf{k} \)

(b) Find the angle between the applied force and the displacement.
4. Consider the following lines of Matlab code:
   \[
   x = 1:7;
   y = 1:0.3:1.7;
   \]
   (a) What is \( x \)?
   (b) What is \( y \)?
   (c) If you call \( \sin(y) \), what will the output be? If this operation is defined, you may leave your answers in terms of trigonometric functions.
   (d) Is \( \text{cross}(x,y) \) defined?

5. What matrix \( A : \mathbb{R}^2 \to \mathbb{R}^2 \) represents projection onto the \( x \) axis followed by projection onto the \( y \) axis?
6. If

\[ A = \begin{pmatrix} 4 & 2 & 0 \\ 2 & 1 & 0 \\ -2 & -1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 3 & 1 \\ 2 & -2 & -2 \\ -1 & 2 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 3 & 1 & -3 \\ 0 & 2 & 6 \\ -1 & 2 & 1 \end{pmatrix} \]

Compute

(a) \( AB \)

(b) \( AC \)

What can you say about \( AB \) and \( AC \)? What does it say about cancellation of matrices? Does \( AB = AC \) imply that \( B = C \)?
7. Given \( T(x) = \begin{pmatrix} -1 & 3 \\ 9 & 4 \end{pmatrix} x \), and \( S(x) = \begin{pmatrix} 3 & -2 & 6 \\ -4 & 6 & 2 \end{pmatrix} x \), compute the following (if defined)

(a) \( T \circ S \)

(b) \( S \circ T \)

(c) \( T \begin{pmatrix} 2 \\ -1 \end{pmatrix} \)

(d) \( S \begin{pmatrix} -2 \\ 4 \end{pmatrix} \)

(e) \( S \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \)
8. (a) Find the matrix $R : \mathbb{R}^2 \to \mathbb{R}^2$ that rotates vectors by 225° counterclockwise
(b) Find the image of (2, 5) under this linear transformation.

9. What matrix has the effect of rotating a vector $v \in \mathbb{R}^2$ through 90° clockwise, and then projecting the result onto the $x$ axis?
10. (**) (a) Find the matrix $R : \mathbb{R}^2 \to \mathbb{R}^2$ that reflects vectors across the line $y = -2x$.
   
   (b) Show that $R^2 = I$.

   (c) Reflect the vector $(-2, 3)$ across the line $y = -2x$. 

11. Set up the augmented matrix $A$ corresponding to this resistor network, with the loop currents in the first three columns.

14) Find the matrix $R$ that reflects vectors across the line $y = -2x$. Show that $R^2 = I$.

Reflect the vector $<-2, 3>$ across the line $y = -2x$.

15) Set up the augmented matrix $A$ with the loop currents in the first three columns, and the potential difference $E$ in the fourth column, that one would rref($A$) in order to solve the system.
12. Consider the linear system for the unknowns $x$, $y$, and $z$.

\[
\begin{align*}
4x + 2y - 3z - 6 &= 0 \\
x - 4y + z + 4 &= 0 \\
-x + 2z - 2 &= 0 
\end{align*}
\]

(a) Write the system in an augmented matrix.

(b) Perform row operations on the augmented matrix to change it to upper triangular form.

(c) Find the solution to the problem from above.
13. Consider the resistor network below:
   
   (a) Set up the augmented matrix $A$ corresponding to this resistor network
   
   (b) Solve the augmented matrix for the currents 1, 2, and 3
   
   (c) Find the voltage $V_0$