Physics 157 Midterm 2 Review Package

UBC Engineering Undergraduate Society

Attempt questions to the best of your ability. Problems are ranked in difficulty as (•) for easy, (••) for medium, and (•••) for difficult. Difficulty is subjective, so do not be discouraged if you are stuck on a (•) problem.

Solutions will be posted at: [https://ubcengineers.ca/tutoring/](https://ubcengineers.ca/tutoring/)

If you believe that there is an error in these solutions, or have any questions, comments, or suggestions regarding EUS Tutoring sessions, please e-mail us at: tutoring@ubcengineers.ca. If you are interested in helping with EUS tutoring sessions in the future or other academic events run by the EUS, please e-mail vpacademic@ubcengineers.ca.

Want a warm up?
These are the easier problems
1, 2, 3

Short on study time?
These cover most of the material
3, 6, 7

Want a challenge?
These are some tougher questions
8, 9, 10

Some of the problems in this package were not created by the EUS. Those problems originated from one of the following sources:

- Exercises for the Feynman Lectures on Physics / Matthew Sands, Richard Feynman, Robert Leighton.
- A Student’s Guide to Entropy / Don Lemons
- Schweettam Thermodynamics / Peter W. Matthews, Charles F. Schwerdtfeger

EUS Health and Wellness Study Tips

- **Eat Healthy**—Your body needs fuel to get through all of your long hours studying. You should eat a variety of food (not just a variety of ramen) and get all of your food groups in.
- **Take Breaks**—Your brain needs a chance to rest: take a fifteen minute study break every couple of hours. Staring at the same physics problem until your eyes go numb won’t help you understand it.
- **Sleep**—We’ve all been told we need 8 hours of sleep a night, university shouldn’t change this. Get to know how much sleep you need and set up a regular sleep schedule.

Good Luck!
1. One mole of gas in a container is initially at a temperature 127°C. It is suddenly expanded to twice its initial volume without heat exchange with the outside. Then it is slowly compressed, holding the temperature constant, to the original volume. The final temperature is found to be −3°C.

(a) What is the coefficient $\gamma$ of the gas?

(b) What net heat transfer, $\Delta Q$, if any, has occurred?

**Solution:**

For any pair of $T$, $V$ during an adiabatic process,

$$T_0 V_0^{\gamma - 1} = TV^{\gamma - 1}$$

So then since $T_0 = 400$ K and $T = 270$ K, we have

$$400V_0^{\gamma - 1} = 270(2V_0)^{\gamma - 1}$$

Thus $400/270 = 2^{\gamma - 1}$. Solving for $\gamma$, we obtain

$$\gamma = 1.57$$

Adiabatic Process During the adiabatic process, there is no heat transfer: $dQ = 0$  

 Isothermal Process For the isothermal process, $\Delta Q = \Delta W$ because internal energy stays constant as long as temperature stays constant. Thus

$$\Delta Q = nRT \ln \left( \frac{V_0}{2V_0} \right)$$

$$= -270R \ln(2)$$

$$= -1.56 \text{ kJ}$$
2. The Solar Constant at Earth’s atmosphere is 1390 W/m². The radius of the Sun is 695 · 10⁶ m, and the average distance between the Earth and the Sun is 150 · 10⁹ m. Find

(a) The temperature of the Sun (assuming it radiates as a black-body)

(b) The equilibrium temperature of Earth

**Solution:**

(a) We start by viewing a sphere centered at the Sun and with a radius equal to the distance between earth and the sun (rₑ). We know that the Solar Constant (in W/m²) is just the Sun’s total power divided by the surface area of this sphere.

\[ P = 1390 \cdot (4\pi rₑ^2) = 3.93 \cdot 10^{26} \text{ W} \]

From the Stefan-Boltzmann Law: \( P = A\sigma eT^4 \). Using the radius of the Sun (rₛ), and knowing that e = 1 for a black body, we get

\[ P = (4\pi rₛ^2)\sigma T_S^4 \]

\[ T_S = \left( \frac{3.93 \cdot 10^{26}}{4\pi r_S^2\sigma} \right)^{1/4} = 5810 \text{ K} \]

Where \( \sigma \) is the Stephan-Boltzmann constant.

(b) The equilibrium temperature of Earth (Tₑ) occurs when the radiation absorbed from the Sun equals the radiation emitted by the Earth. For absorption, we consider the cross-sectional area of the Earth: a 2-dimensional disk. For emission, we use the entire surface area of the Earth. We have the power balance

\[ (1 - \alpha)\pi rₑ^2(1390) = 4\pi rₑ^2\sigma e T_E^4 \]

Here, let’s assume that e = 1 and \( \alpha = 0 \) (full absorption). Simplifying this equation and solving for Tₑ gives us

\[ T_E = \left( \frac{1390}{4\sigma} \right)^{1/4} = 280 \text{ K} \]
3. (**) A gas of coefficient $\gamma$ in a cylinder of volume $V_0$ at temperature $T_0$ and pressure $P_0$ is compressed adiabatically to volume $V_0/2$. After being allowed to come to temperature equilibrium ($T_0$) at this volume, the gas is then allowed to expand slowly and isothermally to its original volume $V_0$. In terms of $P_0$, $V_0$, $\gamma$, what is the net amount of work $W$ the piston does on the gas?

**Solution:** The work is done in two processes. First the adiabatic compression, then the isothermal expansion.

**Adiabatic Compression**

The work for an adiabatic process is

$$W = \frac{C}{1-\gamma} (V_2^{1-\gamma} - V_1^{1-\gamma}),$$

where $C = PV^\gamma$, with values of $P$ and $V$ at any particular time during the process. For convenience we choose $C = P_0V_0^\gamma$.

Plugging in $V_2 = V_0/2$ and $V_1 = V_0$ we obtain

$$W = \frac{P_0V_0^\gamma}{1-\gamma} \left( \frac{V_0^{1-\gamma}}{2^{1-\gamma}} - V_0^{1-\gamma} \right)$$

$$= \frac{P_0V_0}{1-\gamma} (2^{\gamma-1} - 1) \quad (3.2)$$

Note that this work is the work done by the gas, and thus is negative. Since we are looking for the work done by the piston, we instead want the negative of (4.2), i.e.

$$W_1 = \frac{P_0V_0}{\gamma-1} (2^{\gamma-1} - 1) \quad (3.3)$$

**Isothermal Expansion**

For the expansion (isothermal), the work is given by

$$W = nRT \ln(V_f/V_i)$$

Plugging in the given values, we obtain

$$W = nRT_0 \ln(V_0/(V_0/2))$$

$$= P_0V_0 \ln(2) \quad (3.4)$$

Note that this work is positive because the gas did positive work. Since we are looking for the work done by the piston, we want

$$W_2 = -P_0V_0 \ln(2)$$

**Adding it up**

Adding these two works together we obtain the work done by the piston on the gas:

$$\sum W = W_1 + W_2$$

$$= P_0V_0 \left( \frac{2^{1-\gamma} - 1}{\gamma-1} - \ln(2) \right) \quad (3.6)$$
4. Pluto’s diameter is approximately 2000 km and it is 40 times farther away from the Sun than the Earth. The solar constant at the Earth’s atmosphere is 1390 W/m². Assume emissivity is 1. The albedo of Pluto is 0.4.

(a) What is the total power absorbed by Pluto?

(b) What is the temperature of Pluto?

(c) Assume that the atmospheric pressure is half that of Earth’s. What is the density of the molecules on Pluto’s surface? (Hint: use $R = 8.2 \cdot 10^{-5}$ m³ atm/k mol)

Solution:

(a) We know that the intensity of the Sun’s radiation at any distance is the total power of the Sun divided by the surface area of a sphere whose radius is equal to that distance. Applying this, we see

$$P_S = 1390(4\pi r_E^2) = I_P(4\pi r_P^2)$$

$$I_P = \frac{1390(4\pi r_E^2)}{4\pi r_P^2} = \frac{1390r_E^2}{40^2r_E^2} = 0.869 \text{ W/m}^2$$

We use the equation $P_P = I_P\pi r_P^2(1 - a_P)$, where $a_P$ is the albedo. We get

$$P_P = 0.869\pi(1 \cdot 10^6)^2(0.6) = 1.637 \cdot 10^{12} \text{ W}$$

(b) We know that the power in equals the power out:

$$P_P = P_{out} = 4\pi r_P^2 \sigma T_P^4$$

$$T_P = \left( \frac{I_P\pi r_P^2(1 - a_P)}{4\pi r_P^2\sigma} \right)^{1/4} = \left( \frac{I_P(1 - a_P)}{4\sigma} \right)^{1/4} = 38.9 K$$

(c) We know the density of molecules will just be the number of molecules divided by the volume they occupy. We can rearrange $PV = nRT$ to get

$$\frac{n}{V} = \frac{P}{RT}$$

We multiply both sides by Avogadro’s number, $N_a = 6.02 \cdot 10^{23}$

$$\text{density} = \frac{nN_a}{V} = PN_aRT = \frac{0.5 \text{ atm}(6.02 \cdot 10^{23})}{8.2 \cdot 10^{-5}(38.9 K)} = 9.44 \cdot 10^{25} \text{ molecules/m}^3$$
5. An ideal gas with coefficient $\gamma$, is initially at the condition $P_0 = 1$ atm, $V_0 = 1$ litre, $T_0 = 300$ K. It is then:

(i) Heated at constant $V$ until $P = 2$ atm.
(ii) Expanded at constant $P$ until $V = 2$ litres.
(iii) Cooled at constant $V$ until $P = 1$ atm.
(iv) Contracted at constant $P$ until $V = 1$ litre.

(a) Draw a $P$–$V$ diagram for this process.
(b) What work $W$ is done per cycle?
(c) What is the maximum temperature $T_{max}$ the gas attains?
(d) What is the total heat input $\Delta Q$ in steps (i) and (ii) in terms of $\gamma$?

**Solution:**

(a) See figure for $P$–$V$ diagram.

(b) For an isochoric process (processes (i), (iii)) there is no work done. Therefore we just sum up the works from processes (ii) and (iv). For process (ii), work is given by

$$W_{ii} = P\Delta V = 2(2 - 1) = 2 \text{ atm} \cdot \text{litre} = 202.6 \text{ J}$$

For process (iv), work is given by

$$W_{iv} = P\Delta V = 1(1 - 2) = -1 \text{ atm} \cdot \text{litre} = -101.3 \text{ J}$$

Summing up the works, we obtain

$$W = W_i + W_{ii} + W_{iii} + W_{iv}$$
$$= 0 + 202.6 + 0 - 101.3$$
$$= 101.3 \text{ J/cycle}$$
(c) Maximum temperature will be attained when the gas is at largest volume and highest pressure. This will be after process ii (when $P = 2$ and $V = 2$). Since
\[ \frac{PV}{T} = \text{constant} \]
we can calculate the constant with the initial temperature, pressure, and volume values as
\[ \frac{P_0V_0}{T_0} = \frac{1}{300} \]
Plugging in the values $P = 2$ and $V = 2$, we obtain
\[ \frac{1}{300} = \frac{2 \cdot 2}{T_{\text{max}}} \]
Thus
\[ T_{\text{max}} = 1200 \text{ K} \]

(d) We can use the specific heat capacity equations to find the total heat input. Process i is isochoric, so
\[ Q_i = nC_V \Delta T \]
Process ii is isobaric, so
\[ Q_{ii} = nC_P \Delta T' \]
The changes in temperature for each process are
\[ \Delta T_i = 300 \text{ K} \]
and
\[ \Delta T_{ii} = 600 \text{ K} \]
(From the ideal gas law). Thus
\[ \Delta Q = Q_i + Q_{ii} = nC_V \Delta T_i + nC_P \Delta T_{ii} = 300nC_V + 600nC_P \] (5.2)
We want to find a value for $n$ to plug into (5.3). From the ideal gas law,
\[ P_0V_0 = nRT_0 = n(C_P - C_V)T_0 \] (5.4)
we solve for
\[ n = \frac{P_0V_0}{T_0(C_P - C_V)} = \frac{101.3}{300(C_P - C_V)} \] (5.5)
Plugging (5.5) into the expression for $\Delta Q$ (5.3), we obtain
\[ \Delta Q = 300C_V \frac{101.3}{300(C_P - C_V)} + 600C_P \frac{101.3}{300(C_P - C_V)} \] (5.6)
\[ = 101.3 \left( \frac{C_V}{C_P - C_V} + \frac{2C_P}{C_P - C_V} \right) \] (5.7)
\[ = 101.3 \left( \frac{2C_P + C_V}{C_P - C_V} \right) \] (5.8)
Dividing numerator and denominator by $C_V$, we obtain
\[ \Delta Q = 101.3 \left( \frac{2\gamma + 1}{\gamma - 1} \right) \text{ J} \]
6. The first Earth settlers on the moon will have great problems in keeping their living quarters at a comfortable temperature. Consider the use of Carnot engines for climate control. Assume that the temperature during the moon-day is 100°C, and during the moon-night is −100°C. The temperature of the living quarters is to be kept at 20°C. The heat conduction rate through the walls of the living quarters is 0.5 kW per degree of temperature difference.

(a) Find the power \( P_{\text{day}} \) which has to be supplied to the Carnot engine during the day, and

(b) the power \( P_{\text{night}} \) which must be supplied at night.

**Solution:**

(a) During the daytime, the Carnot engine will be run in reverse, because it is cooling (refrigerating) the living quarters. This means that we put in work, and heat is taken from a cold reservoir (indoors) and sent to a warm reservoir (outdoors).

The equation for coefficient of performance for a refrigerator is

\[
\text{COP} = \frac{|Q_C|}{|W|} = \frac{|Q_C|}{|Q_H| - |Q_C|}
\]

In a Carnot engine,

\[
\frac{|Q_C|}{|Q_H|} = \frac{T_C}{T_H}
\]

Thus we arrive at the coefficient of performance for a Carnot refrigerator,

\[
\text{COP} = \frac{|Q_C|}{|Q_H| - |Q_C|} = \frac{|Q_C|}{|Q_H|/|Q_C|} = \frac{1 - |Q_C|/|Q_H|}{T_C/T_H - T_C} = \frac{T_C}{T_H - T_C}
\]

Another way to write the coefficient of performance is

\[
\text{COP} = \frac{|Q_C|}{|W|} = \frac{|Q_C|/t}{|W|/t} = \frac{H}{P}
\]

where \( H \) is the heat current through the walls of the living quarters and \( P \) is the power supplied to the engine.

For the daytime, we have \( T_C = 293 \) K, and \( T_H = 373 \) K, and \( H = 40 \) kW. Combining (7.3) and (7.4), we obtain

\[
\frac{H}{P_{\text{day}}} = \frac{T_C}{T_H - T_C}
\]

Plugging in the appropriate variables into (6.5) and solving for \( P_{\text{day}} \),

\[
P_{\text{day}} = 10.9 \text{ kW}
\]

(b) For the night time, we have \( T_C = 173 \) K, and \( T_H = 293 \) K, and \( H = 60 \) kW. Because we want to heat the interior this time, we will run the Carnot engines forward.
We have two expressions for efficiency:

\[ e = 1 - \frac{T_C}{T_H} = 0.41 \quad (6.6) \]

\[ e = \frac{W}{Q_H} = \frac{W/t}{Q_H/t} = \frac{P}{H} \quad (6.7) \]

Equating (6.6) and (6.7),

\[ \frac{P_{\text{night}}}{H} = 1 - \frac{T_C}{T_H} = 0.41 \quad (6.8) \]

Solving for \( P_{\text{night}} \),

\[ P_{\text{night}} = 24.6 \text{ kW} \]
7. Two samples of gas, A and B of the same initial volume $V_0$, and at the same initial absolute pressure $P_0$, are suddenly compressed adiabatically, each to one half its initial volume.

(a) Express the final pressures ($P_A, P_B$) of each sample in terms of the initial pressure $P_0$, if $\gamma_A = \frac{5}{3}$ (monatomic) and $\gamma_B = \frac{7}{5}$ (diatomic)

(b) Find the ratio of work $W_A/W_B$ required to perform the two compressions described.

**Solution:**

(a) We have, for an adiabatic process,

$$P_0 V_0^{\gamma} = P_A V_A^{\gamma_A} = P_B V_B^{\gamma_B} \quad (7.1)$$

Since both containers are compressed to one-half of the original volume,

$$V_A = V_B = V_0/2$$

We obtain

$$P_0 V_A^{\gamma_A} = P_A \left( \frac{V_0}{2} \right)^{\gamma_A}$$

$$P_0 V_B^{\gamma_B} = P_B \left( \frac{V_0}{2} \right)^{\gamma_B}$$

Thus

$$(P_A, P_B) = (2^{\gamma_A} P_0, 2^{\gamma_B} P_0) = (3.17 P_0, 2.64 P_0)$$

(b) For an adiabatic process, the formulas for work are:

$$W_A = \frac{P_0 V_0^{\gamma_A}}{1 - \gamma_A} \left[ \left( \frac{V_0}{2} \right)^{1-\gamma_A} - V_0^{1-\gamma_A} \right] \quad (7.2)$$

$$W_B = \frac{P_0 V_0^{\gamma_B}}{1 - \gamma_B} \left[ \left( \frac{V_0}{2} \right)^{1-\gamma_B} - V_0^{1-\gamma_B} \right] \quad (7.3)$$

Since

$$V_A = V_B = V_0/2$$

we can plug $V_A = V_0/2$ into (9.2):

$$W_A = \frac{P_0 V_0^{\gamma_A}}{1 - \gamma_A} \left[ \left( \frac{V_0}{2} \right)^{1-\gamma_A} - V_0^{1-\gamma_A} \right] \quad (7.4)$$

$$= \frac{P_0 V_0}{1 - \gamma_A} \left( \frac{1}{2^{1-\gamma_A}} - 1 \right) \quad (7.5)$$

and $V_B = V_0/2$ into (9.3):

$$W_B = \frac{P_0 V_0^{\gamma_B}}{1 - \gamma_B} \left[ \left( \frac{V_0}{2} \right)^{1-\gamma_B} - V_0^{1-\gamma_B} \right] \quad (7.6)$$

$$= \frac{P_0 V_0}{1 - \gamma_B} \left( \frac{1}{2^{1-\gamma_B}} - 1 \right) \quad (7.7)$$
Dividing (9.5) by (9.7), we obtain

\[
\frac{W_A}{W_B} = \frac{1 - \gamma_B}{1 - \gamma_A} \cdot \left( \frac{1}{2^{1/2 - \gamma_A}} - 1 \right)
\]

\[
= \frac{1 - \gamma_B}{1 - \gamma_A} \cdot \left( \frac{2^{\gamma_A - 1} - 1}{2^{\gamma_B - 1} - 1} \right)\]

\[
= 1.1
\]
8. In an ideal reversible engine employing 28 g nitrogen as working substance ($\gamma = 7/5$) in a cyclic operation $a \to b \to c \to d$ without valves, the temperature of the source is 400 K, and the temperature of the sink is 300 K. The initial volume of gas at point $a$ is 6.0 litres and the volume at point $c$ is 18.0 litres.

(a) At what volume $V_b$ should the cylinder be changed from heat input (isothermal expansion) to isolation and adiabatic expansion (from $V_b$ to $V_c$)?

(b) At what volume $V_d$ should the adiabatic compression begin?

(c) How much heat $\Delta Q_{a\to b}$ is put in during the $V_a \to V_b$ part of the cycle?

(d) How much heat $\Delta Q_{c\to d}$ is extracted during the $V_c \to V_d$ part?

(e) What is the efficiency $e$ of the engine?

(f) What change $\Delta S$ in entropy per gram occurs in the working substance during $a \to b$ and $c \to d$?

**Hint.** For a Carnot cycle the expansion ratios $V_b/V_a$ and $V_c/V_d$ are equal. Draw yourself a P-V diagram to help understand the cycle.

**Solution:** Since the question says an ideal reversible engine, this implies that we should set up a Carnot cycle. First, we draw a P-V diagram to help us understand the cycle. We know that the temperature at the upper isotherm is 400 K, and that the temperature at the lower isotherm is 300 K. We also know that $a$ and $c$ have to go in the places they do because the given values tell us the volumes $V_a$ and $V_c$.

![P-V Diagram]

We will first write down all of our known relations, even though we will not need to use some of them. Since we are working with 28 g of Nitrogen, and nitrogen is element 7, 28 grams of nitrogen means 1 mol of nitrogen, so $n = 1$. We also know the volumes of $V_a = 6$ L, and $V_c = 18$ L.

For the ideal gas law at each point, we have (because $n = 1$)

$$P_a V_a = RT_a$$
$$P_b V_b = RT_b$$
$$P_c V_c = RT_c$$
$$P_d V_d = RT_d$$

(8.1) (8.2) (8.3) (8.4)

For the two adiabatic processes, we have

$$T_c V_c^{\gamma - 1} = T_b V_b^{\gamma - 1}$$
$$T_a V_a^{\gamma - 1} = T_d V_d^{\gamma - 1}$$

(8.5) (8.6)
For the two isothermal processes, we have
\[ P_a V_a = P_b V_b \quad (8.7) \]
\[ P_c V_c = P_d V_d \quad (8.8) \]

Because this is a Carnot cycle, we have
\[ V_a V_c = V_d V_b \quad (8.9) \]

Some known values are:
\[ T_a = T_b = 400 \text{ K} \]
\[ T_c = T_d = 300 \text{ K} \]

(a) Plugging in \( T_c = 300 \), \( T_b = 400 \), \( V_c = 18 \) into (9.5) yields
\[ 300V_c^{\gamma-1} = 400V_b^{\gamma-1} \]
\[ 300(18^{7/5-1}) = 953.3 \]
\[ V_b^{2/5} = 2.38 \]
\[ V_b = 8.8 \text{ L} \]

(b) Plugging in \( T_d = 300 \), \( T_a = 400 \), \( V_a = 6 \) to (9.6) yields
\[ 400V_a^{\gamma-1} = 300V_d^{\gamma-1} \]
\[ 400(6^{7/5-1}) = 819 \]
\[ V_d^{2/5} = 2.73 \]
\[ V_d = 12.3 \text{ L} \]

(c) The process \( a \rightarrow b \) is isothermal, so the change in internal energy is 0. By the first law of thermodynamics, we then have \( \Delta U = 0 = \Delta Q - W \). We also have one mole of Nitrogen (\( N_2 \)). Thus
\[ \Delta Q_{a\rightarrow b} = W = nRT_a \ln(V_b/V_a) \]
\[ = 400R \ln(8.8/6) \]
\[ = 1.26 \cdot 10^3 \text{ J} \]

(d) The process \( c \rightarrow d \) is isothermal, so the change in internal energy is 0. By the first law of thermodynamics, we then have \( \Delta U = 0 = \Delta Q - W \). Thus
\[ \Delta Q_{c\rightarrow d} = W = nRT_c \ln(V_d/V_c) \]
\[ = 300R \ln(12.3/18) \]
\[ = -945.9 \text{ J} \]

This means that +945.9 J were removed.
(e) The efficiency of the engine will be the Carnot efficiency, which is

\[ e = 1 - \frac{T_C}{T_H} = 1 - 0.75 = 0.25 \]

Thus the engine is 25% efficient.

(f) During \( a \to b \) (or \( c \to d \), doesn’t matter, it is the same \( \Delta S \)), we have

\[ \Delta S = \frac{\Delta Q_{a\to b}}{T_a} = \frac{1.26 \cdot 10^3}{400} = 3.15 \text{ J/K} \]

To find the entropy per gram, divide by the molar mass of \( N_2 \). Per gram, that is \( 3.15/28 = 0.11 \text{ J/gK} \)

**Remark.** If we had instead used \( \Delta Q_{c\to d}/T_c = 945/300 = 3.15 \) we would have gotten the same answer
9. An insulated container with a movable, frictionless piston of mass $M$ and area $A$, contains $N$ grams of helium gas in a volume $V_1$, as shown. The external pressure is $P$. The gas is very slowly heated by an internal heating coil until the volume occupied by the gas is $2V_1$. What is,

(a) the work $W$ done by the gas?
(b) the heat $\Delta Q$ supplied to the gas?
(c) the change $\Delta U$ in the internal energy of the gas?
(d) the initial temperature $T_i$ and the final temperature $T_f$ of the gas?

Express your answers in terms of the given variables $M, A, P, N, V_1$.

**Solution:**

(a) The piston has area $A$, so then we know that

$$V_1 = hA$$

where $h$ is the distance between the bottom of the container and the piston. Since the piston is moved so that the container is $2V_1$ in volume, we conclude that the piston moved up a distance of $h$ to a new height of $2h$.

The forces on the piston are due to gravity, external pressure $P$, and the pressure of the gas inside. The gas must work against the force of gravity and the external pressure. Those two forces are given by $PA + Mg$. Thus the work is

$$W = (PA + Mg)h$$

$$= \frac{(PA + Mg)V_1}{A}$$

$$= \left( P + \frac{Mg}{A} \right) V_1$$

(b) The piston moves very slowly. This means that the pressure inside is roughly constant throughout the expansion. Thus
\[ \Delta Q = nC_p \Delta T \]  
\[ = nC_p \left( \frac{P \Delta V}{nR} \right) \]  
\[ = \frac{5P \Delta V}{2} \]  
\[ = \frac{5}{2} W \]  
\[ = \frac{5}{2} \left( P + \frac{Mg}{A} \right) V_1 \]  

(c) From the first law of thermodynamics,

\[ \Delta U = \Delta Q - W \]  
\[ = \frac{5}{2} W - W \]  
\[ = \frac{3}{2} \]  
\[ = \frac{3}{2} \left( P + \frac{Mg}{A} \right) V_1 \]

(d) Since the forces on the piston must have been balanced at the start, we have, for some initial pressure \( P_i \),

\[ P_i A = PA + Mg \]  
\[ P_i = P + \frac{Mg}{A} \]  

From the ideal gas law, we have

\[ T_i = \frac{PV_i}{nR} \]  

Since the gas is helium, we know that there are 4 grams per mol, which means that the number of moles \( n \) is given by \( n = N/4 \). Thus, plugging (9.14) into the ideal gas law (9.15),

\[ T_i = 4V_i \frac{Mg}{NR} \left( P + \frac{Mg}{A} \right) \]  
\[ = \frac{4W}{NR} \]

We know that the change in internal energy is

\[ \Delta U = \frac{3W}{2} \]  

and since

\[ \Delta U = nC_V \Delta T \]  

we can equate (9.18) and (9.19) to obtain

\[ \frac{3W}{2} = \frac{N}{4} \frac{3R}{2} (T_f - T_i) \]
Multiplying both sides of (9.20) by $8/(3NR)$,

$$\frac{4W}{NR} = T_f - T_i = T_i$$

Thus

$$T_f = 2T_i$$

$$= \frac{8V_1}{NR} \left( P + \frac{Mg}{A} \right)$$
10. A sample of gas undergoes a transition from an initial state $a$ to a final state $b$ by three different paths, as shown in the $P$-$V$ diagram, where $V_b = 5.00V_i$. The energy transferred to the gas as heat in process 1 is $10P_iV_i$.

(a) How many degrees of freedom does the sample of gas have?
(b) Find the energy transferred to the gas as heat in process 2.
(c) Find the change in internal energy that the gas undergoes in process 3.

Express your answers in terms of $P_i$, $V_i$.

\[ p \]

\[ p_i \]

\[ 3p_i/2 \]

\[ p_f \]

\[ p_f/2 \]

\[ V_i \]

\[ V_b \]

\[ C \]

\[ B \]

\[ A \]

\[ 0 \]

\[ \text{Solution:} \]

(a) We will first try and get more data about path 1. Since it is an isobaric process because pressure is constant, the work done will be

\[ W = P_i(V_b - V_i) = 4P_iV_i \]

From the ideal gas law $V_i/T_i = V_b/T_b$, we can find that

\[ T_b = 5T_i \]

\[ \Delta T = 4T_i \]

The change in internal energy will be

\[ \Delta U = nC_V\Delta T \]

\[ = nC_V(4T_i) \]

\[ = 4C_V(nT_i) \]

\[ = 4C_V(P_iV_i/R) \]

Then we can use the first law of thermodynamics to find a relationship between $C_V$ and $R$.

\[ \Delta U = Q - W \]

\[ 4C_VP_iV_i/R = 10P_iV_i - 4P_iV_i = 6P_iV_i \]

\[ C_V = 3R/2 \]

Thus we conclude that it is a monatomic gas and has 3 degrees of freedom. Note that the three degrees of freedom are simply movement in the $x$, $y$, and $z$ directions.
(b) The work for the first segment of path 2 can be found using the area under the line. We use the formula for the area of a trapezoid.

\[
W = (V_b - V_i)(P_i + 3P_i/2)/2 = 5P_iV_i \quad (10.1)
\]

The change in temperature in the first segment of path 2 can be found using the ideal gas law. We have

\[
P_iV_i/T_i = P_bV_b/T_b(5V_i)(3P_i/2)/(T)
\]

\[
T = 15T_i/2
\]

This means that \( \Delta T = 13/2T_i \).

The change in internal energy will be

\[
\Delta U = nC_V(13/2T_i) \quad (10.2)
\]

\[
= (13/2)C_VP_iV_i/R \quad (10.3)
\]

\[
= (13/2)(3/2)P_iV_i \quad (10.4)
\]

\[
= 39P_iV_i/4 \quad (10.5)
\]

We can use the first law of thermodynamics to find that \( \Delta Q = \Delta U + W \) for this first segment of the path (using (14.1) and (14.5)) is

\[
39P_iV_i/4 + 5P_iV_i = 59P_iV_i/4 \quad (10.6)
\]

For the second (vertical) segment of the path, there is no work done. Thus the heat added (at constant volume) will depend only on the temperature difference. We know that the temperature at \( b \) is \( 5T_i \), so then the change in heat energy for this second segment will be

\[
Q = nC_V(-5/2T_i) = -15P_iV_i/4 \quad (10.7)
\]

Adding these two heats, we find that the total change in heat energy over path 2 is

\[
\Delta Q = 59P_iV_i/4 - 15P_iV_i/4 = 11P_iV_i
\]

(c) Internal energy is independent of path, so then

\[
\Delta U = nC_V(4T_i) = 6P_iV_i
\]
Useful Constants and Conversion Ratios:
- \( R = \text{Ideal Gas constant} = 8.31451 \text{ J/molK} \)
- \( 1 \text{ atm} = 1.013 \times 10^5 \text{ Pa} \)
- \( 1 \text{ atm} \cdot \text{litre} = 101.3 \text{ J} \)
- \( \sigma = \text{Stefan-Boltzmann constant} = 5.67 \times 10^{-8} \text{ W/m}^2 \text{K}^4 \)
- \( \gamma_{\text{air}} = 1.4 \)
- \( C_{V_{\text{air}}} = 20.8 \text{ J/molK} \)
- \( \rho_{\text{water}} = \text{Density of water} = 1 \text{ gram/cm}^3 = 1000 \text{ kg/m}^3 \)

Mechanics:
- Linear Motion: \( x = x_0 + \frac{1}{2}(v_0 + v)t \), \( x = x_0 + v_0 t + \frac{1}{2}a t^2 \), \( v = v_0 + at \), \( v^2 = v_0^2 + 2a(x - x_0) \)
- Circular Motion: \( a_c = \frac{v^2}{r} \)
- Forces: \( \mathbf{F} = m \mathbf{a} = \frac{d}{dt} \mathbf{p} \)
- Friction: \( |\mathbf{F}| = \mu |\mathbf{N}| \)
- Spring: \( \mathbf{F} = -kx \)
- Damping: \( \mathbf{F} = -bv \)
- Work for an isothermal process: \( W = nRT \ln\left(\frac{V_f}{V_i}\right) \)
- Work for an adiabatic expansion: \( TV^{\gamma - 1} \) constant, if the number of moles is constant \( PV^\gamma = C \) where \( C \) is a constant and \( \gamma = C_P/C_V \)
- Heat Transfer: \( Q = mc\Delta T \)

Thermodynamics:
- Thermal Expansion: \( \Delta L = \alpha L_0 \Delta T \)
- Stress and Strain: \( \frac{|\mathbf{F}|}{A} = \frac{Y \Delta L}{L} \)
- Ideal Gas Law: \( PV = nRT \)
- \( K_{\text{av}} = \frac{1}{2}kT \)
- Thermal Conductivity: \( I = \frac{\Delta Q}{\Delta t} = kA \frac{\Delta T}{\Delta x} \)
- Black Body Radiation: \( P = e\sigma AT^4 \), \( \lambda_{\text{max}}T = 2.8977685 \times 10^{-3}m \cdot K \)
- Stephan-Boltzmann Constant: \( \sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \text{K}^4 \)
- Internal Energy: \( U = nC_VT \)
- First Law of Thermodynamics: \( dQ = dU + dW \)
- Work for an isothermal process: \( W = nRT \ln\left(\frac{V_f}{V_i}\right) \)
- Work for an adiabatic expansion: \( TV^{\gamma - 1} \) constant, if the number of moles is constant \( PV^\gamma = C \) where \( C \) is a constant and \( \gamma = C_P/C_V \)
- Heat Transfer: \( Q = mc\Delta T \)

First Law of Thermodynamics: \( dQ = dU + dW \) For an ideal gas, \( dW = PdV \)
- Work for an isothermal process: \( W = nRT \ln\left(\frac{V_f}{V_i}\right) \)
- Work for an adiabatic process: \( W = \int_{V_i}^{V_f} PdV = C \int_{V_i}^{V_f} \frac{dV}{V^{\gamma}} = C \left[ \frac{V^{1-\gamma}}{1-\gamma} \right]_{V_i}^{V_f} \)
- Heat Transfer: \( Q = mc\Delta T \)
- First Law of Thermodynamics: \( dQ = dU + dW \) For an ideal gas, \( dW = PdV \)
- Work for an isothermal process: \( W = nRT \ln\left(\frac{V_f}{V_i}\right) \)
- Work for an adiabatic process: \( W = \int_{V_i}^{V_f} PdV = C \int_{V_i}^{V_f} \frac{dV}{V^{\gamma}} = C \left[ \frac{V^{1-\gamma}}{1-\gamma} \right]_{V_i}^{V_f} \)

Integrals:
- \( \int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1 \)
- \( \int x^{-1} dx = \ln x + C \)

Trigonometry:
- \( \sin \theta_1 + \sin \theta_2 = 2 \cos \left( \frac{\theta_1 - \theta_2}{2} \right) \sin \left( \frac{\theta_1 + \theta_2}{2} \right) \)

Area and Volume:
- Surface Area of a sphere: \( A = 4\pi r^2 \)
- Lateral surface area of a cylinder: \( A = 2\pi rl \)
- Area of a circle: \( A = \pi r^2 \)
- Volume of a cylinder: \( V = l\pi r^2 \)
- Volume of a sphere: \( V = \frac{4}{3}\pi r^3 \)

Oscillations:
- \( \omega = 2\pi f \), \( T = \frac{1}{f} \)
- \( x = A \cos(\omega t + \phi) \), \( \omega^2 = \frac{k}{m} \)
- Damped Oscillations: \( x = A_0 e^{-\frac{\omega}{2m} t} \cos(\omega t + \phi) \), \( \omega = \sqrt{\omega_0^2 - \left( \frac{b}{2m} \right)^2} \)

Energy for damped: \( E = E_0 e^{-\frac{\omega}{\Delta \omega}} \)
Waves:

\[ v = \sqrt{\frac{T}{\mu}}, \quad k = \frac{2\pi}{\lambda}, \quad P = \frac{1}{2} \mu \omega^2 A^2 v, \quad p_o = \rho \omega s_o \]

\[ v = \sqrt{\frac{\gamma RT}{M}}, \quad I = \frac{P_{av}}{4\pi r^2}, \quad \beta = 10 d \log_{10} \left( \frac{L}{l_o} \right), \quad \text{Doppler Effect } f' = f_0 \left( \frac{v \pm v_L}{v \mp v_S} \right) \]

Beats: \( f = f_2 - f_1, \quad y = A \cos(kx + \omega t + \phi) \)

Interference: \( k \Delta x + \Delta \phi = 2\pi n \) or \( \pi(2n + 1), \quad n = 0, \pm 1, \pm 2, \pm 3, \pm 4, \ldots \)

Standing Waves \( f_m = \frac{mv}{2L}, \quad m = 1, 2, 3, \ldots \), \( f_m = \frac{mv}{4L}, \quad m = 1, 3, 5, \ldots \)

Constants:

\[ k = \frac{1}{4\pi \epsilon_0} \approx 9 \times 10^9 \text{Nm}^2/\text{C}^2, \quad \epsilon_0 = 8.84 \times 10^{-12} \text{C}^2/\text{Nm}^2, \quad e = 1.6 \times 10^{-19} \text{C} \]

\[ \mu_0 = 4\pi \times 10^{-7} \text{Tm/A}, \quad c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 299,792, 458 \text{ m/s} \]

Electric potential and potential energy \( \Delta V = V_a - V_b = \int_a^b E \cdot dl = -\int_b^a E \cdot dl \)

\[ E_x = -\frac{dV}{dx}, \quad E = -\nabla V, \quad \Delta U = U_a - U_b = q(V_a - V_b) \]

Maxwell’s Equations:

\[ \int_S E \cdot dA = \frac{Q_{enc}}{\epsilon_0} = 4\pi k Q_{enc} \quad \int_S B \cdot dA = 0 \]

\[ \int_C B \cdot dl = \mu_0 (I_{enclosed}) + \epsilon_0 \mu_0 \frac{d\Phi_E}{dt} \quad \int_C E \cdot dl = -\frac{d\Phi_B}{dt} \]

Where \( S \) is a closed surface and \( C \) is a closed curve. \( \Phi_E = \int E \cdot dA \) and \( \Phi_B = \int B \cdot dA \)

Energy Density:

\[ u_E = \frac{1}{2} \epsilon_0 E^2 \quad \text{and} \quad u_B = \frac{1}{2\mu_0} B^2 \] (energy per volume)

Forces:

\[ F = qE + qv \times B, \quad F = IL \times B \]

Capacitors:

\[ q = CV, \quad U_C = \frac{1}{2} \frac{q^2}{C}, \quad \text{For parallel plate capacitor with vacuum (air): } C = \frac{\epsilon_0 A}{d}, \quad C_{\text{dielectric}} = KC_{\text{vacuum}} \]

Inductors:

\[ \mathcal{L}_L = -L \frac{dl}{dt}, \quad U_L = \frac{1}{2} LI^2, \quad \text{where } L = N\Phi_B/I \text{ and } N \text{ is the number of turns.} \]

For a solenoid \( B = \mu_0 n I \) where \( n \) is the number of turns per unit length.

DC Circuits: \( V^R = IR, \quad P = VI, \quad P = I^2 R \)

For RC circuits: \( q = ae^{-t/\tau} + b, \tau = RC, \) a and b are constants

For LR circuits: \( I = ae^{-t/\tau} + b, \tau = L/R, \) a and b are constants

AC circuits:

\[ X_L = \omega L, \quad X_C = 1/(\omega C), \quad V_C = X_C I, \quad V_L = X_L I \]

\[ V = ZI, \quad Z = \sqrt{(X_L - X_C)^2 + R^2}, \quad P_{\text{average}} = I_{\text{rms}}^2 R, \quad I_{\text{rms}} = \frac{I_{\text{max}}}{\sqrt{2}} \]

If \( V = V_0 \cos(\omega t) \), then \( I = I_{\text{max}} \cos(\omega t - \phi) \), where \( \tan \phi = \frac{X_L - X_C}{R} \)

Addition Equations: \( dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl \times r}{r^3} \)

LRC Oscillations: \( q = A_0 e^{-\frac{R}{2L} t} \cos(\omega t + \phi) \), where \( \omega = \sqrt{\frac{\omega_0^2 - (\frac{R}{2L})^2}{L}} \) and \( \omega_0 = \frac{1}{LC} \)