1. A 2.00m tall refrigerator of mass $m$ has a static coefficient of friction $\mu_s = 0.100$. When a pulling force of 300 N is applied as shown, the refrigerator barely slips and barely tips.

(a) Find $m$ and $x$.

(b) With this information only, is it possible to find $y$? Why or why not?

a) The fact that the refrigerator "barely slips" and "barely tips" is key to providing enough information to make this problem solvable.

$$m = \frac{300}{9.81} \left( \sin(30) + \frac{\cos(30)}{\mu} \right) = 280 \text{kg}$$

$$F_g = 2748 \text{N}$$

$$x = 0.486 \text{m}$$
The clue “barely tips” means that the normal force is acting on the corner of the refrigerator under the applied force. To simplify the question, let’s do the moment balance around this point.

\[ \sum M = 0 \]

\[ 300 \cos(30) = 166.3 \times (0.6 - x) \]

\[ x = 0.266 \text{ m} \]

b) No, it is not possible to find y. Recall that a force can be moved anywhere along its line of action without consequence and \( F_g \) acts entirely in the y direction.
2. The diagram below shows a set of 3 forces and one moment acting on a rigid body.

(a) Find the equivalent force and couple moment acting at point $O$.

(b) Reduce all forces and moments to a single wrench acting on point $P$. Find the resulting force and moment vectors as well as the distances $x$ and $y$.

\[
\begin{align*}
\text{F}_R &= -300 \hat{i} - 500 \hat{j} + 400 \hat{k} \\
M_{RO} &= 200 \hat{i} + (-300 \hat{i} \times 2 \hat{j}) + (-500 \hat{j} \times -3 \hat{i}) = 200 \hat{i} + 0 \hat{j} - 2100 \hat{k} \\
\end{align*}
\]

b) To reduce the system to a wrench, the force and moment vector must be parallel. We can get this line from the resultant force vector since this vector does not change. The unit vector of the resultant force is:

\[
\mathbf{u}_{FR} = -0.424 \hat{i} - 0.707 \hat{j} + 0.566 \hat{k}
\]

Using the unknowns $x$ and $y$ to write the moment equations gives:

\[
\begin{align*}
M_{Ri} &= 200 - 400y = -0.424 \\
M_{Rj} &= -400(3 - x) = -0.707 \\
M_{Rk} &= 500x + 300(2 - y) = 0.566 \\
\end{align*}
\]

Solving this set of equations yields:

\[
\begin{align*}
x &= 1.05 \\
y &= 1.67 \\
M_R &= 1103 \text{Nm}
\end{align*}
\]

a) $x = 1.05$ \quad $y = 1.67$ \quad $M_R = 1103 \text{Nm}$
3. The Diagram below shows a mass supported by three cables which are anchored to fixed supports.

(a) Determine the tension in each of the three cables if the cylinder has a mass of 75 kg.

(b) If each cable can withstand a maximum tension of 1000 N, determine the largest mass that this system can support.

![Diagram](image)

a) Start by expressing each force as a magnitude along with a unit vector:

\[ F_{AB} = 831 \text{ N}, F_{AC} = 35.6 \text{ N}, F_{AD} = 415 \text{ N} \]

b) Based on the results of part a, we know the limiting case is when cable AB supports a load of 1000 N. We could solve the entire system again but it is possible (and much simpler) to scale the results from a according to the new force in the cable.

\[ m_2 = 90.3 \text{ kg} \]
4. Replace the two forces in the diagram below with a single force and couple moment acting at point O.

\[ \mathbf{F}_1 = (-20 \mathbf{i} - 10 \mathbf{j} + 25 \mathbf{k}) \text{ lb} \]
\[ \mathbf{F}_2 = (-10 \mathbf{i} + 25 \mathbf{j} + 20 \mathbf{k}) \text{ lb} \]

First find the resultant force:

\[ \mathbf{F}_R = -30 \mathbf{i} + 15 \mathbf{j} + 45 \mathbf{k} \]

\[ M_{RO} = 80 \mathbf{i} + 87.5 \mathbf{j} + 102.5 \mathbf{k} \]
5. Consider the system below of a cantilevered beam with two forces and one couple moment acting on it.

(a) Determine the equivalent force and moment acting at point A and the I beam.
(b) Can the forces and couple moment acting on this beam be reduced to a single force? If so, determine this force and its location along the beam.
(c) What conditions need to be met in order to reduce a system of forces and moments to a single force? (Hint: consider the wrench problem where the system can at most be reduced to a force and couple moment)

\[
\begin{align*}
F_x &= 26(12/13) - 30 \sin(30) = -5 \text{kN} \\
F_y &= -26(12/13) - 30 \cos(30) = -50 \text{kN} \\
F_R &= \sqrt{(-5)^2 + (-50)^2} = 50.2 \text{kN} \\
\theta &= \tan^{-1}\left(\frac{50}{5}\right) = 84.3^\circ \\
M_{RA} &= -239 \text{kN m} = 239 \text{kN m} \text{ CW}
\end{align*}
\]

b) Find the distance \(d\) along AB where the resultant force produces a moment around A equal to the initial moment.

\[d = 4.79 \text{ m}\]

c) Forces must be coincident, coplanar, or parallel in order to be reduced to a single force with no moment. If this criteria is not met, the wrench is the most that the system can be simplified.