Mathematics 253 Midterm 1 Review Package

UBC Engineering Undergraduate Society

Attempt questions to the best of your ability. This review package consists of 12 pages, including 1 cover page and 24 questions. The questions are meant to be the level of a real examination or slightly above, in order to prepare you for the real exam. Material from lectures and from the relevant textbook sections is examinable, and the problems for this package were chosen with that in mind, as well as considerations based on past examination question difficulty and style. Problems are ranked in difficulty as (*) for easy, (**) for medium, and (***) for difficult. Note that sometimes difficulty can be subjective, so do not be discouraged if you are stuck on a (*) problem.

The solutions to these problems will be posted at the following web address: https://ubcengineers.ca/services/academic/tutoring/. If you believe that there is an error in these solutions, or have any questions, comments, or suggestions regarding EUS Tutoring sessions, please e-mail us at: tutoring@ubcengineers.ca. If you are interested in helping with EUS tutoring sessions in the future or other academic events run by the EUS, please e-mail vpacademic@ubcengineers.ca.

Some of the problems in this package were not created by the EUS. Those problems originated from one of the following sources:

- Schuam's Outline of Calculus 2 ed; Ayres Jr., Frank
- Calculus – Early Transcendentals 7 ed; Stewart, James
- Calculus – 3 ed; Spivak, Michael
- Calculus Volume 1 2 ed; Apostol, Tom

All solutions prepared by the EUS.

Good Luck!
1. Find the partial derivatives $f_x, f_y$ of the function $f$.

$$f(x, y) = [\sin(xy)]^{\cos^3}$$

2. Find all second partial derivatives of $z$.

$$z = x \cos y - y \cos x$$

3. Find the first partial derivatives of the function $g(x, y) = \sin(x \sin y)$
4. Find the partial derivatives \( f_x, f_y \) of the function \( f \).

\[
f(x, y) = x^2 + y^2 \sin(xy)
\]

5. Find the partial derivatives \( f_x, f_y \) of the function \( f \).

\[
f(x, y) = \tan(x^2/y)
\]

6. Let \( v(r, t) = t^n e^{-r^2/(4t)} \). Find the value of the constant \( n \) such that \( v \) satisfies the following equation:

\[
\frac{\partial v}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial v}{\partial r} \right)
\]
7. If \( z = e^{x/y} \sin \frac{z}{y} + e^{y/x} \cos \frac{z}{x} \), show that \( z \) satisfies the following partial differential equation:

\[
x \frac{\partial z}{\partial y} + y \frac{\partial z}{\partial x} = 0
\]

8. (a) Prove that the vectors \( \mathbf{A} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k} \), \( \mathbf{B} = -\mathbf{i} + 3\mathbf{j} + 4\mathbf{k} \), and \( \mathbf{C} = 4\mathbf{i} - 2\mathbf{j} - 6\mathbf{k} \) can form the sides of a triangle.

(b) Find the lengths of the sides of the triangle.

(c) Find the equation of the plane that contains the triangle.

(d) Find a normal vector to the triangle.
9. Find the angles which the vector \( \mathbf{A} = 3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k} \) makes with each of the coordinate axes.

10. Find the projection of the vector \( \mathbf{A} = \mathbf{i} - 2\mathbf{j} + \mathbf{k} \) in the direction of \( \mathbf{B} = 4\mathbf{i} - 4\mathbf{j} + 7\mathbf{k} \).

11. (a) Find the work done in moving an object along a vector \( \mathbf{r} = 3\mathbf{i} + 2\mathbf{j} - 5\mathbf{k} \) if the applied force is \( \mathbf{F} = 2\mathbf{i} - \mathbf{j} - \mathbf{k} \).
   
   (b) Find the angle between the applied force and the displacement.
12. (**)

(a) Find an equation for the plane perpendicular to the vector $\mathbf{A} = 2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$ and passing through the terminal point of the vector $\mathbf{B} = \mathbf{i} + 5\mathbf{j} + 3\mathbf{k}$

(b) Find the distance from the origin to the plane.

13. (**)

Find the minimum distance between the point $(3,2,6)$ and the line $\mathbf{r}(t) = (3t - 2)\mathbf{i} + t\mathbf{j} - (2t + 5)\mathbf{k}$
14. Find the minimum distance between the point (9,0,-2) and the plane \( z = 3x - 2y + 6 \)

15. Find the point of intersection between the line from question 13 and the plane from question 14.
16. Find the equation of the tangent plane to the surface \( f(x, y) = x^4y - 3y^2x + 4y \) at the point \((-1, 2)\).

17. For \( z = x^2 + 3xy + y^2 \), find all second partial derivatives of \( z \).

18. Find all first partial derivatives of the function \( f(x, y) = y^4 \log(x^3 + \sqrt{y}) \).
19. (** Find all first partial derivatives of \( g \).

\[
g(x, y) = \log(y - x) \cdot \int_{x^2}^{y} e^{t^2 - t} \, dt
\]

20. (*) Find all first partial derivatives of \( f \).

\[
f(x, y, z) = \sqrt{\sin(x^2 y) + z^4}
\]
21. Find the equation of the plane tangent to the surface \( h(x, y) = \sqrt{x^4 y^3} - 7x - y \) at the point (2, 1, 1).

22. Find an approximate value for \( g(x, y, z) = \sqrt{xyz} \) at the point (2.05, 1.97, 4.03).
23. Find an approximate value for \( M(2.94, 8.03) \) if \( M(x, y) = \log |x^2 - y| \)

24. Match the following functions with their contour maps.

(i) \( f(x, y) = \log |x \sin y| \)
(ii) \( f(x, y) = x^2 + 5y \)
(iii) \( f(x, y) = \sqrt[3]{y + |x|} \)
(iv) \( f(x, y) = x^2 + xy - y^2 \)
(v) \( f(x, y) = \cos x \sin(y/2) \)
(vi) \( f(x, y) = (2y + 3)(x - 2) \)
(vii) \( f(x, y) = x^2 - xy + y^2 \)
(viii) \( f(x, y) = \sin x + \sin y \)