Mathematics 100 Quiz 1 Review Package –

UBC Engineering Undergraduate Society

Attempt questions to the best of your ability. This review package consists of 20 pages, including 1 cover page and 37 questions. The questions are meant to be the level of a real examination or slightly above, in order to prepare you for the real exam. Material from lectures and from the relevant textbook sections is examinable, and the problems for this package were chosen with that in mind, as well as considerations based on past examination question difficulty and style. Problems are ranked in difficulty as (*) for easy, (**) for medium, and (****) for difficult. Note that sometimes difficulty can be subjective, so do not be discouraged if you are stuck on a (*) problem.

Solutions posted at: [http://ubcengineers.ca/services/academic/tutoring/](http://ubcengineers.ca/services/academic/tutoring/)

If you believe that there is an error in these solutions, or have any questions, comments, or suggestions regarding EUS Tutoring sessions, please e-mail us at: tutoring@ubcengineers.ca. If you are interested in helping with EUS tutoring sessions in the future or other academic events run by the EUS, please e-mail vpacademic@ubcengineers.ca.

The first 13 problems are review of high school material and are highly optional. They cover the basics of the different functions covered in high school.

Some of the problems in this package were not created by the EUS. Those problems originated from one of the following sources:

- Schuam’s Outline of Calculus 2 ed; Ayres Jr., Frank
- Calculus – Early Transcendentals 7 ed; Stewart, James
- Calculus – 3 ed; Spivak, Michael
- Calculus Volume 1 2 ed; Apostol, Tom

All solutions prepared by the EUS.

Good Luck!
Note: For this course, memorizing the standard triangles will be useful. (e.g. Trigonometric outputs for angles in multiples of \( \frac{\pi}{3} \) and \( \frac{\pi}{4} \).)

(*) 1. Evaluate \( \tan \left( \frac{7\pi}{4} \right) \).

(*) 2. Evaluate \( \sec \left( \frac{3\pi}{4} \right) \).

(*) 3. Evaluate \( \cos \left( -\frac{5\pi}{3} \right) \).
4. Evaluate \( \log_2(8192) \).

5. Solve the inequality in terms of intervals of the real line: \( x^2 < 2x + 8 \).
6. Solve the inequality in terms of intervals of the real line: \(|5x - 2| < 6\)

7. Solve the equation for \(x\):
\[
\left| \frac{2x - 1}{x + 1} \right| = 3
\]
8. Fully factor the following polynomial expression: $6x^3 - 14x^2 + 3x - 7$.

9. Fully factor the following polynomial expression: $x^3 + x^2 - 14x - 24$. 
10. Find all real values of $x$ that satisfy the following equation: $2 + \cos(2x) = 3 \cos(x)$.

11. Solve the following equation for $x$:

$$\log_5(x + 3) + \log_5(x - 1) = 1$$
12. Solve the following equation for $x$:

$$16^x = 2^{x+5} - 2^{x+4}$$

13. The equation $2^{x+3} \cdot 3^{2-x} = e^x$ has two real solutions. Find the two solutions.
(⋆) 14. Evaluate the limit. \( \lim_{{x \to 4}} \frac{x - 4}{x^2 - x - 12} \)

(⋆) 15. Evaluate the limit. \( \lim_{{x \to 3}} \frac{x^3 - 27}{x^2 - 9} \)
(**) 16. Evaluate the limit. \( \lim_{x \to 2} \frac{4 - x^2}{3 - \sqrt{x^2 + 5}} \)

(**) 17. Evaluate the limit. \( \lim_{x \to 0} \frac{1}{3 + 2^{1/x}} \)
(∗) 18. Evaluate the limit. \( \lim_{{x \to -1}} \frac{{x^2 + 3x + 2}}{{x^2 + 4x + 3}} \)

(∗) 19. Evaluate the limit. \( \lim_{{x \to 0}} \frac{{3^x - 3^{-x}}}{{3^x + 3^{-x}}} \)
(**) 20. Evaluate the limit. \[ \lim_{x \to 1} \frac{x - 1}{\sqrt{x^2 + 3} - 2} \]

(++) 21. Evaluate the limit. \[ \lim_{x \to 0} \frac{1}{\arctan \left( \frac{1}{2x^3 - 1} \right)} \]
(**) 22. Evaluate the limit \( \lim_{x \to \pi} \frac{\sin x + 2 \tan x}{\sin x} \)

(**) 23. Compute the limit \( \lim_{x \to \infty} x \left( \sqrt{x^2 + 2} - \sqrt{x} \right) \)
(**) 24. Evaluate the limit: \( \lim_{x \to 1} \frac{\sqrt{x} - 1}{x - 1} \)

(*) 25. Compute the limit \( \lim_{z \to \infty} \frac{2z + 7}{3z^2 - 3z + 1} \)
(∗) 26. Compute the limit \( \lim_{y \to \infty} \frac{3y^2 + 2y - 7}{5y^2 + 1} \)

(∗∗) 27. Determine the value of the limit. \( \lim_{x \to \infty} \frac{x + \sin^3 x}{5x + 6} \)
28. Evaluate the limit. \( \lim_{x \to \infty} \left( x^2 + x - x \right) \)

29. Evaluate the limit. \( \lim_{x \to \infty} \frac{x^2(1 + \sin^2 x)}{(x + \sin x)^2} \)
(**) 30. Suppose that
\[ \lim_{x \to a} (f(x) + g(x)) = 2 \]
\[ \lim_{x \to a} (f(x) - g(x)) = 1 \]
find the value of \( \lim_{x \to a} f(x) \cdot g(x) \)

(***) 31. Find a function \( f \) which is discontinuous at 0, ±1, ± \( \frac{1}{2} \), ± \( \frac{1}{3} \), ± \( \frac{1}{4} \) ... but continuous at every other real number.
(**) 32. Find all asymptotes of \( A(x) = \frac{|x - 4|}{x^2 + 7} \)

(**) 33. Find all asymptotes of \( f(x) = \frac{|x|(x + 7)}{x^2 - 4} \)
(**) 34. Find all asymptotes of \( g(x) = \frac{x\sqrt{x^2 + 9}}{x^2 + 3} \)

(**) 35. Find all values of \( a \) such that \( f \) is continuous for all \( \mathbb{R} \):

\[
f(x) = \begin{cases} 
  x + 1 & x \leq a \\
  x^2 & x > a 
\end{cases}
\]
(*** 36. Find one pair of values $a, b$ such that $f(x)$ is continuous at $x = 0$.

$$f(x) = \begin{cases} 
(e^x - 1) \cos \left( \frac{1}{x} \right) & , x < 0 \\
\sin(x + a) + b & , x \geq 0
\end{cases}$$
37. For the function $f(x)$, determine the value of $a$ for which $f$ is continuous at $x = 0$.

$$f(x) = \begin{cases} \frac{\sin^3(x)}{x} & x > 0 \\ a \cos x - 1 & x \leq 0 \end{cases}$$

**Hint 1.** You may use the fact that $\lim_{x \to 0} \frac{\sin(x)}{x} = 1$. 