Mathematics 100 Midterm 2 Review Package – Questions

UBC Engineering Undergraduate Society

Problems are ranked in difficulty as (*) for easy, (**) for medium, and (***) for difficult. Note that sometimes difficulty can be subjective, so do not be discouraged if you are stuck on a (*) problem.

Solutions posted at: http://ubcengineers.ca/tutoring/ If you believe that there is an error in these solutions, or have any questions, comments, or suggestions regarding EUS Tutoring sessions, please e-mail us at: tutoring@ubcengineers.ca. If you are interested in helping with EUS tutoring sessions in the future or other academic events run by the EUS, please e-mail vpacademic@ubcengineers.ca.

The first 7 problems are review of high school material and are highly optional. They cover the basics of the different functions covered in high school.

Some of the problems in this package were not created by the EUS. Those problems originated from one of the following sources:

• Schuam’s Outline of Calculus 2 ed; Ayres Jr., Frank
• Calculus – Early Transcendentals 7 ed; Stewart, James
• Calculus – 3 ed; Spivak, Michael
• Calculus Volume 1 2 ed; Apostol, Tom

Want a warm up? These are the easier problems
1, 3, 15, 23

Short on study time? These cover most of the material
5, 7, 10, 14, 16, 20, 24

Want a challenge? These are some tougher questions
15, 18, 21, 25

EUS Health and Wellness Study Tips

• Eat Healthy—Your body needs fuel to get through all of your long hours studying. You should eat a variety of food (not just a variety of ramen) and get all of your food groups in.

• Take Breaks—Your brain needs a chance to rest: take a fifteen minute study break every couple of hours. Staring at the same physics problem until your eyes go numb won’t help you understand the material.

• Sleep—We have all been told we need 8 hours of sleep a night, university should not change this. Get to know how much sleep you need and set up a regular sleep schedule.

Good Luck!
1. Find the critical numbers of \( r(\theta) = 3\theta - \arcsin \theta \).

2. Find the minimum and maximum values of the function on the interval \([-2, 2]\).
   \[
   f(x) = x^3 - x^2 - 8x + 1
   \]

3. Find the minimum and maximum values of the function on the interval \([-1, 1]\).
   \[
   f(x) = x^5 + x + 1
   \]
4. Suppose that $g'(x) = \sqrt{x^2 + 5}$, and that $g(2) = -4$.
   
   (a) Estimate $g(1.95)$ and $g(2.05)$.
   
   (b) Is each approximation an overestimate or an underestimate?

5. Use a Maclaurin polynomial for $e^x$ to calculate $1/\sqrt[3]{e}$ correct to five decimal places. What degree of the polynomial is required?
6. (** Expand \( \frac{1}{\sqrt{1+x}} \) as a Taylor polynomial. Use this polynomial to estimate \( 1.1^{-1/4} \) correct to three decimal places.

\[ \]
7. Let \( f(x) = 1 - x^{2/3} \). Show that \( f(1) = f(-1) = 0 \), but that \( f'(x) \) is never zero in the interval \([-1, 1]\). Explain how this is possible in view of the Mean Value Theorem.

8. Show that \( x^2 = x \sin x + \cos x \) for exactly two real values of \( x \).
9. (**) Let \( g(x) = \log(x^2 + 1) - \sin(e^{-x^2}) \). Show that there exists a real number \( c \) such that \( g'(c) = 0 \).

10. (**) Let \( h(x) = \sin x - x^2 + \pi x \). Show that there exists a real number \( c \) such that \( h'(c) = 0 \).
11. Find the minimum and maximum values of the function on the interval \([-\frac{1}{2}, 1]\).

\[ f(x) = \frac{1}{x^5 + x + 1} \]

12. Find the global maximum value of \(f(x)\) using any method you can think of, you do not need to be rigorous.

\[ f(x) = \frac{1}{1 + |x|} + \frac{1}{1 + |x - a|} \]
(**) 13. Show that on the graph of any quadratic polynomial the chord joining the points for which $x = a$ and $x = b$ is parallel to the tangent line at the midpoint $x = (a + b)/2$. 
14. For the function \( f(x) = e^{-2x^2 + 4x + 3} \), find (if they exist)

(i) Critical points  
(ii) Maximum points  
(iii) Minimum points  
(iv) Intervals of increase  
(v) Intervals of decrease  
(vi) Points of inflection  
(vii) Intervals of concave upward  
(viii) Intervals of concave downward  
(ix) Sketch the function

15. A culture of bacteria in a petri dish grows exponentially from an initial population of 5000 cells. Let \( b(t) \) be the number of bacteria after \( t \) hours. Then \( b(0) = 5000 \). At \( t = 10 \) hours, scientists remove 4000 cells from the dish. At \( t = 20 \) hours, there are exactly 12000 cells in the dish. How many cells will be present in the dish at \( t = 30 \) hours?
16. A glass of water is left out for a long time at room temperature of 20°C. Someone puts it in the freezer which is held at a constant temperature of −5°C. At 2pm, it is 10°C, and at 3pm its just about to begin freezing. When was it put into the freezer?
17. A body moves along a horizontal line according to the law \( s = f(t) = t^3 - 9t^2 + 24t \).

(a) When is \( s \) increasing and when decreasing?

(b) When is \( v \) increasing and when decreasing?

(c) When is the speed of the body increasing and when decreasing?

(d) Compute the total distance travelled in the first 5 seconds of motion.
18. Two sides of a triangle are 15 and 20 m long, respectively.

(a) How fast is the third side increasing if the angle between the given sides is $\pi/3$ and is increasing at a rate of $\pi/45$ radians per second?

(b) How fast is the area increasing?
19. (***) Two ships sail from the origin at the same time. One sails south at 15 km/hr; the other sails east at 25 km/hr for 1 hour and then turns north. Find the rate of rotation in rad/s of the line joining them after 3 hours.
20. Gas is escaping from a spherical balloon at the rate of 2 cubic feet per minute. How fast is the surface area shrinking when the radius is 12 ft?
(*** 21. Water, at the rate of 10 ft$^3$/min, is pouring into a leaky cistern whose shape is a cone 16 feet deep and 8 inches diameter at the top. At the time the water is 12 feet deep, the water level is observed to be rising 4 inches/min. How fast is the water leaking away?)
22. Compute the differential of the function $y = \sqrt{3 + x^2}$.

23. Use a linear approximation to estimate the value of $29^{-1/3}$.
24. If the third degree Maclaurin polynomial for $g(x)$ is $g(x) \approx T_3,g(x) = 3 + 2x - 5x^2 + x^3/3$ (the subscript $g$ denotes it being the Maclaurin polynomial for $g(x)$), and $f(x) = (g(x))^3$, find the second degree Maclaurin polynomial for $f(x)$.
25. If $11 - 9x + 2x^2$ is the second degree Taylor polynomial to $h(x)$ at $x = 3$, and $f(x) = \frac{h(x)}{\sqrt{x + 1}}$, compute the second degree Taylor polynomial to $f(x)$ at $x = 3$. 