

Lecture 4

Arguments

Consider the following four arguments.

(Dog)

1. If Spot is a dog, then Spot is a mammal.
2. Spot is a dog.
3. Therefore, Spot is a mammal.

(Reptile)

1. If Spot is a dog, then Spot is a reptile.
2. Spot is a dog.
3. Therefore, Spot is a reptile.

(Mammal)

1. If Spot is a dog, then Spot is a mammal.
2. Spot is a mammal.
3. Therefore, Spot is a dog.

(Rock)

1. If Spot is a dog, then Spot is a reptile.
2. Spot is a rock.
3. Therefore, Spot is a fungus.

You might have some intuitions about their quality. The argument **Dog** looks good. The argument **Rock** looks really bad. The arguments **Mammal** and **Reptile** are both problematic, but they're not entirely bad like **Rock**, and maybe it's hard to say exactly what's bad and good about them.

To start saying something about our “good” and “bad” argument intuitions, it's helpful to distinguish between the *premises* and *conclusion* of an argument. The *conclusion* of an argument is always its last line—line 3 in all of the above example arguments—and is often indicated further by some word signaling that it follows from the preceding lines of the argument—like “therefore” or “thus”. It's important to note that if such a signaling word is present, it's not actually part of the conclusion. So, for example, the conclusion of the argument **Dog** is *not* “Therefore, Spot is a mammal.”; the simpler “Spot is a mammal.” is the conclusion. Roughly, the *premises* of an argument are the lines of the argument that come before the conclusion—lines 1 and 2 in all of our example arguments.

Now, we can make our talk of “good” and “bad” arguments more precise with the introduction of two formal concepts: *validity* and *soundness*. Let us consider validity first:

Validity. An argument is *valid* iff: If its premises are true, then its conclusion is true.

We can understand this definition clearly by looking at truth tables for our arguments above. Letting D = “Spot is a dog.” and M = “Spot is a mammal.”, a truth table of all of the lines of the argument **Dog** is

D	M	$D \rightarrow M$
1	1	1
1	0	0
0	1	1
0	0	1

A quick check will reveal that in every line where the premises are both true—those being $D \rightarrow M$ and D , which are only both true on line 1—the conclusion M is true. Thus, as the definition of validity goes, “if the premises are true, the conclusion is true”, is true, and so we can understand the goodness of the argument as corresponding (at least in part) to its validity.

In contrast, we can look at the argument **Mammal** (and continue with the same truth table while doing so). In **Mammal**, the premises are $D \rightarrow M$ and M , while the conclusion is D . In this case it’s *false* that “if the premises are true, the conclusion is true” because there is a line of the truth table—namely, line 3—where the premises are true but the conclusion is false. Mammal’s badness, then, stems from its being *invalid*.

The argument **Reptile**, on the other hand *is valid*. Letting R = “Spot is a reptile.”, and replacing our “therefore”s with a horizontal bar to indicate premises above and conclusion below, we can represent the two arguments in our logical notation as follows:

$$\frac{D \rightarrow M \quad D}{M} \qquad \frac{D \rightarrow R \quad D}{R}$$

The two arguments have the exact same logical structure, and so everything that we said about the truth table in defense of **Dog**’s validity should apply to a truth table for **Reptile** (the reader is encouraged to convince themselves that this is right).

So **Reptile** is good because it is valid. However, we noted that there’s something wrong with it. Specifically, the first premise “If Spot is a dog, then Spot is a reptile.” is false. At least, it’s false in our world. This highlights the second level of goodness for an argument:

Soundness. An argument is *sound* iff: The argument is valid and its premises are true.

Reptile is valid, but unsound. **Dog** (if it’s really true that Spot is a dog) is valid *and* sound. All the lines of **Mammal** are true (since they’re the same lines as **Dog**), which is why we think it seems like an okay argument. But **Mammal** is not valid and, since it’s not valid, it isn’t sound. As far as validity and soundness are concerned, **Mammal** is as bad as the gibberish in **Rock**.

In general, we’ll care a lot more about validity than soundness when doing logic. The latter, of course, is hugely important if, say, we are trying to make an argument for some policy position in government—the validity of your argument that concludes giving everyone

free ice cream would raise GDP 4% doesn't matter if all the premises are false. But, while doing logic on its own, it can be very hard to say what's *actually* true. The argument **Dog** is only sound if it really is true that "Spot is a dog." Is that really true? It's hard to say, since Spot is something I just made up. But, at least, we can say that the argument is valid.