Two coin flips are independent

One coin:
- 50% heads
- 50% tails

Two coins:
- 25% heads, 25% tails, 25% heads, 25% tails

Dirac Notation

We have four possible outcomes.

Recall, in Dirac notation, the probability of some outcome $a$ is $|a|^2$.

We can express this state as...

$\frac{1}{2}|\text{HH}\rangle + \frac{1}{2}|\text{HT}\rangle + \frac{1}{2}|\text{TH}\rangle + \frac{1}{2}|\text{TT}\rangle$

Same probability for each outcome.

2-Qubit Notation

If we measure two qubits, how many possible outcomes are there?

1st qubit 2nd qubit
- $|0\rangle$ $|0\rangle$
- $|0\rangle$ $|1\rangle$
- $|1\rangle$ $|0\rangle$
- $|1\rangle$ $|1\rangle$

As shorthand, we write:

$|1\rangle|0\rangle$ as $|10\rangle$

Combining Two Qubits

Two independent (not entangled) qubits:

$qubit 1: a|0\rangle + b|1\rangle$
$qubit 2: c|0\rangle + d|1\rangle$

The same two qubits, expressed in 2-qubit notation:

$ac|00\rangle + ad|01\rangle + bc|10\rangle + bd|11\rangle$

Try it yourself!

Put these qubits in 2-qubit notation:

$qubit x: \frac{1}{\sqrt{3}}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$
$qubit y: \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$

(Check your answer on the next page!)

Vector Notation

The 2-qubit state from the previous page can also be written as a vector!

$\frac{1}{2\sqrt{3}}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{\sqrt{6}}|10\rangle + \frac{1}{\sqrt{2}}|11\rangle$

Linear Algebra

Matrix multiplication is used to perform gate operations.

C-NOT

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

$\begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{2} \\ \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$

C-NOT operation

$\begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{2} \\ \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$

Try it yourself!

$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

$\begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{2} \\ \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$