Quantitative Investments

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1 June 2018
Recall

Last lecture we discussed fixed income.
- Bond features and innovations;
- Accrued interest;
- Pricing and yields;
- Interest rate sensitivity;
- Risks to bondholders; and,
- Repos.

Today we will talk about yield curves.
Yield Curves

Chapter 12, A Quantitative Primer on Investments with R
This part discusses yield curves. Specifically:

- Yield curves;
- Monetary policy; and,
- Interest rate dynamics.
Term Structure of Interest Rates
For many liquid goods, we can lock in prices at a later time:
- Power; jet; natgas; USD/HKD: trade futures or forwards.
- Interest rates: trade futures, bond, or enter a CD.

Later: get agreed price — or P&L compensates for difference.

Often look at curves: Price or yield vs later dates.

Sometimes call these curves the term structure.

If one date’s price (or yield) is far from nearby dates:
- Trade adjacent dates;
- Store goods/money for less than difference;
- PROFIT! (Maybe?)

This intertemporal smoothing is what market makers, HFT do.
We have assumed constant interest rates. Unrealistic.

Supply and demand affect prices at all maturities.

Thus different maturities have different yields.

A yield curve plots yields versus times to maturity.

- Usually use this terminology for government bond yields.
- However, very related is the LIBOR-based swap curve.
- Also look at credit spreads: yield increase for risky debt.
Yield Curve versus Zero Curve

- Suppose we had zero-coupon bond yields (aka *spot rates*).
- Curve of discount rates often called a *zero curve*.
- Should think about bonds used to make yield curve.
  - On-the-runs (recently-issued) are most liquid; most accurate?
- The yield curve relates to the zero curve like so:

<table>
<thead>
<tr>
<th>Maturity (years)</th>
<th>YTM (%)</th>
<th>Price</th>
<th>Discount Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>3M</td>
<td>0.5</td>
<td>$998.75</td>
<td>0.9988</td>
</tr>
<tr>
<td>6M</td>
<td>1.1</td>
<td>994.53</td>
<td>0.9945</td>
</tr>
<tr>
<td>1</td>
<td>3.0</td>
<td>970.87</td>
<td>0.9709</td>
</tr>
<tr>
<td>2</td>
<td>3.5</td>
<td>966.18</td>
<td>0.9662</td>
</tr>
<tr>
<td>3</td>
<td>4.0</td>
<td>961.54</td>
<td>0.9615</td>
</tr>
<tr>
<td>4</td>
<td>5.0</td>
<td>952.38</td>
<td>0.9524</td>
</tr>
</tbody>
</table>

- If 6M yield calculated as EAR, price = $994.54.
How to Plot a Yield Curve

Plotting a yield curve is not done like other curves.

1. Plot yields of non-indexed govt bonds: most liquid, most tenors.²
2. Consistently plot the same tenors so you can compare across time.
3. Abbreviate tenors: O/N (overnight), 3M (3-month), 10Y (10-year).
4. Plot yields vs time: time is ordered but NOT to scale.

²In bonds, “tenors” refers to maturities and not opera stars.
Some yield curve shapes are associated with macroeconomy.

*Inverted* curve shows investors value long-term bonds highly.
  - Investors may be anticipating an economic slow-down.

*Humped* curve is typical of a currency defense.
  - Government props up mid-term rates to entice investors.

(Gently) *upward-sloping* curve often implies stable economy.

High *front end* of the curve slows short-term growth.
• AU, US: (Gently) upward-sloping curves; stable economic growth.
• AU: Elevated front end of curve; govt trying to cool rapid growth.
What Yield Curves Tell Us: ... The Bad, and the Ugly

- RU: Inverted curve; investors fear slow growth/contraction.
- GR: Humped curve; currency defense, begging for money to stay.
Types of Interest Rates

- Zero-coupon bond yields imply price of money at one time; clean.
- We call such yields *spot rates*, e.g. 6M spot rate $r_{z,6M}$.
- Interest rates are uncertain; helpful to lock-in later rates.
- *Forward rates*: interest rates starting in the future.
  - Future rates can lock in now, e.g. 2Y bond in 1 year $f_{1\rightarrow3}$.
  - Often implied by differences in other bond yields.
- *Short rates*: rates of shorter-term bonds starting now.
  - 6M spot rate above is a short rate.
If rates certain, all bonds to same horizon have same return.

This implies particular certain forward rates.

Thus compounded rate from a 3-year bond should equal:
- 1-year bond reinvested in 2-year bond 1 year forward;
- 2-year bond reinvested in 1-year bond 2 years forward;
- 1-year bonds reinvested in 1-year bonds 1&2 years forward.

In our example zero/yield curve, rates seem to be increasing:
- $2Y\; \text{YTM} = 1Y\; \text{YTM} \times 1Y\; 1-yr-fwd\; \text{YTM}$
  \[1.035^2 = 1.03(1 + r_{1,1}) \implies f_{1\rightarrow 2} = 4.00\%\]
- Quick-n-dirty approximation:
  \[3.5\% \times 2 = 3\% + f_{1\rightarrow 2} \implies f_{1\rightarrow 2} = 4\%.\]
How to Determine the Yield Curve

- We want spot rates for yield curves.
- Coupon bond yields mix in prices of money at multiple times.
- How to find pure yield curve: bootstrap the curve.
- Use spot rates we know to infer those we do not.
- Typically work from front end of curve toward back.
- Example: 1.25% 1.5Y bond yields 1% $\implies P_{1.5Y} = 100.3713$.

\[
100.3713 = \frac{1.25/2}{1 + y_{z,6M}/2} + \frac{1.25/2}{(1 + y_{z,1Y})^2} + \frac{100 + 1.25/2}{(1 + y_{z,1.5Y}/2)^3}
\]  
\[
y_{z,6M} = 0.5\%, \quad y_{z,1Y} = 0.75\% \quad \implies y_{z,1.5Y} = 1.021\%.
\]  

- Then we would find 2Y, etc.
About Forward Rates

- In our example, 1-year rate 1 year forward: \( f_{1 \rightarrow 2} = 4\% \)
- 3Y, 4 yrs fwd: rate for what will be 3Y bond in 4 years, \( f_{4 \rightarrow 7} \).
- If rates were certain, forward rates might be future interest rates.
- With uncertainty, forward rates often above \( E(\text{interest rates}) \).
- Must entice people to commit money in an unknown future state.
- This gets us to theories for the term structure...
Monetary Policy and Yield Curve Dynamics
Term Structure Theories

- A few major theories are used to explain yield curve shapes.
  - *Expectations hypothesis*: Forwards predict later short rates.
  - *Liquidity preference*: must pay people to lock up cash.
    - Short-term lenders value liquidity, dominate market; so,
    - Forward rates above later rates: $f_2 > E(r_2)$.
  - *Market segmentation*: independent investors w/differing horizons.
  - *Preferred habitat*: investors diverge from horizon... for a price.
  - *Interest rate volatility* also seems to affect curve shape.
  - Theories form scale of lender willingness to alter horizon.
  - Also know central banks target yields to implement monetary policy.
Why do we let central banks control monetary policy?

So unemployment and/or inflation do not get out of hand.

Thus the Fed’s dual mandate:
- Maintain price stability (keep inflation low); and,
- Maximize employment.

Fed watchers literally plot these on a bulls-eye chart.

Post-2008: Also tasked w/mitigating financial instability.
- Is preventing bank panics/market crashes a third mandate? Seems so.
How central bankers change rates: *open market operations*.

In other words: they trade bonds to move yields.

Open market operations allow central banks to act quickly.

Most central banks target front end of the curve: overnight rate.

Since 2008: Fed, others have used *non-standard monetary policy*.

- Unleashed open market operations on other parts of yield curve.
- Have also discussed targeting risky bonds/repo markets.
Putting ideas together gives factors which drive yield curve:
- expectations of short rates;
- government targeting of rates;
- mandated investment horizons;
- equilibrium forward rates; and,
- liquidity premia.

Inflation also matters; but, its effect may not be clear.

Curves in the text show some of these ideas.
Really. Never-before seen. (Well, not in this way.)

Curve; S&P 500; curve slope; TED; VIX.
Yield Curve: The Movie!

- Yield curve goes red when inverted.
- TED spread (credit), VIX (equity vol.) indicate crisis.
- Declining yield curve slope often presages SPX declines.
- Litterman-Scheinkman: common movements = shift, twist, bow.

https://www.youtube.com/watch?v=6z8g9xaj3g8
Yield curve models help us assess over-/under-valuation of bonds.

Lots of models; could take a course on just yield curve modeling.

General ideas: yields tend to revert to targeted levels (front end).

- Vašíček model has Ornstein-Uhlenbeck short rate:

\[ dr_t = \lambda (\bar{r} - r_t) dt + \sigma dW_t. \]  

Nelson-Siegel: Short-/medium-/long-term effects.

- Like slope/hump/level; Svensson adds a second hump.

May assume some bonds more liquid; their prices more reliable.

Many models use Litterman-Schenkman factors: level, slope, curve.
Example: Yield Curve Arbitrage

Suppose the following set of UST yields:

- 1-year: 2.0%
- 2-year: 2.1%
- 3-year: 2.3%.

1-year 1 year forward: 2.2%
1-year 2 years forward: 2.2%

Remember: Draw the paths w/yields as slopes.

Does everything look correct?

Can make money: Buy 3Y; pay by selling 2Y & 1Y 2yrs fwd.

What are Sharpe, Sortino ratios of this investment?

How will this affect prices of these bonds?

What if we instead did this trade using GM bonds?
The Road Ahead

We have covered yield curves; on to equity valuation next time!

- Valuation: Equity Valuation;
- Valuation II: Factor Models, Microfoundations, Global Investing, FX;
- Risk Alleviation: Futures, Options, Credit, Structured Products; and,
- All Together Now: Active Portfolios, Investment Firms, Crises.