Recall

Last lecture we discussed the CAPM.
- Single-index model;
- Markowitz curse of dimensionality;
- Equilibrium and capital asset pricing model (CAPM);
- Problems with the CAPM;
- Improving the CAPM;
- Hedging; and,
- The Zero Beta Portfolio.

Today we will talk about the APT and factor models.
Factor Models/
The Arbitrage Pricing Theory

Chapter 15, *A Quantitative Primer on Investments with R*
This part discusses factor models and the arbitrage pricing theory. Specifically:

- Arbitrage pricing theory;
- Trading mispricings;
- Macro risk factors;
- Micro risk factors; and,
- Style factors.
Conditional CAPM, ICAPM show us: one risk factor is not enough.

When we use multiple factors, we get models like:

\[ R_i = \alpha_i + \beta_{i,GDP}(\Delta GDP - r_f) + \beta_{i,rf} \Delta r_f + \eta_i. \]  

Call \( \beta_i \)'s factor sensitivities/loadings... or just betas.

We are now more likely to see negative \( \beta \)'s:

- Maybe instrument is really a hedge for that factor;
- Maybe the model considers a spread between factors; or,
- The risk factors may by multicollinear.
Arbitrage Pricing Theory (APT)

• Arbitrage pricing theory (APT) developed by Ross (1976).

• Three assumptions of APT:
  1. Returns may be described by a factor model;
     \[ r_i = \nu_i + \beta_{i1}f_1 + \ldots + \beta_{ik}f_k + \epsilon_i. \]  
     \( (2) \)
  2. Can diversify away idiosyncratic variance (\( \epsilon_i \) effect); and,

• If these hold, APT makes a bold assertion:
     \[ R_i \approx \beta_{i1}F_1 + \ldots + \beta_{ik}F_k, \quad F_j = f_j - r_f. \]  
     \( (3) \)

• Arbitrage: riskless profits not requiring capital.

• Often trade factor tracking portfolios, e.g. \( T_1 \sim F_1 \).
So... We’re All Done Here?

- APT seems to say: find a factor model and risk goes away.
- Ah, but what is it they say about beautiful theories?²
- Lehmann and Modest: APT does not hold without noise.
  - So that approximation and dropping the error term? Never mind.
- We still need risk management... and risk managers.
- However, the APT offers a very useful perspective.
- Asset management conferences: many talks on risk factors.

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² Much akin to Rushdie’s lament: “To sing (while my dreams are being murdered by facts) / praises of butterflies broken on racks.”
Arbitrage vs. Risk-Return Dominance

What happens when we are in disequilibrium?

- **Dominance argument**: many change holdings slightly.
  - Changes due to differing risk aversion (CAPM).
- **Arbitrage argument**: few take large positions; smash prices together.
  - This implies any disequilibrium closes quickly (APT).

APT-based arguments stronger than CAPM-based arguments.

- **Law of One Price (LOOP)**: economically same = priced same.

What is not arbitrage? Stat arb, risk arb, ... most “arb.”

- We will use **well-diversified portfolios**: $\eta_P \approx 0$.

$$ r_P \approx E(r_P) + \beta_{P,F} F. $$ (4)
A lot of theory for factor models and their validity. Briefly;

- Absence of arbitrage $\iff$ can price all sorts of options.
- Factor model return Sharpe ratio $\leq$ optimal risky portfolio $S_P$.
- Factor model explain less than $R^2$ of true model variation.
- What if a factor model breaks one of these rules?
  - Then we know it is not an accurate model of risk factors.
Rules for Arbitrage

- If two instruments should be equally priced:
  - Cheaper/higher-yielding = cheap; expensive/lower-yielding = rich.
- General rule for arbitrage situations.

  Buy the cheap; sell the rich.\(^3\)

- Amounts must cancel risk factors; use T-bills to balance money.
- This captures difference between their prices/returns.
- Mispricings may be absolute or relative.

\(^3\)This is very punk rock, I know.
Example Relative Mispricing

Suppose we have two well-diversified portfolios $A, B$.

\[ E(r_A) = 0.1 + 1.2F \quad (F \text{ is a risk factor}); \quad \text{and,} \]
\[ E(r_B) = 0.09 + 1.2F \quad \text{where } E(F) = 0. \]

APT/LOOP says these should be priced the same.
- Thus $E(r_A)$ should equal $E(r_B)$... but it does not.
- Thus we have a relative mispricing of $A$ versus $B$.
- Buy cheap ($A$), sell rich ($B$) to cancel out risk.
- Total return on $1$ is then $1(r_A - r_B) = 0.01$.  

E(d_{36})
Example Absolute Mispricing

- Assume \( r_f = 0.02 \); factors \( E(f_1) = 0.05 \), \( E(f_2) = 0.08 \):

\[
r_A = r_f + 1F_1 + 0.5F_2. \tag{7}
\]

- Arb-free \( E(r_A)^* = 0.02 + 1(0.05 - 0.02) + 0.5(0.08 - 0.02) = 0.08 \).

- If observed \( E(r_A) = 0.10 \), \( A \) return > expected: absolute mispricing.

- \( A \) yielding more than expected = cheap.

- Arb portfolio \( G \): Buy \$1 \( A \), \$0.5 T-bills; short \$1 \( F_1 \), \$0.5 \( F_2 \).
Example Absolute Mispricing: The Bookkeeping

- Arb portfolio $G$: Buy $1$ A, $0.5$ T-bills; short $1$ $F_1$, $0.5$ $F_2$.
- Return on $G$ is expectation plus surprises (differences from expected):

  \[ r_G = E(r_A) + 1(f_1 - E(f_1)) + 0.5(f_2 - E(f_2)) \]
  \[ - 1T_1 - 0.5T_2 + 0.5r_f \]
  \[ = E(r_A) - E(f_1) - 0.5E(f_2) + 0.5r_f \]
  \[ = 0.10 - 0.05 - 0.5 \cdot 0.08 + 0.01 = 0.02. \]

- N.B. captured $|arb-free-observed|$ ($|0.08 - 0.10|$).
Example Multi-factor Relative Mispricing

- Suppose the prior setup but we also look at portfolio $B$:

$$r_A = r_f + 1F_1 + 0.5F_2,$$

$$r_B = r_f + 0.8F_1 + 2F_2.$$  \hspace{1cm} (11) \hspace{1cm} (12)

- Arb-free $E(r_B) = 0.02 + 0.8(0.05 - 0.02) + 2(0.08 - 0.02) = 0.164$.
- If observed $E(r_B) = 0.14 < \text{expected}: A$ is cheap, $B$ is rich.
- Arb portfolio $G$: buy $1 A, 1.5 T_2$; short $1 B, 0.2 T_1, 1.3 r_f$. 
Example Multi-factor Relative Mispricing: Bookkeeping

- Arb portfolio $G$: buy $1 \; A$, $1.5 \; T_2$; short $1 \; B$, $0.2 \; T_1$, $1.3 \; r_f$.

$$r_G = E(r_A) + 1(f_1 - E(f_1)) + 0.5(f_2 - E(f_2))$$
$$- E(r_B) - 0.8(f_1 - E(f_1)) - 2(f_2 - E(f_2))$$
$$- 0.2 \; T_1 + 1.5 \; T_2 - 1.3r_f$$

$$= E(r_A) - E(r_B) + 0.2E(f_1) - 1.5E(f_2) - 1.3r_f$$

$$= 0.10 - 0.14 - 0.02 \cdot 0.05 + 1.5 \cdot 0.08 - 0.026 = 0.044.$$  \hspace{1cm} (13)

- N.B. captured mispricing in $A$ (0.02) and $B$ (0.024).
Risk Factors
The one issue we have yet to address: what are useful risk factors?

This gets tricky: factors need to have long-run importance.

In general, we can break factors into three groups:
- Macroeconomic factors;
- Microeconomic factors; and,
- Style factors (which maybe are not real factors).

Many academics made an industry of finding factors.
- That should raise skepticism; many just want to publish.
- \( \text{\textit{Vorsicht!}} \) This is why factor research can be “cookbooky.”
Macro Factors

- Already saw a fixed-income macro model (Litterman-Scheinkman):

\[ r_{jt} = \alpha_j + \beta_{j,lvl} \Delta \text{Level}_t + \beta_{j,slope} \Delta \text{Slope}_t + \beta_{j,curv} \Delta \text{Curv}_t + \eta_{jt}. \] (16)

- Five-factor macro model (Chen-Roll-Ross):

\[ r_{jt} = \alpha_j + \beta_{j,IP} \Delta \% \text{IP}_t + \beta_{j,E} \Delta E_i(t) + \beta_{j,\epsilon} \epsilon_i(t) + \beta_{j,CMG} \Delta \text{CMG}_t + \beta_{j,LMS} \Delta \text{LMS}_t + \eta_{jt}. \] (17)

- \( \epsilon = \) surprise in inflation vs expected; \( i_t - E(i_t) \),
- \( \text{CMG} = \) corporate – govt bond credit spread,
- \( \text{LMS} = \) long–short-term govt yields \( \approx \) YC slope.
Macro Factors: Hedge Fund

- There are three macro factor models often used for hedge funds.
- Hasanhodzic-Lo six-factor model:
  - excess return on market, AA corporate bond index;
  - return on currency, commodity indices; and,
  - corp–govt bond credit spread, implied volatility index (e.g. VIX).
- Fung-Hsieh seven-factor model:
  - excess return on market; small-minus-large-cap return;
  - change in UST 10Y yield; corp–govt bond credit spread; and,
  - trend factors for bonds, FX, and commodities.
- Fung-Hsieh eight-factor model:
  - excess return on US, non-US, emerging market equity indices;
  - return on US, non-US govt bonds and 1M Eurodollars; and,
  - return on gold and trade-weighted USD index.

Commodity index returns often related to unexpected inflation.
Micro Factors: Dynamic

- Micro factors relate to firm-specific behavior.
- Factors are often dynamic or liquidity-based.
- **Engle-Lilien-Robins ARCH-in-mean:**
  - Captures spikes in market volatility w/time series model;
  - Current volatility affects returns: clean risk-return tradeoff.
- **Bollerslev-Engle-Wooldridge GARCH-in-mean:**
  - Like ARCH-in-mean model but allows for cross-correlations.
  - Stock market volatility can affect bond market returns.
Micro Factors: Liquidity, Firm Risk

- Micro liquidity models try to capture effect of market liquidity.
- Liquidity is a very powerful factor; driving force of many crises.
- Amihud-Mendelson model just includes %bid-ask spread surprise:
  \[
  r_{jt} = \alpha_j + \beta_{M,j} R_{Mt} + \beta_{S,j} \left( S_{jt} - E(S_j) \right) + \eta_{jt}. 
  \]

\[\text{(18)}\]

- Acharya-Pedersen Liquidity CAPM relates spread-excess returns:
  \[
  R_{jt} - S_{jt} = \beta_{all}(R_{Mt} - S_{Mt}).
  \]
  - \(\beta_{all}\): components from market, stock returns, spreads.
- Pastor-Stambaugh looks at stock-vs-market outperformance:
  - Looks at Fama-French factors, surprise in dollar volume effect.\(^4\)
- Could also look at firm-specific risk factors:
  - Idiosyncratic volatility, assets vs liabilities, cashflows in vs out.

\[\text{\(Q_{36}\)}\]
Style Factors

- Style factors relate to firm aspect/cross-sectional differences.
- Fama-French three-factor model uses small−big, high−low B/P:

\[ r_{it} = \alpha_i + \beta_{i,M} R_{Mt} + \beta_{i,SMB} SMB_t + \beta_{i,HML} HML_t + \eta_{it}, \quad (19) \]

- Carhart model adds “momentum” winners−losers (WMLt) factor.
- Fama-French five-factor model adds to three factors:
  - \( RMW_t \): robust−weak profitability, and
  - \( CMA_t \): conservative−aggressive portfolios.

\textsuperscript{5} Related: The Morningstar style box.
Are these really risk factors? The right factors?

Research suggests: No. Not risk, not the driving forces.

Palacios-Huerta: adding human capital is better than FF3.

Chan-Chen-Hsieh: SMB related to credit spreads.
  - May also add small-stock effects = more complete market.

Das-King-Sinha: HML only active around earnings announcements!

Sadka: Momentum related to post-earnings announcement drift?

Sinha: News sentiment subsumes momentum.

Wang: initial jobless claims (IJC) surprises explain momentum.

\[ SMB = \text{credit/small-caps}, \quad HML = \text{earnings risk}, \quad WML = \text{news/macro}\]
Modeling Issues

- Factor models are useful; but, modeling questions remain.
- Should we look at levels of variables or changes?
  - Levels if level relates to risk or prices take time to adjust.
  - Changes/surprise if markets efficient, respond to info.
- Excess returns, returns, or surprises?
  - Excess returns for traded instruments w/risk premium;
  - Returns for stationary/mean-zero factors; and,
  - Surprises for macro, strongly-recurring factors.
- Finally, what do we want from factor models?
Coda: Real Risk Factors

- The proliferation of claimed risk factors raises skepticism.
- Harvey-Liu-Zhu note this is like testing multiple hypotheses.\(^6\)
- Must hold models to a stricter standard when many tests.
- They find most-likely factors: market, HML, momentum, and...
  - ...durables consumption, short-term volatility, liquidity.
- Factor actively traded? Also suggests it is a risk factor.

\(^6\)If you roll 1000 dice pairs, are the double-sixes really lucky dice?
The Failure of Arbitrage Pricing

- Huberman-Stanzl: Markets do not allow quasi-arbitrage.
  - Cannot build a sequence of trades that generates money.
  - Cost of exiting APT trades = cost of entry; makes no money.
- “Siamese Twin” shares: strong evidence that APT is not exact.
- Model risk: what if your factor model is wrong?
- Fundamental risk: no idea when you make money, so you sit out.
- Credit, trust: venues check credit; counterparties must trust you.
- Market segmentation: different horizons? different risk pricing.
We covered the APT and factor models; on to Microfoundations next!

- Valuation II: Microfoundations, Global Investing, FX;
- Risk Alleviation: Futures, Options, Credit, Structured Products; and,
- All Together Now: Active Portfolios, Investment Firms, Crises.