Quantitative Investments

Dale W.R. Rosenthal

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Recall

Last lecture we discussed forwards, futures, and swaps.

- Cash/Spot;
- Forwards;
- Futures and the effects;
- Swaps; and,
- Futures curves.

Today we will talk about options basics.
Options Basics

Chapter 20, A Quantitative Primer on Investments with R
This week we will discuss options basics.

In particular, we will discuss:

- Option Markets;
- Option Basics and Payoffs;
- Put-Call Parity;
- Embedded and Exotic Options;
Options

- An option gives holder right to buy/sell underlying asset.
  - In general, right is to buy/sell at strike price.
  - Does not oblige holder to take action, however.
  - Holder exercises right if worthwhile.

- Options reference price, yield, or other underlier “price.”
  - Comparing this price to the strike determines option value.

- The price to buy such an option is called the premium.
Puts, Calls, and Moneyness

- **Seller** (*writer*) of an option cannot choose to exercise.
  - Must provide benefit if exercised; paid *premium* in exchange.

- **Call** options allow buying underlier at strike price.

- **Put** options allow selling underlier at strike price.

- **Moneyness** refers to the value of immediate exercise:

<table>
<thead>
<tr>
<th>Type</th>
<th>$S_t &lt; K$</th>
<th>$S_t = K$</th>
<th>$S_t &gt; K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Put</td>
<td>In the money (ITM)</td>
<td>At the money (ATM)</td>
<td>Out of the money (OOM)</td>
</tr>
<tr>
<td>Call</td>
<td>Out of the money (OOM)</td>
<td>At the money (ATM)</td>
<td>In the money (ITM)</td>
</tr>
</tbody>
</table>

- Options exercised on *expiry date* only are *European*.
  - *American* options may be exercised any time until expiry.
  - Most stock, interest-rate, and futures options are American.
  - Most index and many FX options are European.
Explicitly-traded options more recent than forwards: 1400s in Italy.

Organized options exchanges are even more recent.
  - First: Chicago Board Options Exchange, 1973 (CBOT offshoot).

Two other boosts to options trading in 1973:
  - Black and Scholes discover closed-form option pricing formula;
  - Advent of handheld calculator to allow computation of prices.

Many competing US exchanges for stock and index options.

Globally: stock option use growing; index options most common.

Unlike futures, US options exchanges use independent clearinghouse.
  - Clearinghouse separated from exchanges; facilitates competition.
What is the payoff at maturity for a put or call?

For strike $K$, underlier price $S_t$, expiry at $T$:

- Call expiry value: $C_T = (S_T - K)^+$. 
- Put expiry value: $P_T = (K - S_T)^+$. 
- Remember: We only exercise if payoff is positive.

Payout graphs are typical “hockey² sticks.”

Payoff diagrams may be combined to create basic strategies.

Helpful way to visualize options/combos.

Also holds for early exercise of American options.
We can see put and call diagrams to confirm priors:

- **Protective put**: underlier + put eliminates downside.
- **Covered call**: underlier – call eliminates upside, yields income.
**Options Strategies: Spreads**

- **Bear spread**: long put, short lower-strike put; bet on downside.

![Bear Spread Graph]

- **Bull spread**: long call, short higher-strike call; bet on upside.

![Bull Spread Graph]
Options Strategies: Collars

- **Collars** can model protection offered in merger terms.
  - Described by behavior in a range of acquirer stock prices.
- **Fixed-price collar**: offers fixed price unless buyer stock moves.
  - If buyer’s stock drops/spikes, acquirer pays less/more, aka “Travolta.”
- **Fixed-ratio collar**: offers price with floor+ceiling, aka “Egyptian.”

Graphs showing:
- "Travolta" Value vs. Underlier Price $S_T$
- "Egyptian" Value vs. Underlier Price $S_T$
We can also use options to express a view on volatility.

- **Straddle**: same-strike call + put; **Strangle**: similar, but $K_{put} < K_{call}$.

- **Butterfly**: long call, short 2 mid-strike calls, long high-strike call.
- **Condor**: like butterfly, but mid-strike calls have different strikes.
Even without pricing an option, we know they have value.

Also know: a call struck at $30 has value if stock is at $29.99.

More volatile stock $\implies$ call has more potential value.

*Time value*: difference between option price and terminal payoff.
  - Sort of like taking a smoothing/kernel average of nearby prices.

Some strategies are all about time value, *e.g.* calendar spread:
  - Buy longer-dated option, short shorter-dated option.
  - Effect is like a skewed butterfly spread.
Put-Call Parity: Payoff Diagram

- Long call (---), short put (···): like shifting underlying by $PV(K)$.
- We call this relationship put-call parity.
Put-Call Parity

- Thus the five prices we use are related:
  - Prices of calls and puts expiring at $T$ ($C_{t,T}, P_{t,T}$);
  - Prices of underlier and strike ($S_t, K$); and,
  - Price of money — interest rate $t \rightarrow T$ ($r$).

- Arbitrage argument (with all typical assumptions):
  - At $t$: buy underlier @ $S_t$, put @ $P_{t,T}$; sell call @ $C_{t,T}$.
  - At $T$: get $S_T + (K - S_T)^+ - (S_T - K)^+ = K$.
  - No arbitrage price $K$: $Ke^{-r(T-t)}$ or $\frac{K}{(1+r)^{(T-t)}}$.
  - Should also account for omitted dividend yield $d$.

- Put-call parity: $C_{K,t,T} - P_{t,T} = S_t e^{-d(T-t)} - Ke^{-r(T-t)}$. 
Embedded Options

- Many contracts have embedded options, option-like behavior.
- *Callable bonds* are short a call at some yield.
- *Convertible bonds* (CBs) allow converting bond to stock.
  - Specified conversion ratio implies # shares, strike price.
  - Stock is issued out of company treasury; dilution results.
- Bonds with *detachable warrants* are very much like CBs.
- *Collateralized loans*: like risk-free bond less collateral put.

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3 Options on yields appear to be “mirror imaged”.

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Exotic Options

- American/European puts/calls often called *vanilla options*.
- In contrast, *exotic options* involve different payoffs.
  - Payoffs may involve extra optionality or price history.
  - Payoffs may be on oddly-behaved underliers like spreads\(^4\).
- Exotic options include:
  - *Asian options*: based on average underlier price;
  - *Swing options*: allow deviations from constant-delivery rate;
  - *Exchange options*: allow switching underlying assets;
  - *Spread options*: may hedge marginal production cost;
  - *Quanto options*: hedge FX and underlier risk;
  - *Barrier options*: knock-in, knock-out; and,
  - *Lookback options*: payoff based on recent past best/worst price.

\(^4\)Spreads behave oddly because they can assume negative values.
Vanilla options respond in certain ways to key variables:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Call value</th>
<th>Put value</th>
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<tbody>
<tr>
<td>$S_t$↑</td>
<td>↑</td>
<td>↓</td>
</tr>
<tr>
<td>$K$↑</td>
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<td>↑</td>
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<tr>
<td>$\sigma$↑</td>
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<tr>
<td>$T$↑</td>
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</tr>
<tr>
<td>$d$↑</td>
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</table>
Early Exercise

- Does European vs. American option difference matter?
  - Sometimes, but in most circumstances not.
- Would not exercise OOM options early: negative value.
- Would not exercise ATM options early: gives up option value.
- Without dividends, we would not exercise a call option early.
- Immediately pre-dividend, might exercise ITM call early.
- For an American put option, early exercise may be optimal:
  - If $S_t = 0.01$, $K = 30$: why lose time value of profit?
  - Thus below some $S^*$, $P_{t,T}^{Am} = K - S_t > P_{t,T}^{Eu} > Ke^{-r(T-t)} - S_t$. 

$Q_{36}$
We covered option basics; on to option valuation next!

- Risk Alleviation: Option Valuation, Credit, Structured Products; and,
- All Together Now: Active Portfolios, Investment Firms, Crises.