Quantitative Investments

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Last lecture we discussed statistical modeling.

- Types of analysis;
- Data issues;
- Estimation and diagnostics; and,
- Game-theoretic concerns.

Today we will talk about fixed income.
Fixed Income

Chapter 11, A Quantitative Primer on Investments with R
This part discusses fixed income. Specifically:

- Bond features and innovations;
- Accrued interest;
- Pricing and yields;
- Interest rate sensitivity;
- Risks to bondholders; and,
- Repos.
A *bond* is a security relating to a loan.

- Typically, the bond is the rights to loan repayment.
- The *par (face)* value is the amount borrowed.
  - Often, par is repaid in whole at *maturity date*.
- Bonds may receive regular *coupon* (interest) payments.
  - Annual interest = *coupon rate*; $\times$ par.
- *Zero coupon bonds* pay only par at maturity.
- Par, coupon, maturity, other details listed in *bond indenture*. 
Some features entice investors.

*Floating rate bonds* pay interest based on current rates.
  - Bond prices less affected by interest rate movements.

*Indexed bonds*: par values change with inflation.

*Interest rate puts*: lenders force repurchase if rates ↑.
  - Reduces interest rate risk of investors.

*Convertible bonds*: firm repays in equity if investor chooses.
Other features help issuers reduce risk.

*Interest rate calls* let firm buy bond if rates ↓.

*Sinking fund provisions:* retire bonds over time.
  - Smoothes cashflow over time vs one big outflow.

*Inverse floaters* pay less when rates rise, more when rates drop.

*Asset-backed bonds* often backed by mortgages, other loans.
  - Let lenders shift loans/receivables to other investors. (even Bowie!)

*Catastrophe bonds* pay back less if certain rare events occur.
  - *e.g.* severe hurricane/earthquake; a method of reinsurance.
Pricing a bond is theoretically simple:

\[
\text{bond value} = E(\text{PV of par}) + E(\text{PV of coupons}). \tag{1}
\]

If bond matures in \( T \) years, constant rates, just after coupon:

\[
\text{bond value} = \frac{\text{par}}{(1 + r)^T} + \sum_{t=1}^{T} \frac{\text{coupon}}{(1 + r)^t} \tag{2}
\]

\[
= \frac{\text{par}}{(1 + r)^T} + \frac{\text{coupon}}{r} \left( 1 - \frac{1}{(1 + r)^T} \right). \tag{3}
\]

- \( \text{PV(Perpetuity)} = \frac{\text{notional}}{r} \Rightarrow \text{PV(coupons)} = \frac{\text{coupon}}{r} - \frac{\text{coupon}}{r(1+r)^T} \).
- Between coupons, also add accrued interest.
Accrued Interest

- Bonds in most countries quoted *clean* (without interest).
  - Buyers of bonds between coupons are owed a partial coupon.
- Thus the actual price paid includes accrued interest.
  - *Invoice price* (aka *dirty price*) quote includes accrued interest.
- Why we, most countries quote prices clean? Comparability.
- How to calculate accrued interest (AI)?

\[
\tau = \frac{\text{days from last coupon}}{\text{days, last to next coupon}}
\]

\[
AI_\tau = \frac{\text{coupon rate}}{\text{coupons/year}} \times \tau.
\]

- Daycount for most government bonds is 30/360.
  - Many corporates, some countries use Actual/365.
Slightly Better Bond Pricing

- These formulas work if the bond pays interest annually.
- For a bond which pays interest $n$ times/year, we can do better.
- If bond matures in $T$ years, just after a coupon payment:

$$\text{price} = \frac{\text{par}}{(1 + \frac{r}{n})^{nT}} + \sum_{t=1}^{nT} \frac{\text{coupon}}{(1 + \frac{r}{n})^t}$$  \hspace{1cm} (6)

- Between coupons: fractional discounting — accrued interest.

$$\text{dirty price} = \frac{\text{par}}{(1 + \frac{r}{n})^{\lfloor nT \rfloor - \tau}} + \sum_{t=1}^{\lfloor nT \rfloor} \frac{\text{coupon}}{(1 + \frac{r}{n})^{t-\tau}},$$  \hspace{1cm} (7)

$$\text{clean price} = \text{dirty price} - AI\tau.$$  \hspace{1cm} (8)
• **Yield to maturity**: annual return if bond held to maturity.
  - Just invert the pricing formula (i.e. solve for $r$).
    
    $$\text{YTM} = \{ r : \text{bond market price} = \text{bond value} \text{ (6) or (7)} \}.$$  
    
  • **Current yield** = annual coupons/current price.
  • May also consider other payoff scenarios: *yield to call, yield to put*. 

Yields based on current prices; what if rates change?
- We would like to know realized compound return (RCR).

Problem: We don’t know where rates are going.
- Horizon analysis considers RCR for various scenarios, holding periods.

How to compute log-returns for bonds?

$$r_{t \rightarrow \tau} = \log \left( \frac{P_{\tau} + AI_{\tau} + C_{\tau}}{P_t + AI_t} \right)$$ (10)

where $C_{\tau}$ captures coupons paid from time $t$ to time $\tau$. 
Even with constant rates, bond prices change over time.

*Interest rate risk:* when interest rates rise, bond prices fall.

This is important to remember; bond prices affect gains:
- You can lose money if you sell risk-free bonds at a low price.
- Furthermore: longer-term bonds = more sensitive to interest rates.
- Long-term bonds (e.g. 30Y UST) can be as volatile as the S&P 500.
- Bonds tend to be least volatile when issued and near maturity.

If YTM ↑, holding period return ↓
Look at price vs yield for varying maturities of 4.5% coupon bonds:

- Price of 4.5% bond yielding 4.5%: 100.
- Bond prices change inversely with interest rates.
- Longer-dated bonds more sensitive to interest rates.
- Interest rate sensitivity higher at lower interest rates.
Look at price vs yield for varying coupons of 20Y bonds:

- Zero-coupon bond always below 100 if rates >0: *discount bond*.
- Yield >, < coupon? Bond trades at a *discount, premium*.
- Bond prices decrease as coupon decreases.
- Lower-coupon bonds are more %-sensitive to rates.
We can directly analyze interest rate sensitivity:

\[ P_0 = \sum_{t=1}^{T} \frac{CF_t}{(1 + y)^t} \quad \text{or} \quad \sum_{t=1}^{nT} \frac{CF_t}{(1 + \frac{y}{n})^t}, \]

\[ \frac{dP_0}{dy} = \sum_{t=1}^{T} -t \frac{CF_t}{(1 + y)^{t+1}} \quad \text{or} \quad \sum_{t=1}^{T} -n \frac{t}{n(1 + \frac{y}{n})^{t+1}}. \]

**DV01**: dollar value of a 1 basis point yield change.\(^2\)

\[ \text{DV01} = \frac{dP_0}{dy} \cdot 0.0001. \]

One basis point scale is much more sensible.

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\(^2\)One basis point is \(\frac{1}{100}\)-th of 1% = 0.01%.
Interest Rate Sensitivity: Duration

- Can formalize idea that maturity/cashflow timing relate to sensitivity.
- **Duration**: PV-weighted average time of cashflows.

\[
D = \sum_{t=1}^{T} \frac{t \cdot CF_t / (1 + y)^t}{P_0} = -\frac{(1 + y)}{P_0} \left. \frac{dP_0}{dy} \right|_{y_0}. \tag{14}
\]

- Modified duration \(D^*\): gives us % bond price change.

\[
D^* = \left. -\frac{1}{P_0} \frac{dP}{dy} \right|_{y_0}. \tag{15}
\]

- Sensitivity, cashflow timing, maturity... all wrapped up together.
- **N.B.** \(DV01 = -D^*P_0 \cdot 0.0001\).
Interest rates $\uparrow$? Bond prices $\downarrow$.

- Bonds trade at premium/discount based on coupon vs interest rates.
- Duration of a $T$-year zero-coupon bond? $T$.
- Lower-coupon bonds more sensitive to interest rates.
- Interest rate sensitivity is not linear: more sensitive at low rates.
From earlier observations come simpler rules:

- Duration of zero coupon bond = maturity (in years).
- For same maturity, lower coupon $\Rightarrow$ higher duration.
- For same coupon, duration usually $\uparrow$ as maturity $\uparrow$.
  - If bond is not trading at a discount, this is always true.
- All else equal, duration of coupon bond $\uparrow$ as YTM $\downarrow$.
- Duration of a perpetuity: $D_{\text{perp}} = \frac{1+y}{y}$.
Note that we mentioned how duration changes.

Q: Are these changes linear?

Q_A: Would $\Delta YTM = 2\epsilon$ affect prices twice as much as $\Delta YTM = \epsilon$?

A: Usually, no. This is due to convexity.

Recall plots of prices vs. $\Delta YTM$ were curved.

- Modified duration related to current derivative wrt $y$.
- Convexity related to second derivative wrt $y$:

\[
\text{Convexity} = \frac{1}{P_0} \frac{d^2 P}{dy^2} = \frac{1}{P_0(1 + y)^2} \sum_{t=1}^{T} \left( t^2 + t \right) \frac{CF_t}{(1 + y)^t}.
\]\n
(16)
Convexity: A Complication

- Convexity seems like trouble:
  - Durations must be recalculated after every yield change.
  - Formula seems complicated; isn’t duration alone sufficient?
- But convexity is helpful:
  - Interest rate cuts benefit bondholders more than hikes hurt.
  - Allows us to better measure interest rate sensitivity:
    \[
    \frac{\Delta P}{P_0} = -D^* \Delta y + \frac{\text{convexity}}{2}(\Delta y)^2.
    \] (17)
  - And convexity is a reality; you should understand it.
Convexity of Callable Bonds

- Recall that some bonds may be called by issuer if rates drop.  
- Thus callable bonds rarely trade much above the call price.  
  - Creates *negative convexity*: rate drops affect prices less.
- Problem: Called bond is extinguished before maturity.
- Solution: Compute **effective duration** \( D_{\text{eff}} = -\frac{\Delta P/P_0}{\Delta r} \).
- Mortgages are similar: if rates drop, holders refinance loans.
  - But people often wait to refinance (minimize fees; “dumbos”).
  - Thus mortgages more likely than corps to trade above principal balance.

![Diagram showing callable and non-callable bond prices with yield on the x-axis and bond price on the y-axis. The call price is 102, and the call trigger is 4%. The callable bond is extinguished at the call price.]
Discount Bonds

- **Original-issue discount bonds** issued with low coupons.
  - OID bonds are less common than *par bonds*.
  - Includes zero-coupon bonds (“zeros”) such as T-bills.
  - Zero-coupon bonds only pay at maturity.
- Longer term zeros are often created by *stripping* a bond.
  - Each payment becomes a unique zero-coupon bond.
  - These bonds are called *strips*.
Default

- A borrower who misses bond payment is in *default* = serious.
- Default may happen for a number of reasons:
  - *Balance sheet insolvency*: liabilities > assets.
  - *Cashflow insolvency*: cannot make a payment.
  - *Strategic default*: purposeful default, may be negotiating tactic.
- Corporations, municipalities, even federal governments may default.
- Creditors may seize specific assets if pledged as bond *collateral*.
- Otherwise, creditors may force sale of general firm assets.
- Often cheaper, better: *debt workout* (a compromise).
Protective Covenants

- Bond indentures have *protective covenants* to assure lenders.
- **Subordination clauses**: “No cutting in line.”
  - Later borrowing is subordinated = paid later if bankruptcy.
- **Cross-default clauses**: “All for one, one for all.”
  - Cannot default on some bonds and pay others to play games.
- **Dividend restrictions**: “Don’t spend more than you make.”
  - Typical: Cannot pay more in dividends than earnings.
- **Asset withdrawal restrictions**: no selling assets to pay shareholders.
- **Asset substitution restrictions**: do not change to make firm riskier.
- **Underinvestment clauses**: must also invest in lower-risk projects.
- **Force majeure clause**: (rarer) excuses default in war, disasters, etc.
Credit Spreads

- One of the most informative measures: credit spread.
- Basically, yield difference from similar-maturity government bond.
- Short-term: interbank – govt bills, eg TED spread.
  - Very liquid; top credit vs “risk-free”.
  - Can indicate when economy nearing crisis.\(^3\)
- Intermediate term: Corporate bonds – notes/bonds.
  - Less liquid; sign of spreading crisis.
  - Exemplar: 10-year BBB/Baa-rated corporates – UST 10Y.
- High-yield rates vs investment-grade rates.

Unrated/sub-investment-grade bonds are *high yield bonds.*

- Also known as “junk” bonds.

Pre-1977: All junk was “fallen angels” (downgraded issues).

1977: Drexel and Milken premier first original-issue junk.

- Investors found diversified high-yield portfolios attractive.

High yield often trades somewhat like equity.

- Equity receives residual cash after paying bondholders.
- High yield should trade similarly if assets < liabilities.
Liquidity Effects

- New bonds like puppies: most quickly find a “forever home.”
- Newly-issued bonds are liquid for a short period.
- After that, they become illiquid; few circulate and little trading.
- Insurers, endowments, pensions hold for long-term to hit targets.
- Government bonds: on-the-run vs off-the-run.
- Yield difference can be tens–hundreds of basis points.
- Not free money! Liquidity risk is vicious type of risk.
- May become more liquid near certain remaining maturities.
**Repos**

- *Repurchase agreement* (repo): sale+later buy of assets.
- Pledge asset as collateral in exchange for loan of cash.
- Lender holds back some of the proceeds (the *haircut*).
- Why repo assets?
  - Frees up cash in case needed for day-to-day operations.
  - A form of leverage; allows making other investments.\(^4\)
- Special rates quoted if many people want to short asset.
- Repos are an important part of liquidity management.
- Repos have declined; Basel III/Dodd-Frank made them costly.

\(^4\)This does increase your risk, however.
We have covered fixed income; on to yield curves next time!

- Valuation: Yield Curves, Equity Valuation;
- Valuation II: Factor Models, Microfoundations, Global Investing, FX;
- Risk Alleviation: Futures, Options, Credit, Structured Products; and,
- All Together Now: Active Portfolios, Investment Firms, Crises.