Quantitative Investments

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Last lecture we discussed yield curves.

- Yield curves;
- What yield curves tell us;
- Monetary policy; and,
- Interest rate dynamics.

Today we will talk about equity valuation.
Today we will discuss equity valuation. Specifically:

- Sector and industry analysis;
- Fundamental analysis and comparables;
- Issues with discounted cashflow (DCF) models;
- Dividend discount models (DDMs);
- Free cashflow methods;
- The value of growth; and,
- Thoughts on the pricing kernel.
Sector and Industry Analysis

- Below economy, must consider sectors and industries.
  - Investing well is easier in well-performing industries.
  - Sectors/industries have different sensitivities to economy.
  - Firms within an industry are exposed to similar risk factors.

- Economic sensitivity metric: degree of operating leverage.

\[
DOL = \frac{\% \text{ change in profits}}{\% \text{ change in sales}} = 1 + \frac{\text{fixed costs}}{\text{profits}}. \tag{1}
\]

- Pure borrowing (leverage) also increases cycle sensitivity.

- Thus we expect certain ranges of DOL for different industries, e.g.:
  - Software: high fixed cost, tiny variable costs = high DOL; vs
  - Construction: medium fixed cost, large variable costs = low DOL.
Sectors often respond coherently to the business cycle.
Suggests choosing sector weights in light of business cycle.
*Rotation* reweights sectors optimally given business cycle.
Thus if we are entering a contraction, we might:
- overweight defensive stocks; and,
- underweight cyclical stocks.
How to define sectors and industries?

- Barra has defined national/world sectors, industries.
- S&P also has their own definitions.
- Nat’l/Int’l industry codings, *e.g.* NAICS, UKSIC, NACE, ISIC.

National/international codings use a hierarchy: SSIIi...i

- SS = sector (1–2 characters); II = industry;
- i...i = characters/symbols for sub-industry.
- *e.g.* NAICS: 20 sectors; 92 industries (sub-sectors).
- Often use code substrings SS or SSII as categorical variable.
⚠️ Periklu! Accounting measures may be “smoothed” or “managed.” This make them less reliable.

- Fundamental analysts compare companies using:
  - financial filings (in US: from SEC’s EDGAR); and,
  - derived measures, *e.g.* from Compustat.

- We often compare firms by *comparables*:
  - Price ratios: earnings (P/E), book (P/B), even website clicks;
  - Per-share ratios: earnings, book, sales, cashflow.

- Idea: Similar companies should have similar measures.

- All imply prices the analyst takes into consideration.
Criticisms of P/E analysis abound, among them:

- Earnings is an accounting construct.
  - Economic earnings may diverge (e.g. depreciation).
- Companies manage (inflate, smooth) earnings.
- Assumes earnings rise smoothly; ignores lumpiness.
- P/E for cycicals is less stable.
- P/E may be meaningless for start-ups.
Price-to-Earnings: Alternatives

- Can use P/E and forecasted dividends to predict target price.
  - Feed this into DCF models to get intrinsic value.
- Can look at other ratios:
  - *Price/book*: how aggressively market values assets.
  - *Price/cashflow*: avoids earnings management.
  - *Price/sales*: useful for valuing firms without earnings.
Limitations of Book Value

- Equity holders are “residual claimants.”
  - But, what are assets (and hence residual) worth?
- Problem: book value = original cost less depreciation.
- We might instead ask: What would assets sell for?
  - This is critical for debt analysts.
  - If liquidation value\(^2\) exceeds market cap, expect raiders.
- Consider replacement cost of assets less liabilities.
- Leads to Tobin’s \(q\): \(q = \frac{\text{market price of firm}}{\text{replacement cost of assets}}\).
  - \(q\) should tend to 1 due to competition.
  - If \(q > 1\), competitors can replicate firm for cheaper.

\(^2\)Money left after selling assets and repaying debt.
Basic approach for a certain time horizon:
- For now, assume we have prediction of price at year-end.

\[
\text{Expected HPR} = \frac{E(\text{annual div.})}{P_0} + \frac{E(\text{annual } \Delta \text{price})}{P_0} = E(\text{div. yield}) + E(\text{cap gain yield})
\]  

Is expected HPR attractive for risk? Think about expected return.
- Think of \textit{stochastic discount factor }d\textit{ for return factor }R
  - We then expect that competitive pricing yields \(E(dR) = 1\).
- We often think in terms of \textit{pricing kernel }k = 1/d - 1.
When discounting cashflows, we use the pricing kernel \( k \).

However, if \( k \) is uncertain, that biases the valuation.

For a convex \( f \), Jensen’s Inequality says:

\[
E(f(k)) > f(E(k)) \quad \forall f : f'' > 0
\]  

(4)

This is trouble; cannot just plug in averages. Must correct:

\[
E(f(k)) \approx f(E(k)) + \frac{1}{2} f''(E(k)) \sigma_k^2
\]  

(5)

Thus if we discount cashflows:

\[
\tilde{V} = \sum_t \frac{C_t}{(1 + k)^t} + \frac{1}{2} \sum_t t(t + 1) C_t \sigma_k^2
\]  

(6)

We will come back to this shortly.
Intrinsic Value

- *Intrinsic value* uses price target, pricing kernel $k$.
  - Computes what would be a fair price for stock.

- One-year horizon, annual dividend $D_1$, EOY target price $P_1$:

  $$V_0 = \frac{E(D_1) + E(P_1)}{1 + k} \quad (7)$$

- For a two-year investment horizon:

  $$V_0 = \frac{E(D_1)}{1 + k} + \frac{E(D_2) + E(P_2)}{(1 + k)^2} \quad (8)$$

- This prepares us for the world of DDMs.
Dividend Discount Model (DDM)

- Intrinsic value hints at valuing stock at any horizon.
- Basic *dividend discount model* (DDM) for $H$-year horizon:\n
$$V_0 = \frac{D_1}{1 + k} + \frac{D_2}{(1 + k)^2} + \cdots + \frac{D_H + P_H}{(1 + k)^H}$$ \hspace{1cm} (9)

- If dividends constant $D$, annuity price yields $P_H, V_0$:\n
$$P_H = \frac{D}{k} \Rightarrow V_0 = \sum_{t=1}^{\infty} \frac{D}{(1 + k)^t} = \frac{D}{k}$$ \hspace{1cm} (10)

- Thus we value stock as the sum of all discounted dividends.
- But for estimated $\hat{k}$, we adjust for Jensen’s Inequality:\n
$$V_0 = \frac{D}{k} + \frac{D}{k^3} \sigma_k^2.$$ \hspace{1cm} (11)

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3 We suppress most of the expectations from here forward.
Gordon-Shapiro Constant-Growth DDM

- Reasonable extension: dividends which grow at rate $g$.  
- This yields the constant-growth dividend discount model:

$$V_0 = \frac{D_0(1 + g)}{1 + k} + \frac{D_0(1 + g)^2}{(1 + k)^2} + \ldots = \frac{D_0(1 + g)}{k - g} = \frac{D_1}{k - g} \quad (12)$$

- Instant implication: dividend growth at/above $k$ is unsustainable.
- We can also invert this formula to estimate $k$:

$$k = \frac{D_1}{P_0} + g = \text{div. yield} + \text{capital gains rate.} \quad (13)$$

- For estimated $\hat{k}$ and $\hat{g}$, we Jensen correct to get:

$$\hat{V}_{0,G-S} = \frac{\hat{D}}{\hat{k} - \hat{g}} + \frac{\hat{D}}{(\hat{k} - \hat{g})^3} (\hat{\sigma}_k^2 + \hat{\sigma}_g^2 - 2 \text{Cov}(\hat{k}, \hat{g})). \quad (14)$$

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$^4$Discovered by Gordon and Shapiro (1956).
Reinvestment, Gordon-Shapiro, and DuPont

Does Gordon-Shapiro suggest companies pay out all dividends?
- No; Gordon-Shapiro shows this may be unwise.

Suppose we lower our dividend payout ratio.
- Reinvest fraction $b$ of dividends in company.\(^5\)
- If company's $ROE > k$, this seems sensible.

Then, dividends grow at rate $g = b \cdot ROE$, implying

$$V_0 = \frac{E_1 (1 - b)}{k - g} = \frac{E_1 (1 - b)}{k - b \cdot ROE}.$$  \hfill (15)

The DuPont model decomposes $ROE$ into separate tasks:

$$ROE = \frac{\text{net profits}}{\text{sales}} \times \frac{\text{sales}}{\text{average assets}} \times \frac{\text{average assets}}{\text{average equity}}.$$  \hfill (16)

\(^5\)Unpaid dividends are also called retained earnings.
Multistage Growth DDMs

- Constant ROE sounds great; contradicts observed life cycles.
- Instead, use a life cycle model and basic DDM to value firm.
- Basic idea: value modeled dividends via discounting.
  - Near dividends are discounted individually.
  - Assume decreasing dividend growth during maturity stage.
  - Then use Gordon model for dividends thereafter.
  - This also handles one-time special dividends.
Multistage Growth DDM Example

Insert example here valuing an actual company.

Except that multi-stage modeling was an exercise in frustration.

- The multistage growth DDM is flexible — maybe too flexible.
- You can work hours trying to make a nice example of a real firm.
- But when the model works, it gives highly varying prices.
- If you torture the model, it will confess.
Free Cashflow Models and WACC

- Instead of dividends, can value free cashflow to all/part of firm.
- Discount by firm’s *weighted average cost of capital* (WACC).
- Funding: consider sources of funding for cashflow destination.
  - Firm? Use equity, bond, bank loans in fractions $w_e + w_b + w_\ell = 1$.
- For funding sources having rates of return $r_e, r_b, r_\ell$:
  \[
  WACC = w_e r_e + (w_b r_b + w_\ell r_\ell)(1 - \tau). \quad (17)
  \]
- Why multiply bonds, loans by $1 - \tau$? Tax advantage of debt.
Free Cashflow to the Firm

Find cost of replicating free cashflow to the firm (FCFF):

$$FCFF = EBIT (1 - \tau) + \text{depreciation} - \text{capex} - \Delta NWC$$ \hspace{1cm} (18)

where $\tau$ is tax rate, NWC is net working capital.

Then discount FCFF by WACC:

$$\text{Firm value} = F_0 = \sum_{t=1}^{T} \frac{FCFF_t}{(1 + WACC_t)^t} + \frac{F_T}{(1 + WACC_T)^T},$$ \hspace{1cm} (19)

or $F_0 = \frac{FCFF_1}{WACC - g}$. \hspace{1cm} (20)

Subtract debt, compute per-share value: $V_0 = \frac{\text{firm value} - \text{debt}}{\text{shares outstanding}}$. 

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If we estimate FCFF and WACC \((i.e. r_e = \hat{k})\), firm value is:

\[
F_0 = \sum_{t=1}^{T} \left( \frac{\hat{\text{FCFF}}_0 (1 + \hat{g})^t}{(1 + \text{WACC}_t)^t} + \frac{t(t+1)\hat{\text{FCFF}}_0 (1 + \hat{g})^t}{2(1 + \text{WACC}_t)^{t+2}} \hat{\sigma}_{\text{WACC}-g}^2 \right) + \frac{\hat{\text{V}}_T}{(1 + \text{WACC}_T)^T},
\]

\[
\hat{\text{V}}_T = \frac{\hat{\text{FCFF}}_0 (1 + \hat{g})^T}{\text{WACC} - \hat{g}} + \frac{\hat{\text{FCFF}}_0 (1 + \hat{g})^T}{(\text{WACC} - \hat{g})^3} \hat{\sigma}_{\text{WACC}-g}^2, \text{ where}
\]

\[
\hat{\sigma}_{\text{WACC}-g}^2 = \hat{\sigma}_{\text{WACC}}^2 + \hat{\sigma}_g^2 - 2 \text{Cov}(\text{WACC}, \hat{g}).
\]
Free Cashflow to Equity

- Find cost of replicating *free cashflow to equity* (FCFE).

\[
FCFE = FCFF - \text{interest}(1 - \tau) + \Delta\text{net debt}. \quad (24)
\]

- Find change in firm leverage from time 0 → t: \( l_t = \frac{1 + \text{debt}_t(1 - \tau)}{1 + \text{equity}_t(1 - \tau)} \).

- Adjust \( k \) for change in leverage: \( k_{E,t} = r_f + l_t(k - r_f) \).

- Discount cashflows at \( k_{E,t} \) and apportion per share:

\[
V_t = \frac{FCFE_t}{k_{E,t} - g}, \quad \text{or} \quad \hat{V}_t = \frac{\hat{FCFE}_t}{\hat{k}_{E,t} - \hat{g}} + \frac{\hat{FCFE}_t}{(\hat{k}_{E,t} - \hat{g})^3} \hat{\sigma}_t^2 k_{E,t-g}, \quad (25)
\]

\[
V_0 = \frac{\sum_{t=1}^{T} FCFE_t}{(1+k_{E,T})^T} + \frac{V_T}{(1+k_{E,T})^T} \text{ shares outstanding} \quad \text{or estimated version}.... \quad (26)
\]
In theory, these valuation models should all be equivalent.

- *Modigliani-Miller Theorem* says dividends, debt irrelevant.
- Thus DDM, G-S, FCFF, FCFE, P/E should all yield same values.

Why do these valuations differ then?

- Uncertainty in assumptions. (*e.g.* When is firm “mature?”)
- Uncertainty in direct inputs.
- Non-constancy of inputs (*r_f* varies).
- M&M ignores taxes, creditor rights, costs, other frictions.
- \(WACC_T - g\) and \(k_{E,T} - g\) near 0 \(\Rightarrow\) fat-tailed valuations.

A good analyst would consider all these models.
Growth Opportunities and P/E

- If $ROE > k$, re-investing is better than value without: $\frac{E_1}{k}$.
- Difference = present value of growth opportunities (PVGO):

$$PVGO = P_0 - \frac{E_1}{k} \quad (27)$$

- Can rearrange PVGO formula to see P/E relationship:

$$\frac{P_0}{E_1} = \frac{1}{k} \left(1 + \frac{PVGO}{E_1/k}\right). \quad (28)$$

- Thus if PVGO is 0, stock trades like an annuity.
- As growth opportunities become more valuable, P/E rises.
- What is $PVGO / (E_1/k)$? Ratio of growth value to asset value.
  - If growth value high relative to assets, P/E will be high.
Recall Gordon-Shapiro model: \( P_0 = \frac{D_1}{k-g} \).

Substituting in \( D_1 = E_1(1 - b) \) and \( g = b \cdot \text{ROE} \):

\[
P_0 = \frac{E_1(1 - b)}{k - b \cdot \text{ROE}} \Rightarrow \frac{P_0}{E_1} = \frac{1 - b}{k - b \cdot \text{ROE}}. \tag{29}
\]

Note effect of \( b, \text{ROE} \) on P/E for \( k = 10\% \):

<table>
<thead>
<tr>
<th>Retained earnings ( b )</th>
<th>ROE</th>
<th>0%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7%</td>
<td>10.0</td>
<td>9.1</td>
<td>7.7</td>
<td>5.3</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>10.0</td>
<td>10.0</td>
<td>10.0</td>
<td>10.0</td>
</tr>
<tr>
<td></td>
<td>13%</td>
<td>10.0</td>
<td>11.1</td>
<td>14.3</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Rough guide: \( \text{P/E} \approx g \Rightarrow \frac{\text{P/E}}{g} < 1 \) attractive.
Big question: what *really* is the pricing kernel $k$?

Efficient markets: $k = r_f + $ some multiples of risk premia.

But then we estimate $k$, yielding $\hat{k}$.

Why not use average excess returns of a stock? Could, but . . .

1. We think that model reduces noise (and thus valuation variation);
2. We can correct for time trends; and,
3. We can exclude alpha — which allows for mispricings.
4. Also nice: it helps reveal relevant economic forces.

Use risk-free rate based on our investment/decision horizon.

Probably should allow for multiple risk factors.\(^6\)

\(^6\)Serious foreshadowing here.
We have covered equity valuation; on to the CAPM next time!

- Valuation II: CAPM, Factor Models, Microfoundations, Global;
- Risk Alleviation: Futures, Options, Credit, Structured Products; and,
- All Together Now: Active Portfolios, Investment Firms, Crises.