Recall

Last lecture we discussed option valuation.

- Binomial Tree Option Pricing;
- The Black-Scholes-Merton Option Model;
- Divergences from Black-Scholes;
- Recovering the Risk-Neutral Density; and,
- Real Options.

Today we will talk about credit.
Chapter 22, A Quantitative Primer on Investments with R
Today we will discuss credit.

In particular, we will discuss:

- Basics, Credit Measures, and Issues;
- Credit Derivatives;
- The Merton Model of the Firm;
- Structural Credit Models;
- Accounting-based Credit Models;
- Default Intensity Credit Models; and,
- Credit Instruments for Modeling and Trade.
Credit Basics

- Basic idea of credit: entity’s ability to borrow money/value.
- **Credit risk**: risk of *default* (missing promised cashflow/repayment).
- Credit is both absolute and relative.
  - Entities with better/worse credit pay less/more to borrow.
  - When credit is *loose/tight*, everyone pays less/more.
- Credit is *procyclical*: looser in expansion, tighter in recession.
- Duality of lender perceptions leads to two equilibria:
  - If troubles look temporary: lend more, market self-heals
  - If troubles look long-lasting: lend less, crisis ensues.
- If lenders refuse to lend, we may end up in a *credit crunch*. 
Measures of Credit and Yields

- There are many ways we can measure/compare creditworthiness.
- Can use yields, ratings, default probabilities, credit scores.
- Yields are commonly-used: higher yield means more risk.

\[
\text{Promised YTM} = \text{Expected yield} + \text{Risk penalty.} \tag{1}
\]

- We sometimes watch *credit spreads* = yield default premia:
  - Default premium = Promised YTM − Same-tenor govt YTM.
  - TED spread separates peaceful (TED <48bp), crises (TED >48bp).
  - TED = 3MUSDLIBOR − 3M US T-bills YTM
- Lending rates often quoted at idiosyncratic spread, e.g. “LIBOR+110”
Rating agencies try to estimate credit risk of bonds, issuers.

Four agencies recognized by ECB for assessing borrower collateral.
- By size: Standard & Poor’s (S&P), Moody’s, Fitch, DBRS.

Range of (long-term debt) credit ratings:

<table>
<thead>
<tr>
<th>Agency</th>
<th>Highest Grade</th>
<th>High Grade</th>
<th>Speculative Grade</th>
<th>Very Poor Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P</td>
<td>AAA, AA+→AA-</td>
<td>A+→BBB-</td>
<td>BB+→B-</td>
<td>CCC…D</td>
</tr>
<tr>
<td>Moody’s</td>
<td>Aaa, Aa1→Aa3</td>
<td>A1→Baa3</td>
<td>Ba1→B3</td>
<td>Caa1…C</td>
</tr>
<tr>
<td>Fitch</td>
<td>AAA, AA+→AA-</td>
<td>A+→BBB-</td>
<td>BB+→B-</td>
<td>CCC, DDD, DD, D</td>
</tr>
<tr>
<td>DBRS</td>
<td>AAA, AAH→AAL</td>
<td>AH→BBBL</td>
<td>BBH→BL</td>
<td>CCCH→CL</td>
</tr>
</tbody>
</table>

Short-term debt ratings are simpler, coarser.
Default probabilities (PDs): not easily mapped from ratings.
  - Ratings are relative: ratings’ PDs vary over business cycle.

This is $\mathbb{P}$-measure; discount with (risky) kernel $k$.

Rating-implied PDs also vary for sovereign vs corporate bonds.
  - Sovereign 5-year PDs: 0% (Aaa) to 1.3% (Baa) to 38% (Caa–C).
  - Corporate 5-year PDs: 0.1% (Aaa) to 3.7% (Baa) to 36% (Caa–C).

Complication: *seasoning* = default rate falls after a few years.
Bank lending may rely on other information, recourse:
- Bank may have borrower as a banking/payments customer.
- Bank may require more restrictive covenants (e.g. collateral).
- Repayment is more likely to be amortized.

Banks often use *credit scores* for lending decisions.

Credit scoring models: typically quantitative, more objective.

Credit scoring tends to increase lending to small businesses.
- Some research suggests credit scoring reduces adverse selection.
Credit Issues

- Credit analysis may be complicated by issues/portfolio effects.
- **Absolute priority**: repayment order in liquidation (by *seniority*).
  - Most bonds say new bond issues cannot be higher priority
  - Priority: derivatives → bank loans → bonds by seniority → equity.
  - Covenants, info asymmetries make funding sources differ.
- Borrowers who share risk factors may have *correlated defaults*.
  - These are difficult to simulate; copulas are not enough.
- Defaults, recovery rates may correlate: more defaults = lower recovery.
- Mandate effects: safe asset holdings affect capital adequacy.
Credit Derivatives

- Credit derivatives protect buyer from default.
- Asset swap: swap bond for most of face+buyback option.
  - This gives default protection and repo-like financing.
- Credit default swap: in default, exchange bond for face.
- Credit default swaps also exist on indices:
  - CDX (N. Am., Em. Mkt) and iTraxx (Europe, Asia).
We saw that Bachelier, Bronzin modeled stock prices as:

\[ dS_t = \mu dt + \sigma dW_t. \]  \hspace{1cm} (2)

Also noted a flaw: stocks could go below 0. Is it a flaw?

Limited liability: Equity = (Assets - Liabilities)⁺.

Merton idea: Equity is a call option on the value of the firm.

Model firm w/assets \( A_t \), liabilities \( L_t \), asset vol \( \sigma_A \):

\[ dA_t = \mu A_t dt + \sigma_A A_t dW_t, \]  \hspace{1cm} (3)

\[ S_t = (A_t - L_t)^+. \]  \hspace{1cm} (4)
Equity as a call on firm assets explains some stylized facts:
- Some asymmetry (negative skewness) of equity returns.
- Leverage effect: volatility rises when price falls.

Should we estimate volatilities for assets instead of equities?
This led to a whole range of capital-structure-based models.
Structural Credit Models: Merton, Moody’s KMV

- Structural credit models use model of firm capital structure.
- Merton model: one zero-coupon bond, maturing at $\tau$.
- Callable bonds? Consider all permutations of early calls.
- Moody’s KMV models: equate equity and option value, volatility:

$$S_t = A_t e^{-\delta(\tau-t)} \Phi(d_1) - L_t e^{-r_f(\tau-t)} \Phi(d_2),$$

$$\sigma_S S_t = \sigma_A A_t \Phi(d_1),$$

where $d_1, d_2$ are as in Black-Scholes-Merton for asset vol $\sigma_A$.
- Moody’s KMV infers $\hat{L}_t$; may be unknown/shadow liabilities.
Problem: When should equity call option expire? (Never, probably.)

Samuelson and McKean considered a perpetual American option.

\[ A_t^* = \frac{L_t \eta}{\eta - 1} \] (early exercise boundary) \hspace{1cm} (7)

\[ \eta = \frac{r_f - \delta + \frac{\sigma^2_A}{2} + \sqrt{(\delta - r_f - \frac{\sigma^2_A}{2})^2 + 2\delta\sigma^2_A}}{\sigma^2_A} \] \hspace{1cm} (8)

\[ S_t = C_{L_t,t,\infty} = \begin{cases} (A_t^* - L_t)(\frac{A_t}{A_t^*})^\eta & A_t \leq A_t^*, \\ A_t - L_t & A_t > A_t^*. \end{cases} \] \hspace{1cm} (9)

Equity might not seem to be an American option, however:

- Shareholders can choose to buyout lenders at any time.
We could (and do) build much more complicated firm models.

Key outcome of structural models: distance to default.

\[ d_{\text{default}} = \frac{\log(A_t/L_t)}{\sigma_A} \]  

or,

\[ d_{\text{default}}^Q = d_{2, \text{Merton}} = \frac{\log(A_t/L_t) + (r_f - \delta - \frac{\sigma_A^2}{2})(T - t)}{\sigma_A \sqrt{T - t}}. \]  

Distance to default is used in other credit/factor models.

Could also use equity options and vol curve to infer \( \sigma_A, L_t \).
Accounting-Based Models: Ratios

- Accounting-based models look at “structural-ish” measures.
- Often use accounting ratios: how many times can X pay for Y?
- **Coverage ratios**: measure earnings vs fixed costs.
  - *Times-interest-earned ratio* = \( \frac{\text{EBIT}}{\text{interest}} \).
  - *Fixed-charge coverage ratio* = \( \frac{\text{EBIT}}{\text{interest + leases + sinking funds}} \).
- **Liquidity ratios**: measure bills vs liquid assets.
  - *Current ratio* = \( \frac{\text{current assets}}{\text{current liabilities}} \).
  - *Quick ratio* = \( \frac{\text{current assets} - \text{inventory}}{\text{current liabilities}} \).
- **Cashflow-debt ratio** measures need for short-term funding.
- **Leverage ratios**: indicate excessive debt (debt- or assets-to-equity).
- **Profitability ratios**: measure returns vs capital.
  - *Return on assets* = ROA = \( \frac{\text{EBIT}}{\text{total assets}} \).
  - *Return on equity* = ROE = \( \frac{\text{net income}}{\text{equity}} \).
Accounting-Based Models: Ratio Models

- Beaver first put ratios in linear models; used market-wide coefficients.
- **Altman’s Z-score**: $Z > 3$ is safe, $Z < 1.8$ is distressed.

\[
Z = 1.2 \frac{\text{working capital}}{\text{total assets}} + 1.4 \frac{\text{retained earnings}}{\text{total assets}} + 3.3 \frac{\text{ROA}}{\text{sales}} + 0.6 \frac{\text{equity value}}{\text{total liabilities}}.
\]

- **Ohlsen’s O-score**:
  \[
  O = -1.3 - \text{size (0.4 assets/GNP)} - \text{profitability index} - \text{liquidity index} + \text{leverage index}
  \]
  \[
P(\text{Bankruptcy}) = \frac{1}{1 + e^{-O}}.
\]

- **Piotrowski’s F-score**: uses 9 indicator variables.
  - 4 profitability, 3 financial performance, 2 operating efficiency.
Default Intensity Models

- *Default intensity models* estimate default rate/P(default) (aka PD).
- Idea: model *time to default* which may be *censored* (unobserved).
  - Left censoring: people too risky do not get loans.
  - Right censoring: loan may be repaid before default occurs.
- *Survival analysis* used for handling censored data.
- Often model time to default as $\text{Exp}(\lambda)$ or $\text{Gamma}(m, \lambda)$.
  - Gamma better if multiple risk sources/default after multiple failures.
- Then, we build models for conditional $\lambda$, $m$.
  - Compound exponential/gamma model or gamma GLM.
- Recent work uses Hawkes processes to get correlated defaults.
Can also look at what market prices of credit derivatives imply.

Market prices: especially nice since they are forward-looking.

Credit default swap (CDS) prices are especially informative.
  - CDSs yield nearly direct measures of risk-neutral PDs.

Some research suggests stock returns may also be informative.

And yet... we can model CDS prices.

So perhaps some combination of methods would work best?
  - Research suggests: Yes; combine different approaches.
Finally: useful to see how credit works in arduous situations.

In international trade, parties often use letters of credit (LoCs, LCs).
- LoC is a promise by issuer’s bank to pay seller’s banks for goods.
- Complex set of handshakes, sign-offs to reduce risk.
- *Bill of lading* (BoL) is claim for goods at end of transport.

LoCs, BoLs used even w/very low-credit counterparties.

A diagram is informative to show interactions.
Letters of Credit: Diagram

1. Negotiate deal

Buyer

2. Apply for LoC
3. Issue LoC

Issuing Bank

4. Advised LoC

Advising Bank

5. Ship goods
6. BoL

Carrier

7. Goods in transit
8. BoL, confirmed LoC
9. Funds released

Seller

10. BoL, confirmed LoC
11. Funds released
12. Receive BoL
13. BoL
14. Claim goods

N.B. 8–9–10–11 may proceed as 8–10–11–9.
We covered credit; on to structured products and PE next!

- Risk Alleviation: Structured Products and Private Equity; and,
- All Together Now: Active Portfolios, Investment Firms, Crises.