

uFloat Dynamic Model

What is the uFloat?

The uFloat is a low-cost instrument package for near-shore measurements. It is designed to drift freely with the water and to actively control only the depth of the instrument.

Dynamic Model

The dynamic model for uFloat depth tracking is based on a force balance equation and includes terms for weight, buoyancy, damping, added mass:

$$m_{float}\ddot{z} = F_w - F_b - F_d - F_{am}$$

The forces of weight and buoyancy are calculated from measured physical properties (mass, volume, gravity and water properties).

The forces of drag and added mass are defined with the Morison force equation for a solid body accelerating relative to fluid flow direction [1].

The complete dynamics model includes measured states, physical properties, and two unknown coefficients (C_d , C_{am}) in a nonlinear differential equation:

$$m_{float}\ddot{z} = m_{fl}g - V_{fl}\rho g - \frac{1}{2}\rho A_{cs}C_d|\dot{z}|\dot{z} - C_{am}V_{fl}\rho g\ddot{z}$$

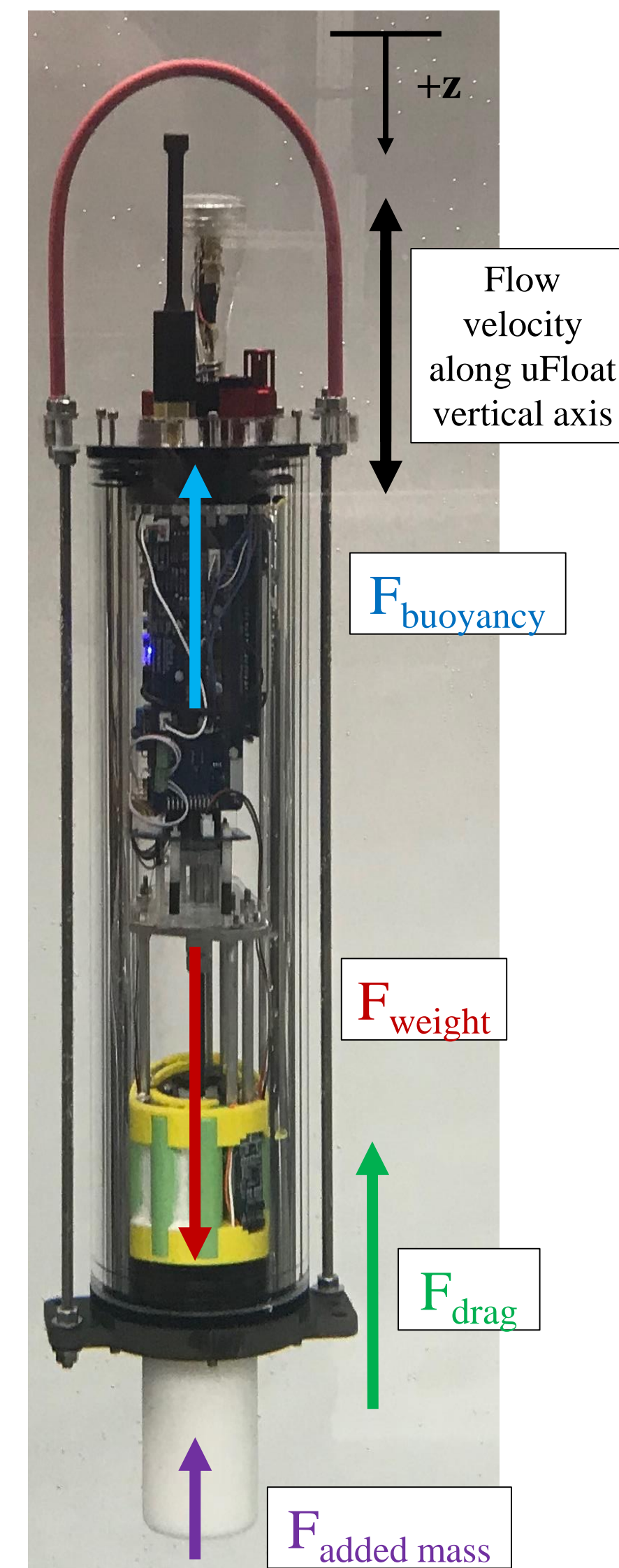


Figure 1. Free body diagram to define the uFloat dynamic equation.

Experimental Data

Experimental Data

Data was collected during the March 8, 2019 deployment of the prototype uFloat (S/N 000) in Lake Washington. The water velocity field is assumed to be zero.

The uFloat was programmed for long descents and ascents, providing subsets of time with vertical travel at a steady state velocity. Several subsections in gray were used for system identification. (Fig 2)

Test anomalies

- uFloat had a GoPro and recovery line attached (adjusted mass, volume, drag, added mass)
- In subsets 6, 7 recovery line went taught and applied an external force, may have also occurred in other subsets

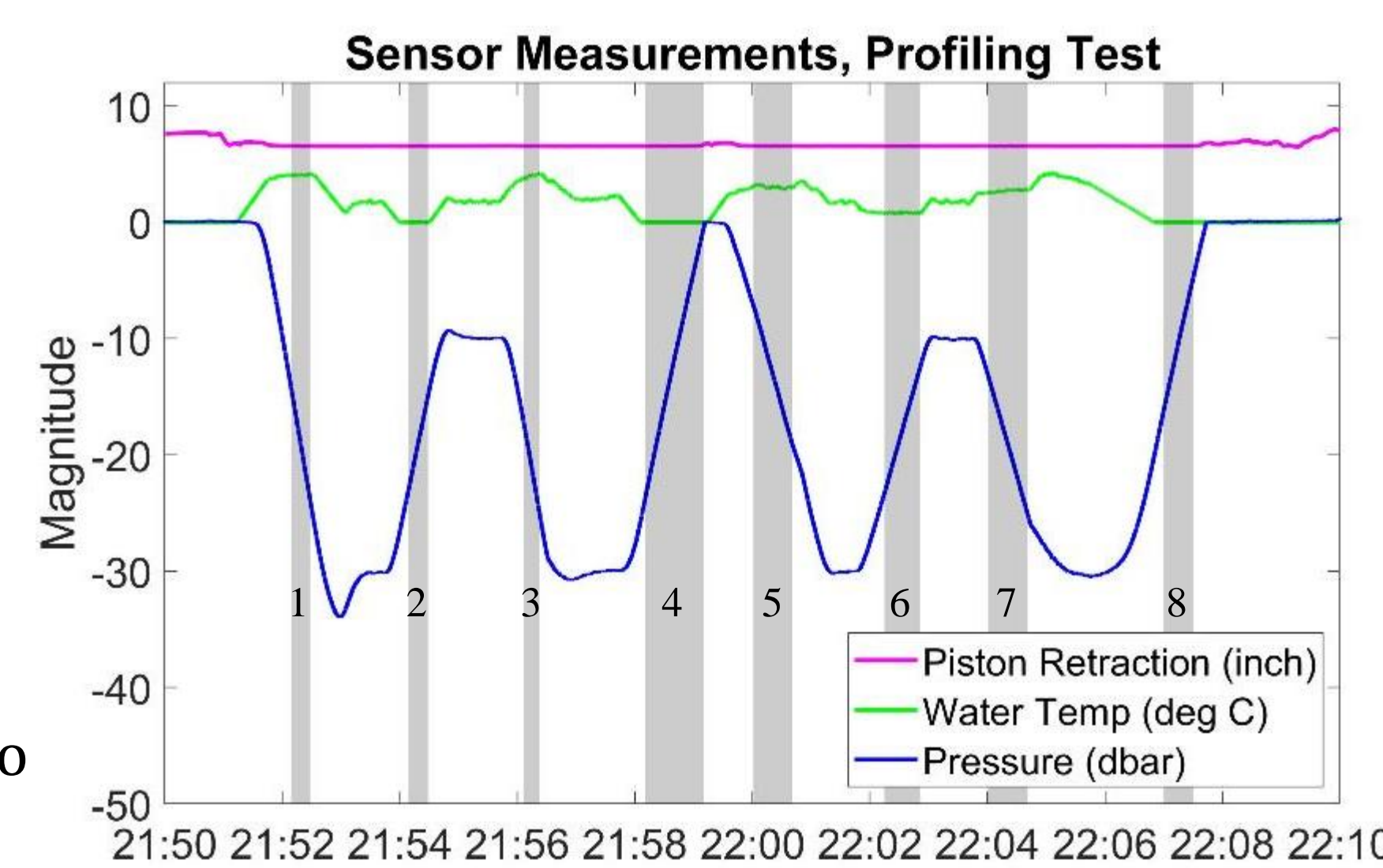


Figure 2. Subsets of profiling test used for system identification.

System Identification

Why is System Identification needed?

The uFloat nonlinear dynamics model includes two unknown coefficients related to drag and added mass that must be estimated with experimental data.

Method

1. Simplify Dynamics Equation

Using the data subsets at steady state velocity gives zero acceleration (added mass term goes to zero) leaving a simplified equation with only C_d , \dot{z} as unknown:

$$0 = m_{fl}g - V_{fl}\rho g - \frac{1}{2}\rho A_{cs}C_d|\dot{z}|\dot{z}$$

2. Find Steady State Velocity

A simple Linear Least Squares Kalman Filter estimates a constant velocity (\dot{z}) for each subset to substitute into the force balance equation for each subset.

3. Solve for Drag Coefficient

With all other properties measured or estimated, solve the algebraic equation for C_d for each subset, take a time-weighted average to find a single C_d value.

4. Solve for Added Mass Coefficient

The C_{am} value is found using the EKF observer. (Fig 4) The values of C_{am} are varied to find the best fit to the data.

Calculated Drag Coefficients, Descending uFloat								
Figure Subset Number	Time Start (hh:mm:ss)	Time End (hh:mm:ss)	Duration (s)	Mean Retraction (m)	Std Dev Retraction (\sqrt{m})	Std Dev as % of Mean	Estimated Velocity (m/s)	Calculated C_d (unitless)
1	21:52:09	21:52:29	20	0.10282	0.00147040	1.4 %	0.518	0.723
3	21:56:06	21:56:23	17	0.10107	0.00343860	3.4 %	0.483	0.801
5	22:00:01	22:00:41	40	0.07702	0.00309020	4.0 %	0.310	0.953
7**	22:04:01	22:04:41	40	0.06855	0.00251850	3.7 %	0.298**	0.649**

Time weighted average of C_d , not including Subset 7: **0.86**

** Data excluded from weighted average. Recovery line applied forcing to uFloat, zero acceleration assumption not valid.

Calculated Drag Coefficients, Ascending uFloat								
Figure Subset Number	Time Start (hh:mm:ss)	Time End (hh:mm:ss)	Duration (s)	Mean Retraction (m)	Std Dev Retraction (\sqrt{m})	Std Dev as % of Mean	Estimated Velocity (m/s)	Calculated C_d (unitless)
2*	21:54:09	21:54:29	20	0.00011	0.00011301	102.7 % *	0.416	1.240
4	21:58:10	21:59:10	60	0.00004	0.00000000	0.0 %	0.398	1.359
6**	22:02:15	22:02:51	36	0.02027	0.00136400	6.7 %	0.300**	1.494**
8	22:07:00	22:07:30	30	0.00001	0.00000000	0.0 %	0.391	1.405

Time weighted average of C_d , not including Subset 11: **1.35**

* Piston had slight retraction and extension, introducing slight accelerations during this period.

** Data excluded from weighted average. Recovery line applied forcing to uFloat, zero acceleration assumption not valid.

Results

The expected drag coefficient for a smooth cylinder with similar proportions to the uFloat is 0.85 [2]. The uFloat was close to this in descent (0.86) but shows higher drag when ascending (1.35). Similar asymmetry was observed for the added mass coefficient in ascent (0.0) and descent (3.0).

The initial small data set produced reasonable estimates. The experiment will be repeated in Lake Washington with multiple floats to produce a larger data set without interferences from a GoPro or safety line.

Nonlinear Observer

Why use an observer?

An observer can use real time sensor measurements to estimate state values that are not directly measured but are useful in control decisions. An observer also smooths measurement noise.

Method

A nonlinear Extended Kalman Filter (EKF) observer was used on the uFloat data. The EKF takes the most recent measurements and integrates the system forward in time to predict the system states at the next time step. At the next time step, it takes a difference between the expected and actual state (error covariance matrix) which is used to update a gain value (K) to reduce the error between the observer and the actual system performance. This method assumes system noise is Gaussian white noise with a normal distribution.

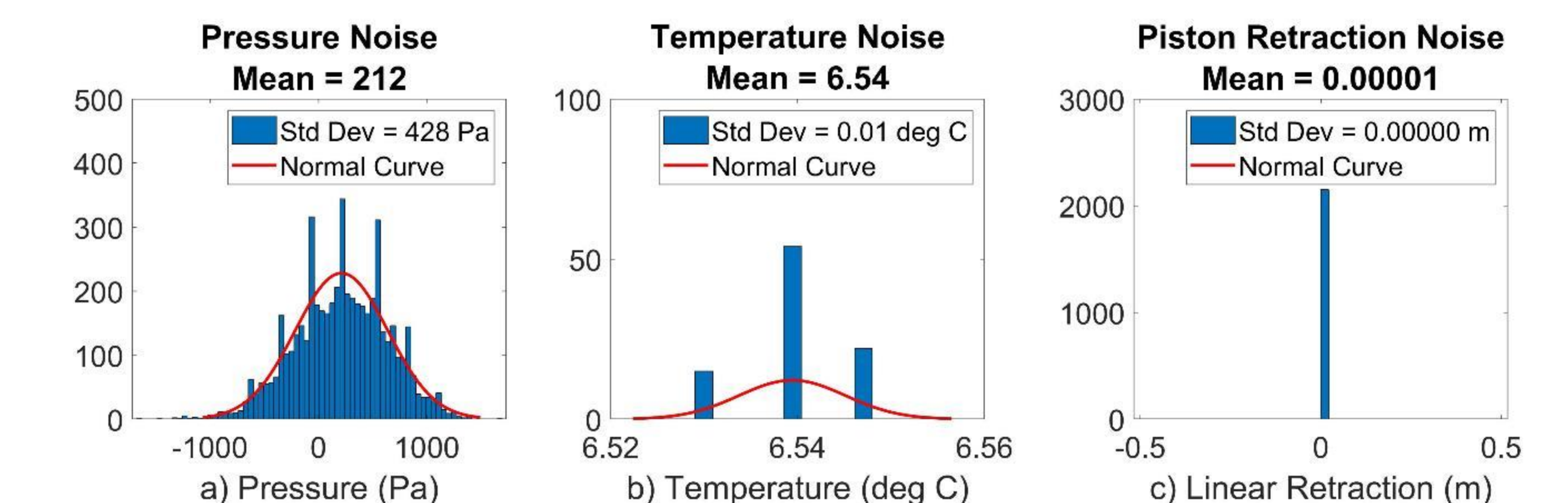


Figure 3. Statistical analysis of uFloat sensor measurements to confirm approximately normal distribution.

Results:

The EKF observer is sensitive to disturbances in the float properties (volume, mass, cross-sectional area) that will occur in normal use by adding external instruments or capturing debris in a tidal flow. The interference of the recovery line demonstrated a sensitivity to external forcing, which will occur in turbulent water. A method with better disturbance rejection may be needed.

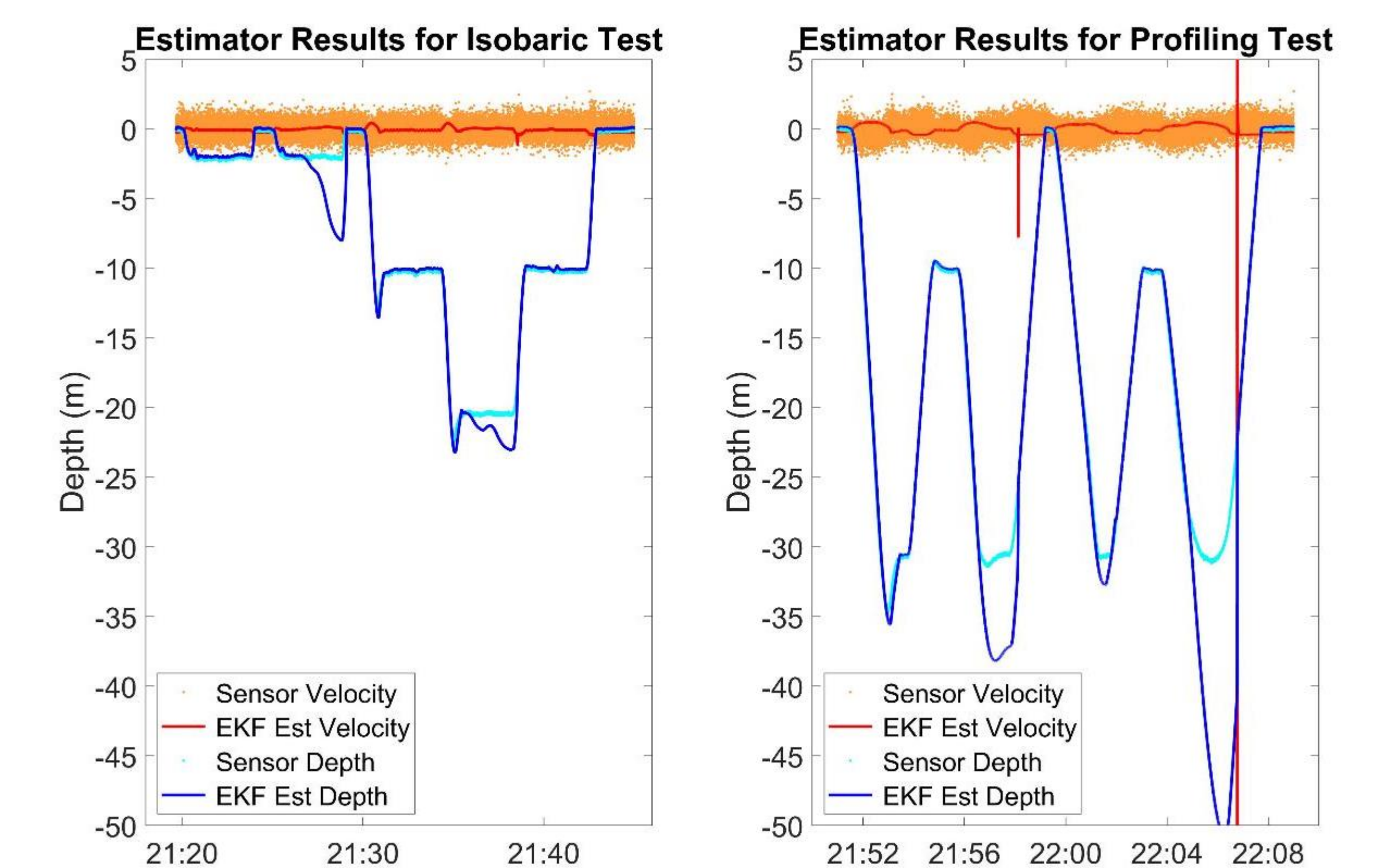


Figure 4. Comparison of EKF estimated depth, velocity with sensor measurements of depth, velocity.