# Bounding the Gains from Trade*

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**Abstract**

I propose a new approach for quantifying the gains from trade based on bounds instead of point estimates. First, I show that many specialized inputs models, where different exports use different inputs, fit any input-output dataset. Second, I show that the domestic input share in domestically sold goods determines the gains from trade. Third, I show how to compute bounds on these counterfactuals. Fourth, using the World Input-Output Database I find the bounds are wide, increasing in trade openness, and larger in multi-sector models. Fifth, I argue that measuring supply chain networks more accurately helps narrow the bounds.

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# 1 "No Nation Was Ever Ruined by Trade"[1]

But how big are the gains? Indeed, one of the quintessential questions in economics is that of quantifying the gains from trade. This entails a formidable challenge: Historical episodes featuring countries transitioning rapidly to or from autarky are few, while designing scientific experiments to identify these gains in reality are prevented by moral and practical considerations. Trade economists have thus acquiesced to measuring these gains in structural models calibrated to the world economy. Further, the field has coalesced around a tacit methodological agreement where counterfactuals are reported as point estimates obtained by shocking a specific model.

I propose a new approach based on computing bounds across a continuum of models that perfectly fit the observable data. I begin by showing that any input-output bilateral trade database can be explained by many specialized inputs models − production processes in which the use of inputs depends on the downstream use of output. These degrees of freedom imply that while different parameterizations deliver the same aggregate moments such as GDP, gross output, and bilateral trade, they might feature vastly different supply chain networks. Further, input-output data cannot identify which parameterization is closest to the true data generating process since it contains no information on which inputs are used for which output *within* a country-industry.[2]

I show that the gains from trade in specialized inputs models depend on the supply chain network as proxied by the change in the expenditure shares on domestic inputs used in the production of domestically sold goods. These shares are inverse measures of specialization that capture how many additional domestic inputs need to be called on across all stages of the supply chain to produce domestic output when trade breaks down. Hence, conditional on a given dataset and some estimate for the trade elasticity, the welfare gains may differ vastly across different parameterizations of the specialized inputs model.

I then develop an optimization procedure for computing exact lower and upper bounds on the gains from trade. In the autarky case, this approach finds the extremal domestic supply chain networks delivering the lowest and highest possible domestic input expenditure shares in domestically sold goods consistent with a given input-output dataset. These bounds contain the point estimates based on roundabout models − production processes in which all output uses the same input mix − which correspond to the knife-edge case of no input specialization. In this case, the domestic input share in domestically sold goods equals the aggregate domestic input expenditure share and the gains from trade are given by the formula of Arkolakis et al. (2012) (ACR henceforth).[3]

I implement the bounds approach using the World Input-Output Database for 2014 and show

[1] "... even seemingly the most disadvantageous," wrote Benjamin Franklin in 1774.

[2] For example, the data does not reveal how many of Mexico's imported car parts are used to produce cars sold on the domestic market and how many domestically produced car parts are used in exports.

[3] An innovative recent approach by Adao et al. (2017) computes counterfactuals non-parametrically based on trade in factor services. However, in the presence of intermediate inputs, computing the factor content of trade requires assumptions on the supply chain network. For example, Costinot and Rodríguez-Clare (2018) follow Johnson and Noguera (2012) and compute factor services imports with input-output analysis which is equivalent to assuming roundabout production technology. The bounds approach can be extended to this setting by searching for the factor services that maximize/minimize the gains from trade across all supply chain networks consistent with an input-output database.

that the bounds are wide. I document two stylized facts. First, the range of values is increasing in trade openness. For example, using a one-sector model with a trade elasticity of $-5$, the gains from trade for the U.S. (relatively closed) lie between $1.2-3.1\%$, for Germany (moderately open) between $3-13\%$, while the range for Taiwan (very open) is $3-45\%$. The intuition is that in a very open economy there are enough imports and exports to rationalize supply chain networks in which goods sold on the domestic market have very few or many domestic inputs, while in relatively closed economies there are only so many imported inputs that can be used to produce domestically sold goods. Second, the bounds are much higher and wider in multi-sector models. This is reminiscent of the similar well known effect for roundabout point estimates (Ossa 2015) but the bounds increase by relatively more for most countries. For example, with multiple sectors, the range increases to $2.8-4.0\%$ for the U.S., to $9-61\%$ for Germany, and $13-112\%$ for Taiwan.

Lastly, I show that better measurement of supply chain networks is useful for narrowing the bounds on the gains from trade. By leveraging information on supply chain linkages, such as microdata at the firm level, researchers can discard the estimates corresponding to parameterizations of specialized inputs models that, while consistent with the input-output data, are at odds with the underlying supply chain networks. This paper is thus complementary to research showing how to use the general theory of specialized inputs to improve measurement (see de Gortari 2017).

The case for specialized inputs is supported by various recent studies. Within-industry exports vary across destinations due to quality (Bastos and Silva 2010), trade regime (Dean et al. 2011), and credit constraints (Manova and Yu 2016). Likewise, the use of imports varies across firm size (Gopinath and Neiman 2014, Blaum et al. 2017a, 2017b, Antràs et al. 2017), multinational activity (Hanson et al. 2005), firm capital intensity (Schott 2004), and the quality of output (Fieler et al. 2017). Further, recent research has made explicit connections between imports and exports through quality linkages (Bastos et al. 2018), trade participation (Manova and Zhang 2012), rules-of-origin (Conconi et al. 2018), and manufacturing supply chains (de Gortari 2017). This evidence suggests that the use of inputs depends on the downstream use of output *within* a country-industry. Finally, production processes vary also in terms of the intensity of labor inputs. Processing trade firms export lower-cost labor assembly goods (De La Cruz et al. 2011, Koopman et al. 2012) while firms exporting to richer countries hire higher-skilled workers (Brambilla et al. 2012, Brambilla and Porto 2016). Thus, value-added shares may also differ depending on the destination of output.

While this paper maintains several restrictive assumptions, mainly that of perfect competition, conceptually it argues for a bottom-up approach in which counterfactuals are studied under initially broad sets of plausibly accurate models that are then refined as more information becomes available. When little is known about an important dimension $-$ the supply chain network in this case $-$ computing bounds avoids taking a top-down approach where strong theoretical assumptions are imposed outright. Indeed, parting agnostically from a more general theory and then narrowing the bounds through better measurement often yields new insights (see Popper 1959).

## 2 Specialized Inputs Meet Armington

I define specialized inputs as production processes in which the use of inputs varies across goods sold to different markets or industries. To make the exposition as clear as possible, I ignore the industrial dimension in the main text and focus on the simplest microstructure as given by Armington with perfect competition.[4] There are $\mathcal{J}$ countries and each produces $\mathcal{J}$ differentiated varieties − each tailored to a specific market. Production features constant returns to scale so that the unit price of a good sold by $j'$ to $j$ equals its marginal cost

$$p\left(j',j\right) = \left(w\left(j'\right)\right)^{\beta(j',j)} \left(\sum_{j''\in\mathcal{J}} a\left(j''\left|j',j\right.\right)\left(p\left(j'',j'\right)\tau\left(j'',j'\right)\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}(1-\beta(j',j))}. \quad (1)$$

I impose the standard unit elasticity assumption between labor and intermediate inputs, so that $\beta\left(j',j\right)$ is the value-added share and $w\left(j'\right)$ is the price of equipped labor. Intermediate inputs from different sources are imperfectly substitutable with $\sigma \geq 1$ denoting the elasticity of substitution.

Production is specialized in that $j'$ puts in specific shares of domestic value-added and inputs from each source $j''$ into its exports to each market $j$. Note, however, that input specificity is *eroded* as goods flow down the supply chain. That is, $j'$ has access to specific inputs from each source $j''$, available at the unit cost $p\left(j'',j'\right)$ times an iceberg trade cost shifter $\tau\left(j'',j'\right) \geq 1$, but can use them to produce new goods for any destination $j$. The input expenditure shares are disciplined by the parameters $a\left(j''\left|j',j\right.\right)$ and the *standard* (roundabout) Armington model with no specialized inputs is the knife-edge case with $a\left(j''\left|j',j\right.\right) = a\left(j''\left|j'\right.\right)\ \forall j \in \mathcal{J}$. I interpret this heterogeneity as a simple way of (exogenously) capturing the interdependencies across different stages and countries of the supply chain observed empirically in the aforementioned studies.[5]

I keep the final demand side as simple as possible and assume that $\varphi\left(j'\left|j\right.\right)$ disciplines the share of imports from $j'$ in the consumption of $j$. The final consumption price index equals

$$P\left(j\right) = \left(\sum_{j'\in\mathcal{J}} \varphi\left(j'\left|j\right.\right)\left(p\left(j',j\right)\tau\left(j',j\right)\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}},$$

and welfare is given by real income: $W\left(j\right) = w\left(j\right)L\left(j\right)/P\left(j\right)$.

### 2.1 A Perfect Fit to the Data

There is a continuum of parameterizations of this model that perfectly replicate the observed input-output data. While all deliver the same aggregate moments (gross output, GDP, bilateral trade) by construction, supply chain linkages vary significantly and this affects the gains from trade.

[4] Generalizing to multiple sectors is straightforward and in the appendix. The Armington microstructure is not overly restrictive since other perfect competition models have similar implications (see Antràs and de Gortari 2017). However, studying the implications of specialized inputs with imperfect competition is an important avenue for future research.

[5] For example, even if American and Chinese inputs command the same prices in Mexico, the former might be more compatible for exports to the U.S. (thus delivering lower quality-adjusted prices). These forces can be unpacked in richer microfoundations with similar implications: Antràs and de Gortari (2017) develop a Ricardian specialized inputs setting in which the difference in firms' supply chains arises endogenously. Alternatively, incorporating *trilateral* trade costs $\tau\left(j'',j',j\right)$ yields analogous results and can be used to capture modern forms of trade policy such as rules-of-origin.

To start, I build the model's world input-output table. From the price index, the expenditure share in country $j$ on final goods from each source $j'$ is given by

$$\pi_F\left(j'\left|j\right.\right) = \frac{\varphi\left(j'\left|j\right.\right)\left(p\left(j',j\right)\tau\left(j',j\right)\right)^{1-\sigma}}{\sum_{i'\in\mathcal{J}}\varphi\left(i'\left|j\right.\right)\left(p\left(i',j\right)\tau\left(i',j\right)\right)^{1-\sigma}}, \tag{2}$$

so that bilateral final good flows equal these shares times aggregate consumption or GDP

$$F\left(j',j\right) = \pi_F\left(j'\left|j\right.\right) w\left(j\right) L\left(j\right).$$

The intermediate input side is constructed by noting that a share of the exports to a given market is attributed to the inputs embedded in them. From the unit prices in equation (1), the share of inputs from source $j''$ used in the exports of $j'$ to $j$ equals

$$\pi_X\left(j''\left|j',j\right.\right) = \frac{a\left(j''\left|j',j\right.\right)\left(p\left(j'',j'\right)\tau\left(j'',j'\right)\right)^{1-\sigma}}{\sum_{i''\in\mathcal{J}}a\left(i''\left|j',j\right.\right)\left(p\left(i'',j'\right)\tau\left(i'',j'\right)\right)^{1-\sigma}}. \tag{3}$$

Aggregate intermediate input exports from $j''$ to $j'$ are then defined implicitly as

$$X\left(j'',j'\right) = \sum_{j\in\mathcal{J}}\pi_X\left(j''\left|j',j\right.\right)\left(1-\beta\left(j',j\right)\right)\left(X\left(j',j\right)+F\left(j',j\right)\right), \tag{4}$$

where the right-hand side traces the value of inputs from $j''$ by eliminating value-added in the exports of $j'$ to $j$ through the term $1-\beta\left(j',j\right)$ and then breaking up aggregate input purchases into input purchases from source $j''$ through the input shares $\pi_X\left(j''\left|j',j\right.\right)$. The summation then adds up all inputs from $j''$ used by $j'$ in exports to all markets. In practice, input flows can be computed, conditional on a set of input shares, using this equation as a fixed point.[6]

This model has enough degrees of freedom to fit the data perfectly. Conditional on any vector of iceberg trade costs $\tau\left(j',j\right)\geq 1$ and any elasticity of substitution $\sigma\geq 1$, the parameters $\varphi\left(j'\left|j\right.\right)$ adjust to match final good flows, the input shares $a\left(j''\left|j',j\right.\right)$ adjust to match intermediate input flows, and the value-added shares $\beta\left(j',j\right)$ adjust to match GDP. More importantly, there are too many degrees of freedom and so there is a continuum of parameterizations that fit the data.

The roundabout model corresponds to the knife-edge case of no specialization in which exports to all markets use the same input mix.[7] With these restrictions, equation (4) implies the well-known

[6]Alternatively, they can be computed directly with linear algebra through $\boldsymbol{X} = \boldsymbol{\pi_X}\left(\boldsymbol{1}-\boldsymbol{\beta}\right)\left[\mathbb{I}-\boldsymbol{\pi_X}\left(\boldsymbol{1}-\boldsymbol{\beta}\right)\right]^{-1}\boldsymbol{F}$. This approach is reminiscent of the Leontief inverse matrix but requires a matrix of size $\mathcal{J}^2\times\mathcal{J}^2$ instead of size $\mathcal{J}\times\mathcal{J}$.

[7]More precisely, I use the term roundabout when referring to production processes in which all output uses the same input mix *and* in which the model is implemented literally in that the sectors in the theory are mapped one-to-one to the sectors in the data (for example Costinot and Rodríguez-Clare 2014, Caliendo and Parro 2015, and Caliendo et al. 2017). More generally, the above specialized inputs model can also be interpreted as a multi-sector roundabout model in which country $j'$ has $\mathcal{J}$ sectors and in which each sector $j$ only exports to country $j$. The mapping to the data is not one-to-one, however, since the theory has $\mathcal{J}$ sectors whereas the data has one. This paper is by no means the first to take issue with the aggregation in input-output data. Rather, I show how to use the same data in new ways by constructing bounds that take the (potential) aggregation concerns into account. In the future, the advent of firm-to-firm data will make the cutting-edge approaches of Bernard et al. 2018, Lim 2017, and Tintelnot et al. 2017 more widely applicable.

property of roundabout models, and of input-output analysis more generally, that input shares are proportional to bilateral trade shares. That is

$$\pi_X\left(j''\left|j',j\right.\right)=\pi_X\left(j''\left|j'\right.\right)\ \forall j,\quad \Rightarrow \pi_X\left(j''\left|j'\right.\right)=\frac{X\left(j'',j'\right)}{\sum_{i''\in\mathcal{J}}X\left(i'',j'\right)}.$$

Hence, while roundabout models may fit the data perfectly, this *cannot* be interpreted as evidence *for* the roundabout approach since many other specialized inputs models also fit it perfectly. Moreover, input-output data contains no information identifying which networks are most accurate.

## 2.2 The Gains from Trade

Similar to ACR, specialized inputs models admit *sufficient statistics* formulas.[8] The change in welfare following an arbitrary shock to trade barriers is given by

$$\hat{W}\left(j\right)=\left[\hat{\pi}_F\left(j\left|j\right.\right)\hat{\pi}_X\left(j\left|j,j\right.\right)^{\frac{1-\beta(j,j)}{\beta(j,j)}}\right]^{\frac{1}{1-\sigma}}. \tag{5}$$

Conditional on a trade elasticity, welfare depends on: $(i)$ the change in domestic final consumption shares, $(ii)$ the change in domestic input expenditure shares in domestically sold goods, and $(iii)$ the domestic value-added shares in domestically sold goods. In the autarky case, the first share is observed, i.e. $\hat{\pi}_F\left(j\left|j\right.\right)=1/\pi_F\left(j\left|j\right.\right)$, but the latter two are not. The intuition is that domestic expenditure shares proxy how many domestic inputs can be called on to make up for the lost foreign inputs while the value-added share proxies the importance of inputs in domestically sold goods along all stages of the supply chain.

Given an input-output dataset, counterfactual predictions thus vary across different parameterizations of specialized inputs models. In the knife-edge case of roundabout production this formula delivers point estimates and can be implemented with directly observable data

$$\hat{W}\left(j\right)=\left[\hat{\pi}_F\left(j\left|j\right.\right)\hat{\pi}_X\left(j\left|j\right.\right)^{\frac{1-\beta(j)}{\beta(j)}}\right]^{\frac{1}{1-\sigma}}. \tag{6}$$

If symmetry in input and final good shares is also imposed, i.e. $\varphi\left(j'\left|j\right.\right)=a\left(j'\left|j\right.\right)$, this becomes the standard ACR formula. More generally, computing bounds is useful whenever there is no information about the underlying supply chain network and a researcher wishes to take a broader perspective on the range of counterfactuals consistent with a given dataset.

## 2.3 The Import Demand System Is Not CES

Before delving deeper into counterfactuals it is helpful to pause and analyze why specialized inputs imply that aggregate expenditure shares are insufficient for tracing the implications of changes in trade barriers. In a nutshell, this occurs because supply chains play a role in propagating trade

[8] This was first shown by Antràs and de Gortari (2017) in a multi-stage Ricardian setting.

shocks and specialized inputs determine the structure of these trade linkages. In words, if Ford Mexico exports cars to the U.S. built with domestic inputs while Volkswagen Mexico exports to Germany using Chinese imports then changes in Mexican trade costs with different partners will have asymmetric effects on input suppliers depending on the structure of Mexican supply chains.

Formally, this can be stated in terms of the restriction imposed by ACR concerning how trade shocks pass through into relative imports with third countries. To dig deeper into this, it is useful to define the auxiliary variable denoting the dollar value of inputs from source $j''$ that country $j'$ uses to produce exports for market $j$.

$$\mathcal{X}\left(j''\left|j',j\right.\right)=\pi_X\left(j''\left|j',j\right.\right)\left(1-\beta\left(j',j\right)\right)\left(X\left(j',j\right)+F\left(j',j\right)\right).$$

From equation (4), the use of inputs across all sales equals aggregate input purchases $X\left(j'',j'\right)=\sum_{j\in\mathcal{J}}\mathcal{X}\left(j''\left|j',j\right.\right)$. With these variables in hand, the partial elasticity of imports in $j'$ from source $j''\neq j'$ relative to domestic input purchases with respect to changes in trade costs with a third country $i''\neq j'$ equals

$$\frac{\partial\ln X\left(j'',j'\right)/X\left(j',j'\right)}{\partial\ln\tau\left(i'',j'\right)}=\left(1-\sigma\right)1_{[j''=i'']}+\sum_{j\in\mathcal{J}}\left(\frac{\mathcal{X}\left(j''\left|j',j\right.\right)}{X\left(j'',j'\right)}-\frac{\mathcal{X}\left(j'\left|j',j\right.\right)}{X\left(j',j'\right)}\right)\frac{\partial\ln\mathcal{X}\left(j'\left|j',j\right.\right)}{\partial\ln\tau\left(i'',j'\right)}. \quad (7)$$

The first term captures the direct effect on relative imports present when $j''=i''$. In roundabout models this is the only effect. More generally, however, supply chains play a role. The partial elasticity $\partial\ln\mathcal{X}\left(j'\left|j',j\right.\right)/\partial\ln\tau\left(i'',j'\right)$ captures the change in domestic input purchases due to both a substitution effect capturing a shift in imports from $i''$ to domestic inputs in exports to $j$ and a supply chain effect derived from the change in downstream production as proxied by the changing level of exports to $j$. That is

$$\frac{\partial\ln\mathcal{X}\left(j'\left|j',j\right.\right)}{\partial\ln\tau\left(i'',j'\right)}=-\left(1-\sigma\right)\pi_X\left(i''\left|j',j\right.\right)+\frac{\partial\ln\left(X\left(j',j\right)+F\left(j',j\right)\right)}{\partial\ln\tau\left(i'',j'\right)}.$$

Further, the term in parenthesis in equation (7) amplifies/dampens the effect on relative imports depending on the differential importance of each export market $j$ for inputs from $j''$ relative to $j'$.

In words, if Mexico's exports to Germany use mostly Chinese inputs, then a reduction in shipping costs between Mexico and Germany will reduce both imports from China and domestic input sales following the substitution towards more German inputs. But imports from China will fall relatively more since Germany was a more important export market for Chinese inputs than for Mexican inputs. However, this is counteracted by the supply chain effect which increases Chinese imports relatively more following the increase in exports to Germany.

Hence, the supply chain effect illustrates how changes in trade barriers with third countries affect imports asymmetrically depending on the depth of supply chain integration. In contrast, the roundabout model is the knife-edge case in which supply chain linkages are symmetric. In other words, when Mexico uses the exact same input mix to produce all exports this model satisfies what

ACR define as the import demand system being CES in which case the asymmetric effect of trade costs on relative imports operating through supply chain linkages disappears. Formally

$$\frac{\mathcal{X}\left(j''\left|j',j\right.\right)}{X\left(j'',j'\right)}=\frac{\mathcal{X}\left(j'\left|j',j\right.\right)}{X\left(j',j'\right)}\ \forall j\quad\Rightarrow\frac{\partial\ln X\left(j'',j'\right)/X\left(j',j'\right)}{\partial\ln\tau\left(i'',j'\right)}=\left(1-\sigma\right)1_{\left[j''=i''\right]}.$$

The empirical evidence on supply chain linkages suggests that specialized inputs play a crucial role in propagating trade shocks. For example, Barrot and Sauvagnat (2016), Carvalho et al. (2016), and Boehm et al. (2018) show that supply chain disruptions due to natural disasters are propagated by input specificity through trade networks. Increases in suppliers' marginal costs mostly affect tightly-linked firms, rather than entire industries symmetrically as in roundabout models.

Finally, note that the gravity equation's empirical success is not grounds in favor of the roundabout model. I show in the appendix that gravity regressions fare well across simulations of the specialized inputs model even though structural gravity does not hold. While third country trade costs do shift bilateral trade flows, on aggregate the bilateral terms dominate. The overall effect is to attenuate the trade elasticity since this model misspecification, i.e. incorrectly assuming structural gravity, is similar to introducing classical measurement error. This suggests that gravity-based trade elasticity estimates are biased downwards when deep supply chain linkages are pervasive.

## 3 Bounding the Gains from Trade

I now develop a practical procedure for computing bounds on the welfare gains from trade.

### 3.1 Bounds with Common Value-Added Shares

I begin with a restricted version with common value-added shares given by the data as $\beta\left(j'\right)=GDP\left(j'\right)/GO\left(j'\right)$. Conditional on an input-output dataset $X\left(j',j\right)$ and $F\left(j',j\right)$, the lowest gains from trade relative to autarky for country $j'$ in any model delivering a welfare equation as in equation (5) are found through

$$\begin{aligned}\underset{\left\{\pi_X\left(j''\left|j',j\right.\right)\right\}_{j'',j\in\mathcal{J}}}{\text{minimize}}\quad & -\ln\pi_X\left(j'\left|j',j'\right.\right),\\ \text{subject to}\quad & X\left(j'',j'\right)=\sum_{j\in\mathcal{J}}\pi_X\left(j''\left|j',j\right.\right)\left(1-\beta\left(j'\right)\right)\left(X\left(j',j\right)+F\left(j',j\right)\right),\ \forall j'',\\ & \sum_{j''\in\mathcal{J}}\pi_X\left(j''\left|j',j\right.\right)=1,\ \forall j,\\ & \pi_X\left(j''\left|j',j\right.\right)\geq 0,\ \forall j'',j.\end{aligned}\tag{8}$$

The objective function minimizes the gains from trade in equation (5) where the endogenous variables are the input shares from each source country for exports to each destination, i.e. $\pi_X\left(j''\left|j',j\right.\right)$ for each $j'',j\in\mathcal{J}$. The constraints restrict the search to expenditure shares that perfectly fit the data. This optimization problem is relatively easy to solve since the objective function is wellbehaved and the constraints are linear.[9] The upper bound is found through maximization.

Crucially, computing the bounds requires only zooming in on all possible supply chain networks *within* country $j'$. That is, while the world economy depends on $\mathcal{J} \times \mathcal{J} \times \mathcal{J}$ shares, the optimization solves only for $\mathcal{J} \times \mathcal{J}$ endogenous variables. This occurs because this model features little specialization in that country $j'$ buys specific inputs from $j''$, but can then use them to produce exports to any market. Hence, the linkages through specialized inputs of country $j'$ extend at most from its immediate import suppliers to its direct export markets. In other words, the observed bilateral trade flows to and from country $j'$ curtail its specialized network and computing the bounds requires only searching for extremal domestic supply chain linkages.[10] This is illustrated for a simple two-country network in figure 1.

### 3.2 Bounds with Destination-Specific Value-Added Shares

In reality, value-added shares might also vary across exports to different markets. Computing the bounds is more intricate though since input shares in exports to different markets translate into different dollar amounts depending on how much domestic value is added directly to each type of exports. This complicates the optimization since now both the input and value-added shares need to be solved for. However, I show in the appendix that solving for the input shares net-of-value-added defined as $\tilde{\pi}_X(j'' | j', j) = \pi_X(j'' | j', j)(1 - \beta(j', j))$ is sufficient and that the original variables can then be recovered from the latter. This trick reduces the size of the problem from $\mathcal{J} \times (\mathcal{J} + 1)$ endogenous variables to only $\mathcal{J} \times \mathcal{J}$, but, more importantly, it transforms the bilateral trade constraints in equation (4) from quadratic to linear in the endogenous variables.

### 3.3 Bounds with Respect to Arbitrary Changes in Trade Barriers

The bounds approach can also be extended to arbitrary changes in trade barriers. However, this is far more challenging numerically since it involves optimizing a nonlinear function subject to nonlinear constraints. This occurs because computing the bounds across two trade equilibria requires solving for the counterfactual equilibrium as well.

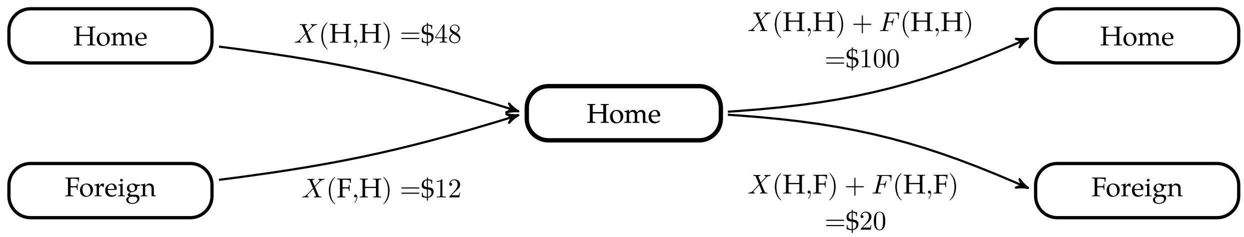
Following the hat-algebra spirit of Dekle et al. (2007) the change in expenditure shares equals

$$\hat{\pi}_X\left(j'' \left| j', j\right.\right) = \frac{\left(\hat{p}(j'', j')\,\hat{\tau}(j'', j')\right)^{1-\sigma}}{\sum_{i'' \in \mathcal{J}} \pi_X(i'' | j', j)\left(\hat{p}(i'', j')\,\hat{\tau}(i'', j')\right)^{1-\sigma}}. \tag{9}$$

In the case of autarky, the counterfactual equilibrium is known since $\hat{\tau}(j'', j') = \infty$ for all $j'' \neq j'$ implies $\hat{\pi}_X(j'' | j', j) = 1/\pi_X(j'' | j', j)$ when $j'' = j'$ and zero otherwise. However, with finite changes in trade costs, computing the change in expenditure shares requires imposing nonlinear

[9] Actually, in this simple case it is quicker to minimize $\pi_X(j' | j', j')$ directly. I keep the $\ln$ here to note that it is required for computing bounds in the more general cases discussed below.

[10] This breaks down with a higher degree of specialization. For example, suppose that instead $a(j''' | j'', j', j)$ disciplines the share of inputs from $j'''$ used by country $j''$ to produce specialized inputs for $j'$ that are further tailored for the downstream market $j$. This would deliver wider bounds, but comes at a cost in dimensionality (see de Gortari 2017).




| | Roundabout | Lower Bound | Upper Bound |
|---|---|---|---|
| $\pi_X$ (H \|H, H) | 80% | 96% | 76% |
| $\pi_X$ (H \|H, F) | 80% | 0% | 100% |
| $\mathcal{X}$ (H \|H, H) | $40 | $48 | $38 |
| $\mathcal{X}$ (H \|H, F) | $8 | $0 | $10 |
| $\hat{W}$ (H) | 7.6% | 3.7% | 8.7% |

**Figure 1:** Domestic Networks in a Simple Home vs Foreign Example: The value-added share equals $\$60/\$120 = 0.5$. Note that home is a relatively closed economy and so the upper bound is mechanically close to the roundabout point estimates. That is, the roundabout shares assign a lot of domestic inputs into all output and so many domestic inputs can be shifted out of exports into domestically sold goods ($8) but few domestic inputs can be shifted into exports from domestically sold goods ($2). The gains from trade are relative to autarky with $1 - \sigma = -5$.

constraints that implicitly define the changes in unit prices and wages. Hence, whereas the autarky bounds are given by the supply chain networks delivering the lowest/highest *shares* of domestic inputs in domestically sold goods, here the bounds depend on the supply chain networks delivering the lowest/highest *changes* in these shares. While this optimization problem is well-defined, in practice it may be hard to solve numerically.[11] The appendix reports the full derivation.

# 4 Bounds in the 2014 World Input-Output Database

I illustrate the bounds approach to counterfactuals using the World Input-Output Database (WIOD) for the year 2014 (Timmer et al. 2015). This data provides input-output bilateral trade data for 44 countries and 56 sectors per country. Throughout I focus on the gains relative to autarky.

## 4.1 Bounds with a Single Sector

I start by computing the bounds when aggregating the data to a single sector per country. Note the optimization problems do not depend on the trade elasticity, but the latter is necessary for transforming the solutions into bounds. It is well known that estimating elasticities involves many

[11] The bounds approach is not only applicable to the gains from trade. For example, bounds can be computed on the extremal partial trade elasticities in equation (7) consistent with a given dataset.

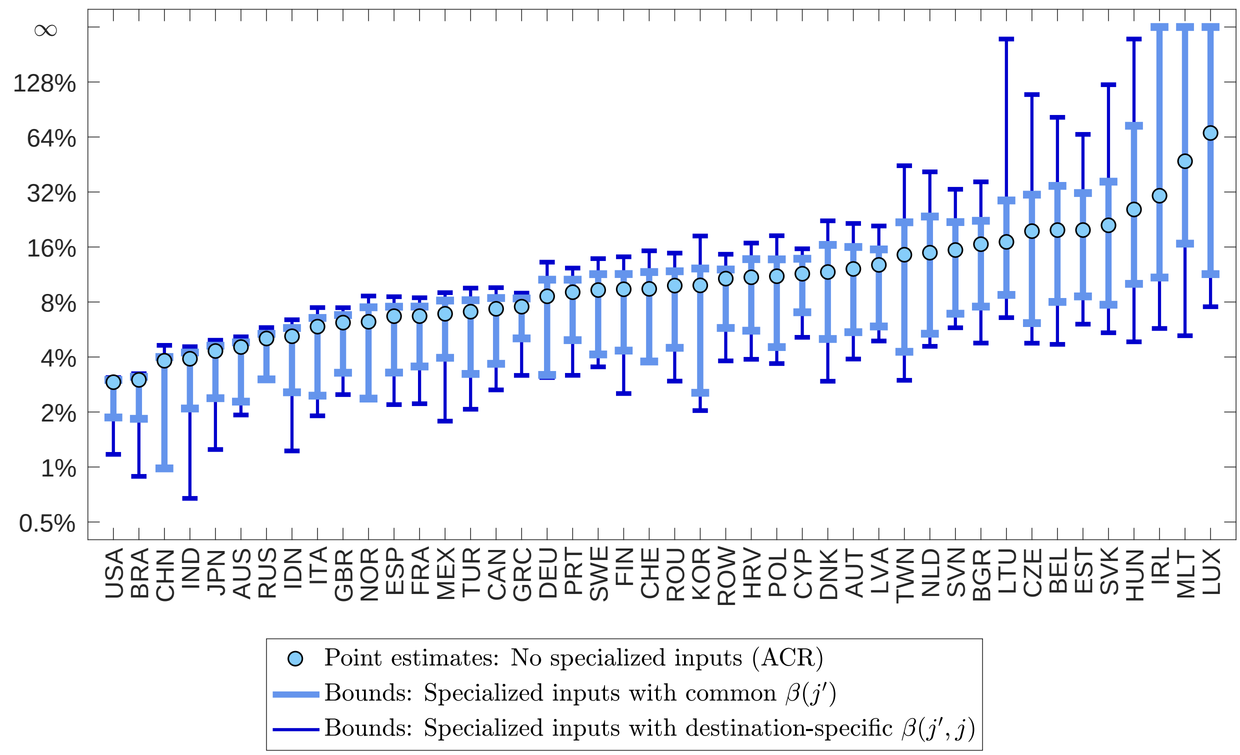



**Figure 2:** Welfare Gains from Trade Relative to Autarky: Data is from the WIOD for 2014 and is aggregated to the country level. All counterfactuals use a roundabout trade elasticity of $1 - \sigma = -5$. Bounds correspond to specialized inputs models as in equation (5). Point estimates correspond to the roundabout model in equation (6). Note the log scale.

identification threats and, more importantly, that the elasticity is *model dependent* (Melitz and Redding 2015). While measuring elasticities in specialized inputs models is a fascinating future research topic, it is beyond this paper's scope and so I simply set a roundabout trade elasticity of $1 - \sigma = -5$, in line with mainstream estimates (Anderson and van Wincoop 2003, Costinot and Rodríguez-Clare 2014, Head and Mayer 2014).

Figure 2 plots the gains from trade relative to autarky in the roundabout model (ACR) and in specialized inputs models with both common and destination-specific value-added shares (note the log scale). Since the latter class of models nest the former the bounds are wider. Any value within the bounds is feasible since the optimization constraints are linear and any convex combination of the lower and upper bounds is a possible initial trade equilibrium. Further, the reader can transform any of these numbers $x$ to her preferred elasticity $1-\sigma$ through $(1+x)^{(1-6)/(1-\sigma)} - 1$.

The bounds on the gains from trade are wide and increasing in trade openness. For example, the ACR gains for the U.S., a relatively closed economy with only $10\%$ of its total inputs imported from abroad, are low at $2.9\%$ while the range with destination-specific value-added shares lies between $1.2 - 3.1\%$ indicating that the gains might actually be $60\%$ lower or $10\%$ higher. The range is relatively small, however, with a ratio between the upper and lower bounds of $2.6$. In contrast, very open economies are consistent with a wide range of domestic supply chain networks since one can find both trade equilibria in which goods sold domestically use either mostly domestic inputs or almost no domestic inputs. For example, Taiwan imports about $40\%$ of its inputs from abroad and has a bounds ratio of $45\%/3\% = 15$. The full results are reported in the appendix.

The main takeaway is that the bounds approach lets researchers study counterfactuals using input-output data without having to rely on the strong assumptions of the roundabout model. Importantly, since the data is always matched perfectly, it contains no further information on which set of specialized inputs models best captures the world's supply chain networks. This suggests that focusing on point estimates may lead to specific biases as is exemplified in figure 2 through the mechanical correlation where distance between the ACR gains and the upper bound increases with trade openness. This occurs because trade equilibria in which domestically sold goods use arbitrarily few domestic goods (a high upper bound) can only be found in countries that trade a lot (figure 1 provides further intuition). In practice, extremely open economies like the small European markets on the right of figure 2 feature upper bounds that are quite literally off the charts.

### 4.2 Bounds with Multiple Sectors

It is well documented that multi-sector models deliver larger gains from trade (Ossa 2015, Costinot and Rodríguez-Clare 2014). I now extend the bounds approach to this setting; I discuss only the welfare formula in the main text and relegate all other derivations to the appendix.

There are $\mathcal{K}$ sectors per country so that every country-sector produces $\mathcal{J}$ differentiated varieties, one for each market.[12] The change in welfare in country $j$ following a shock to trade costs equals

$$\hat{W} = \prod_{k \in \mathcal{K}} \left( \hat{\pi}_F\left(k\right)^{\frac{1}{1-\sigma(k)}} \prod_{k' \in \mathcal{K}} \prod_{k'' \in \mathcal{K}} \hat{\pi}_X\left(k'' \left| k' \right.\right)^{\frac{\gamma(k'',k')\delta(k',k)}{1-\sigma(k'')}} \right)^{\alpha(k)}. \tag{10}$$

All terms are $j$-specific but I omit the explicit dependence to lighten notation. I impose the standard assumption that both production and final consumption is Cobb-Douglas across sectors with $\gamma\left(k'', k'\right)$ the share of expenditure on sector $k''$ inputs when producing sector $k'$ goods (in country $j$) and with $\alpha\left(k\right)$ the final expenditure share on sector $k$ goods (in country $j$). I focus on common shares across destinations since computing bounds with destination-specific shares in a multi-sector setting involves highly nonlinear optimization. The term $\delta\left(k'', k'\right)$ summarizes all direct and indirect domestic production paths between sectors $k''$ and $k'$ and is defined with linear algebra as $\boldsymbol{\delta} = \left[\mathbb{I} - \boldsymbol{\gamma}\right]^{-1}$. Varieties from different sources are imperfectly substitutable with elasticity $\sigma\left(k\right) \geq 1$. Finally, the relevant expenditure changes are $\hat{\pi}_F\left(k\right)$, the share of sector $k$ final consumption purchased domestically, and $\hat{\pi}_X\left(k'' \left| k' \right.\right)$, the share of sector $k''$ inputs purchased domestically for the production of sector $k'$ goods that are sold domestically. As before, the final expenditure shares are directly observable but the input expenditures for domestically sold goods are not.

Figure 3 shows that the multi-sector bounds are wide and (mostly) increasing in trade openness in the thirty largest economies in the WIOD.[13] The previous mechanical correlation, however, has

[12]More generally, each country-sector could produce a differentiated variety for every other country-sector. This delivers wider bounds but requires taking a stand on how consumers aggregate varieties produced for different sectors.

[13]In practice, I only let the input shares be as low as $10^{-8}$ or else tiny sectors often lead to infinite upper bounds.

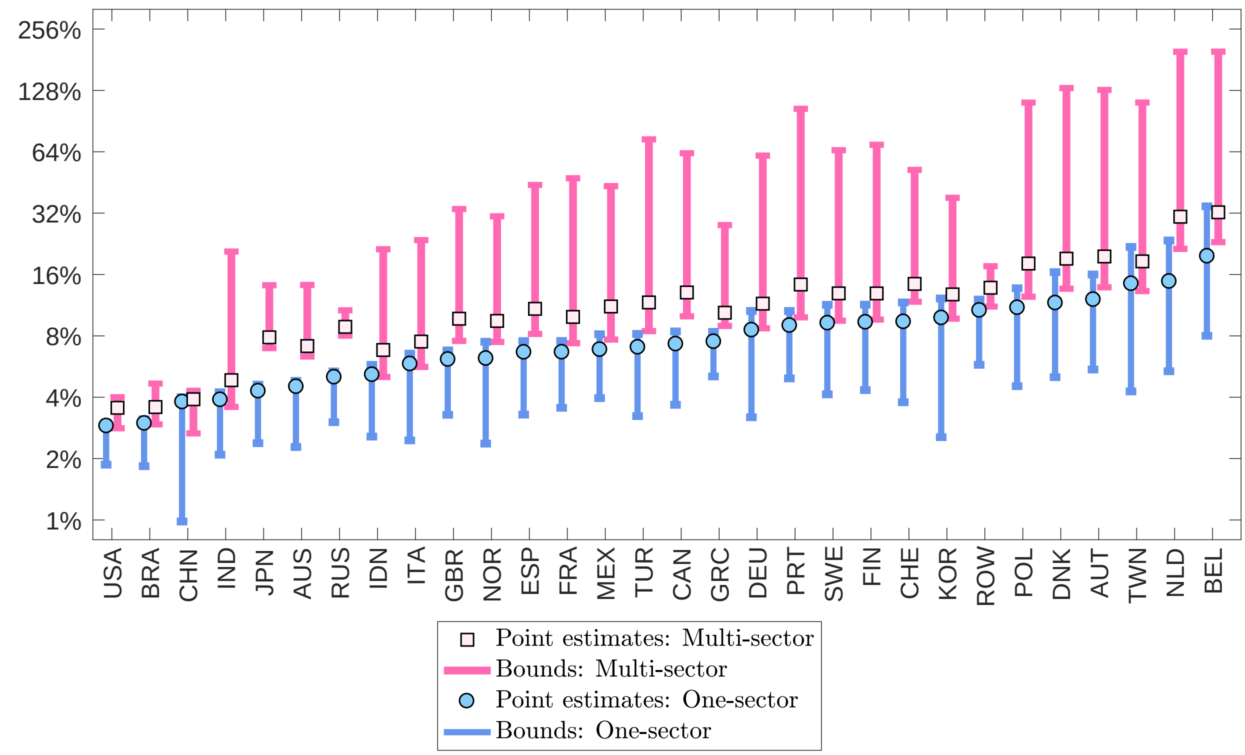



**Figure 3:** Welfare Gains from Trade Relative to Autarky, Multiple Sectors: Data is from the WIOD for 2014 and is aggregated to $\mathcal{K} = 45$ sectors per country (see appendix). All counterfactuals use common roundabout trade elasticities $1 - \sigma(k) = -5$ in all sectors. Bounds correspond to multi-sector specialized inputs models as in equation (10). Point estimates correspond to the roundabout model. Note the log scale.

disappeared and now the point estimates are close to the lower bound.

The multi-sector bounds are much larger and overlap little with the one-sector bounds because heterogeneity in openness across sectors leads to disproportionate effects on the gains from trade. To see why, note that I imposed a common roundabout trade elasticity $1 - \sigma(k) = -5$ across all sectors since this implies that if the input-output data across all pairs of sectors were identical (in shares) to the aggregate one-sector data then the bounds would be exactly the same. This does not occur since even countries that are relatively closed on aggregate often have some very open sectors consistent with many different supply chain networks, thus implying wide bounds. That cross-sector heterogeneity matters is reminiscent of Ossa (2015), who emphasizes that sectors with low elasticities of substitution are highly sensitive to trade costs. In future work, computing bounds with elasticities measured in the context of specialized inputs may produce even wider bounds.

## 5 Narrowing the Bounds with Better Measurement

Computing bounds on counterfactuals is not the endgame, but rather a first step to be complemented by better measurement. In previous research (de Gortari 2017), I showed how researchers

Alternatively, the bounds could be computed with a smaller number of sectors to avoid this issue.

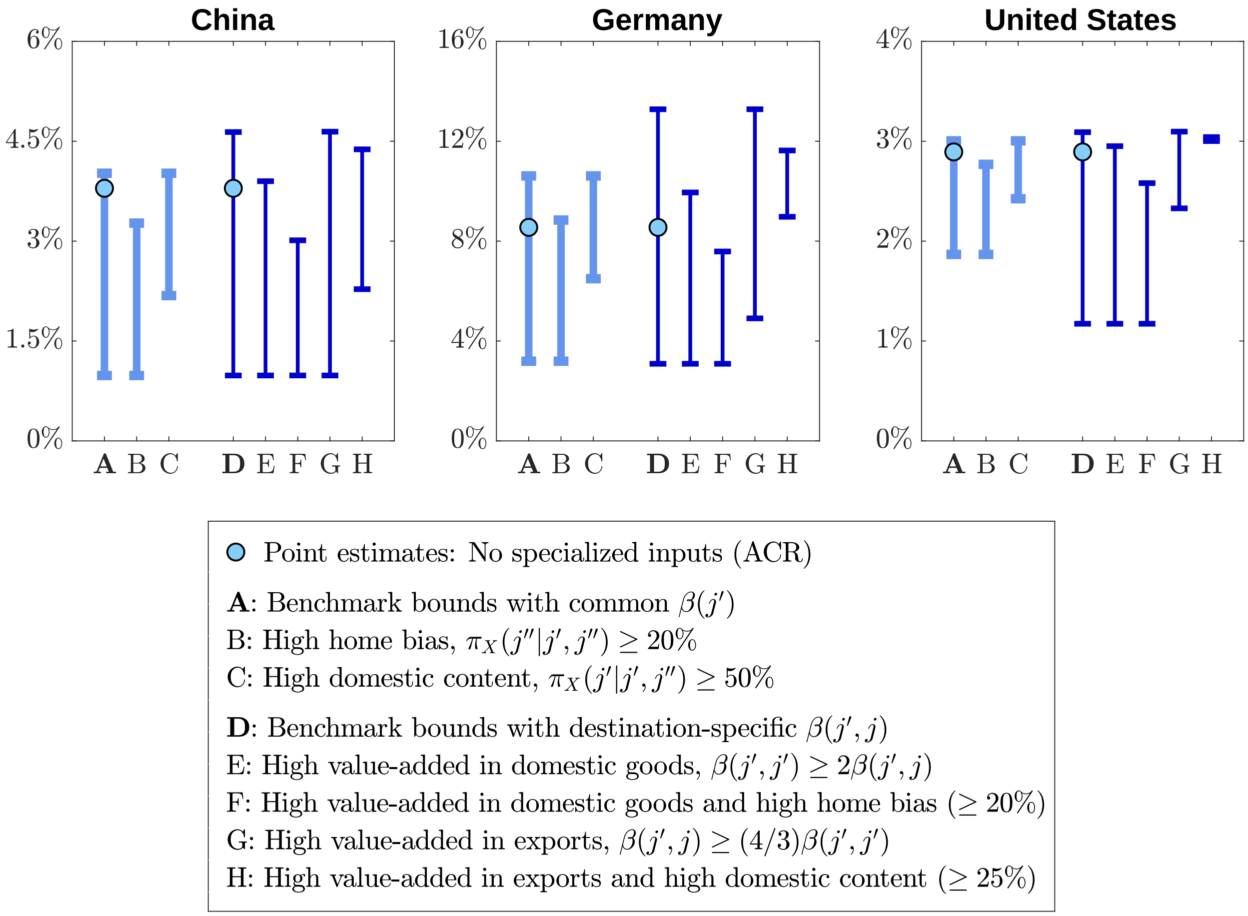



**Figure 4:** Narrowing the Bounds on the Welfare Gains from Trade Relative to Autarky: The benchmark bounds in **A** and **D** correspond to the ranges in figure 2. The bounds in B and C impose common value-added shares and include additional restrictions. The bounds in E, F, G, and H incorporate destination-specific value-added shares and include additional restrictions. In every exercise, I omit the restrictions on the domestic shares $\pi_X(j'|j',j')$ in order to not restrict the ranges directly. Further, in order to get feasible solutions, I only apply the restrictions to the top trading partners.

can use the theory of specialized inputs to measure supply chain networks more accurately. Without delving into *how* to do better measurement, I now show *why* it is useful. Again, for clarity, I revert back to the single-sector bounds.

Suppose that after thoughtful and meticulous measurement, new information about the supply chain networks underlying input-output data is revealed. This can be used to narrow the bounds on counterfactuals by informing *which* specialized inputs parameterizations to reject.

Take China, Germany, and the United States – the largest economies of the American, Asian, and European continents. Figure 4 shows how figure 2's ranges become smaller when imposing additional restrictions into the optimization problem in (8). For example, suppose that after careful analysis of firm-level data, it becomes clear that exports to a given market use a high share of imports from that same market. Column B shows that the upper bounds tend to fall, relative to the benchmark in $\mathbf{A}$, when imposing a *home-bias* share, $\pi_X(j''|j', j'')$, of at least $20\%$ in exports since now fewer intermediate input imports can be put into goods sold on the domestic market.[14] Alternatively, suppose firms offshore production to countries that produce particularly suitable inputs. Column C shows that imposing a share of at least $50\%$ of domestic inputs in all exports increases the lower bound since now a certain amount of domestic inputs must be exported.

Measuring value-added shares more accurately also delivers sharper bounds. For example, De La Cruz et al. (2011) and Koopman et al. (2012) show that when processing trade is pervasive, exports often contain little domestic value-added. Column E presents the ranges when imposing twice as much domestic value-added in domestically sold goods than in exported goods, while column F further restricts the range by imposing a high share of inputs from the same market to which goods are exported to. The upper bound falls dramatically because both restrictions require that each dollar of exports contain a high share of imported inputs. Alternatively, Kee and Tang (2016) have documented an upward trend in the share of domestic value in Chinese exports over the last decade as China has begun exploiting its comparative advantage in high domestic content industries. Column H shows that the lower bounds increase when imposing a high domestic value-added share and also a high domestic input share in exports since disrupting trade becomes more costly once domestic consumption becomes more tightly linked with international trade.

## 6 Concluding Thoughts

This paper's message is twofold. First, I proposed a bounds approach to counterfactuals based on the fact that input-output data is consistent with a continuum of supply chain networks. In practice, I showed how to measure the gains from trade within a class of specialized inputs models and developed a practical way for constructing bounds conditional on a trade elasticity.

Second, I argued that better measurement and knowledge of supply chain networks can help reduce the range of possible estimates. Moreover, in practice the type of restrictions imposed matters. For example, seven out of nine ranges in columns B, F, and H in figure 4, and three out of three for the U.S., do not cover the benchmark roundabout (ACR) point estimates.

Finally, while in this first pass I have implemented the latter idea only illustratively in figure 4 there are two important things to note. First, since the data contains no information on which supply chain network is closest to the real world's data generating process, it strikes me that a priori any point within the ranges, including the roundabout estimates, should be treated with the equal amount of confidence or skepticism. Second, the roundabout approach is inherently theoretically-based while the bounds approach is partly empirically-based when backed by better measurement. I hope that this discussion has highlighted the versatility and usefulness of applying this procedure to more finely tuned restrictions based on systematic empirical analysis in future research.

[14]This could be driven by multinational firms offshoring a production stage within otherwise domestic supply chains (Hanson et al. 2005), or by compatibility in quality (Bastos et al. 2018), or rules-of-origin (Conconi et al. 2018).

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# A Data and Results

## A.1 WIOD Industries

In order to compute the multi-sector bounds I aggregate the WIOD's 56 sectors slightly in order to eliminate some very small sectors. Otherwise, the input-output data features several zero domestic input flows which drives the gains from trade to infinity; note that this is an issue with CES production and also drives the roundabout gains to infinity. The aggregation is described in the following table.

| Industries | Raw WIOD | | Aggregated WIOD | |
|---|---|---|---|---|
| | # | % of GDP | # | % of GDP |
| Crop, animal production, hunting, related service activities | 1 | 3.05 | 1 | 3.05 |
| Forestry and logging | 2 | 0.23 | 2 | 0.50 |
| Fishing and aquaculture | 3 | 0.27 | 2 | 0.50 |
| Mining and quarrying | 4 | 3.70 | 3 | 3.70 |
| Manufacture of food products, beverages and tobacco | 5 | 4.27 | 4 | 4.27 |
| Manufacture of textiles, wearing apparel and leather | 6 | 1.73 | 5 | 1.73 |
| Manufacture of wood and of products of wood and cork | 7 | 0.60 | 6 | 1.55 |
| Manufacture of paper and paper products | 8 | 0.63 | 6 | 1.55 |
| Printing and reproduction of recorded media | 9 | 0.31 | 6 | 1.55 |
| Manufacture of coke and refined petroleum products | 10 | 2.39 | 7 | 2.39 |
| Manufacture of chemicals and chemical products | 11 | 2.65 | 8 | 2.65 |
| Manufacture of basic pharmaceutical products | 12 | 0.77 | 9 | 0.77 |
| Manufacture of rubber and plastic products | 13 | 1.10 | 10 | 1.10 |
| Manufacture of other non-metallic mineral products | 14 | 1.23 | 11 | 1.23 |
| Manufacture of basic metals | 15 | 2.80 | 12 | 2.80 |
| Manufacture of fabricated metal products | 16 | 1.54 | 13 | 1.54 |
| Manufacture of computer, electronic and optical products | 17 | 2.51 | 14 | 2.51 |
| Manufacture of electrical equipment | 18 | 1.47 | 15 | 1.47 |
| Manufacture of machinery and equipment n.e.c. | 19 | 2.21 | 16 | 2.41 |
| Manufacture of motor vehicles, trailers and semi-trailers | 20 | 2.81 | 17 | 2.81 |
| Manufacture of other transport equipment | 21 | 0.91 | 18 | 0.91 |
| Manufacture of furniture; other manufacturing | 22 | 0.76 | 19 | 0.76 |
| Repair and installation of machinery and equipment | 23 | 0.20 | 16 | 2.41 |
| Electricity, gas, steam and air conditioning supply | 24 | 3.31 | 20 | 3.31 |
| Water collection, treatment and supply | 25 | 0.23 | 21 | 0.58 |
| Sewerage; waste collection, treatment and disposal | 26 | 0.35 | 21 | 0.58 |
| Construction | 27 | 7.53 | 22 | 7.53 |
| Wholesale and retail trade and repair of motor vehicles | 28 | 0.87 | 23 | 0.87 |
| Wholesale trade, except of motor vehicles and motorcycles | 29 | 4.86 | 24 | 4.86 |
| Retail trade, except of motor vehicles and motorcycles | 30 | 3.14 | 25 | 3.14 |
| Land transport and transport via pipelines | 31 | 2.56 | 26 | 2.56 |
| Water transport | 32 | 0.42 | 27 | 0.42 |
| Air transport | 33 | 0.48 | 28 | 0.48 |
| Warehousing and support activities for transportation | 34 | 1.00 | 29 | 1.00 |
| Postal and courier activities | 35 | 0.23 | 30 | 0.23 |

| Industries | Raw WIOD | | Aggregated WIOD | |
|---|---|---|---|---|
| | # | % of GDP | # | % of GDP |
| Accommodation and food service activities | 36 | 2.37 | 31 | 2.37 |
| Publishing activities | 37 | 0.40 | 32 | 0.85 |
| Motion picture, video, television, sound recording, music | 38 | 0.45 | 32 | 0.85 |
| Telecommunications | 39 | 1.50 | 33 | 1.50 |
| Computer programming, consultancy, information service | 40 | 1.30 | 34 | 1.30 |
| Financial service, except insurance and pension funding | 41 | 2.89 | 35 | 4.74 |
| Insurance, reinsurance and pension funding | 42 | 1.32 | 35 | 4.74 |
| Activities auxiliary to financial services and insurance | 43 | 0.52 | 35 | 4.74 |
| Real estate activities | 44 | 5.38 | 36 | 5.38 |
| Legal, accounting; head offices; management consultancy | 45 | 2.20 | 37 | 2.20 |
| Architectural and engineering; technical testing and analysis | 46 | 0.71 | 38 | 2.25 |
| Scientific research and development | 47 | 0.50 | 38 | 2.25 |
| Advertising and market research | 48 | 0.34 | 38 | 2.25 |
| Other professional, scientific and technical; veterinary | 49 | 0.71 | 38 | 2.25 |
| Administrative and support service | 50 | 2.29 | 39 | 2.29 |
| Public administration, defense, compulsory social security | 51 | 5.44 | 40 | 5.44 |
| Education | 52 | 2.31 | 41 | 2.31 |
| Human health and social work activities | 53 | 4.04 | 42 | 4.04 |
| Other service activities | 54 | 2.10 | 43 | 2.22 |
| Activities of households as employers | 55 | 0.12 | 43 | 2.22 |
| Activities of extraterritorial organizations and bodies | 56 | 0.00 | 43 | 2.22 |

**Table 1:** World Input-Output Database Industrial Classification: The shares refer to percent of world GDP.

## A.2 The Gains from Trade

| Country | % of World GDP | Aggregate Domestic Share | One-Sector | | | | | Multi-Sector | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | ACR | Common Bounds | | Dest.-Spec. Bounds | | ACR | Common Bounds | |
| **AUS** | 1.8 | 87.9 | 4.5 | 2.3 | 4.8 | 1.9 | 5.2 | 7.1 | 6.3 | 14.2 |
| **AUT** | 0.5 | 68.6 | 12.0 | 5.5 | 16.0 | 3.9 | 21.5 | 19.6 | 13.9 | 128.7 |
| **BEL** | 0.7 | 57.6 | 19.6 | 8.0 | 34.6 | 4.7 | 82.0 | 32.3 | 23.1 | 198.3 |
| **BGR** | 0.1 | 67.3 | 16.4 | 7.5 | 22.3 | 4.8 | 36.4 | 26.5 | 19.7 | 223.0 |
| **BRA** | 3.0 | 88.2 | 3.0 | 1.8 | 3.1 | 0.9 | 3.2 | 3.6 | 3.0 | 4.7 |
| **CAN** | 2.3 | 78.3 | 7.3 | 3.7 | 8.4 | 2.6 | 9.6 | 13.1 | 10.0 | 63.0 |
| **CHE** | 0.9 | 76.9 | 9.3 | 3.8 | 11.7 | 3.8 | 15.3 | 14.4 | 11.8 | 52.1 |
| **CHN** | 13.8 | 93.6 | 3.8 | 1.0 | 4.0 | 1.0 | 4.6 | 3.9 | 2.7 | 4.3 |
| **CYP** | 0.0 | 68.5 | 11.3 | 7.0 | 13.8 | 5.1 | 15.6 | 19.9 | 18.2 | 51.6 |
| **CZE** | 0.3 | 64.4 | 19.3 | 6.1 | 30.9 | 4.8 | 109.3 | 31.4 | 21.1 | 261.2 |
| **DEU** | 4.8 | 76.0 | 8.5 | 3.2 | 10.6 | 3.1 | 13.2 | 11.5 | 8.7 | 61.3 |
| **DNK** | 0.4 | 65.1 | 11.6 | 5.0 | 16.4 | 2.9 | 22.3 | 19.1 | 13.6 | 131.5 |
| **ESP** | 1.7 | 80.3 | 6.6 | 3.3 | 7.5 | 2.2 | 8.6 | 10.9 | 8.2 | 44.0 |
| **EST** | 0.0 | 60.3 | 19.6 | 8.6 | 31.6 | 6.1 | 66.2 | 55.9 | 37.9 | 287.9 |
| **FIN** | 0.3 | 74.1 | 9.3 | 4.3 | 11.4 | 2.5 | 14.1 | 12.9 | 9.6 | 69.3 |
| **FRA** | 3.5 | 79.0 | 6.6 | 3.5 | 7.6 | 2.2 | 8.4 | 9.9 | 7.4 | 47.5 |
| **GBR** | 3.7 | 82.1 | 6.1 | 3.3 | 6.8 | 2.5 | 7.4 | 9.7 | 7.6 | 33.5 |
| **GRC** | 0.3 | 75.1 | 7.5 | 5.1 | 8.4 | 3.2 | 8.9 | 10.4 | 9.0 | 28.0 |
| **HRV** | 0.1 | 69.7 | 10.8 | 5.6 | 13.7 | 3.9 | 16.8 | 24.9 | 20.5 | 121.6 |
| **HUN** | 0.2 | 48.6 | 25.4 | 10.1 | 73.7 | 4.8 | 814.7 | 39.9 | 22.7 | 474.5 |
| **IDN** | 1.2 | 81.8 | 5.1 | 2.6 | 5.8 | 1.2 | 6.4 | 6.8 | 5.0 | 21.3 |
| **IND** | 2.8 | 83.9 | 3.9 | 2.1 | 4.2 | 0.7 | 4.6 | 4.9 | 3.6 | 20.7 |
| **IRL** | 0.3 | 39.7 | 30.2 | 10.9 | $\infty$ | 5.7 | $\infty$ | 37.7 | 26.2 | 262.6 |
| **ITA** | 2.6 | 83.4 | 5.8 | 2.5 | 6.5 | 1.9 | 7.5 | 7.5 | 5.6 | 23.6 |
| **JPN** | 6.0 | 85.3 | 4.3 | 2.4 | 4.6 | 1.2 | 4.9 | 7.9 | 7.0 | 14.2 |
| **KOR** | 1.8 | 78.2 | 9.8 | 2.5 | 12.2 | 2.0 | 18.4 | 12.8 | 9.7 | 38.1 |
| **LTU** | 0.1 | 55.8 | 16.9 | 8.7 | 28.8 | 6.6 | 606.4 | 26.3 | 19.8 | 176.2 |
| **LUX** | 0.1 | 41.1 | 66.7 | 11.4 | $\infty$ | 7.5 | $\infty$ | 99.5 | 65.4 | 3788.2 |
| **LVA** | 0.0 | 74.6 | 12.6 | 5.9 | 15.5 | 4.9 | 20.9 | 32.3 | 26.0 | 140.3 |
| **MEX** | 1.6 | 71.6 | 6.8 | 4.0 | 8.1 | 1.8 | 9.0 | 11.2 | 7.7 | 43.4 |
| **MLT** | 0.0 | 41.1 | 46.7 | 16.7 | $\infty$ | 5.2 | $\infty$ | 59.9 | 42.9 | 770.9 |
| **NLD** | 1.1 | 63.2 | 14.7 | 5.4 | 23.5 | 4.6 | 41.3 | 30.7 | 21.4 | 198.1 |
| **NOR** | 0.6 | 78.0 | 6.2 | 2.4 | 7.5 | 2.4 | 8.6 | 9.5 | 7.5 | 30.9 |
| **POL** | 0.7 | 74.6 | 11.0 | 4.5 | 13.7 | 3.7 | 18.4 | 18.1 | 12.5 | 111.5 |
| **PRT** | 0.3 | 74.3 | 9.0 | 4.9 | 10.6 | 3.2 | 12.3 | 14.3 | 9.9 | 104.0 |
| **ROU** | 0.2 | 75.4 | 9.7 | 4.5 | 11.8 | 3.0 | 14.8 | 14.0 | 11.6 | 51.8 |
| **RUS** | 2.3 | 90.6 | 5.0 | 3.0 | 5.4 | 3.0 | 5.8 | 8.9 | 8.0 | 10.7 |

| Country | % of World GDP | Aggregate Domestic Share | One-Sector | | | | | Multi-Sector | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | ACR | Common Bounds | | Dest.-Spec. Bounds | | ACR | Common Bounds | |
| **SVK** | 0.1 | 59.5 | 20.8 | 7.7 | 36.5 | 5.4 | 123.7 | 40.9 | 28.4 | 318.5 |
| **SVN** | 0.1 | 65.1 | 15.3 | 6.9 | 21.9 | 5.8 | 33.1 | 24.4 | 16.3 | 185.1 |
| **SWE** | 0.7 | 74.5 | 9.2 | 4.1 | 11.4 | 3.5 | 13.8 | 12.9 | 9.5 | 65.1 |
| **TUR** | 1.0 | 78.7 | 7.0 | 3.2 | 8.2 | 2.1 | 9.5 | 11.7 | 8.4 | 73.6 |
| **TWN** | 0.7 | 66.4 | 14.4 | 4.3 | 21.8 | 3.0 | 44.6 | 18.6 | 13.3 | 111.6 |
| **USA** | 23.1 | 89.7 | 2.9 | 1.9 | 3.0 | 1.2 | 3.1 | 3.6 | 2.8 | 4.0 |
| **ROW** | 14.2 | 79.5 | 10.6 | 5.8 | 12.1 | 3.8 | 14.6 | 13.8 | 11.1 | 17.6 |
| **Mean** | 2.3 | 72.0 | 12.8 | 5.2 | 14.9 | 3.4 | 56.2 | 20.6 | 14.9 | 201.9 |
| **Weighted** | 10.6 | 83.8 | 6.3 | 3.0 | 7.2 | 2.1 | 10.9 | 8.8 | 6.8 | 31.8 |

**Table 2:** Welfare Gains from Trade Relative to Autarky: Data is from the WIOD for 2014. The aggregate domestic share refers to the aggregate share of inputs purchased domestically and is a good proxy for trade openness. Common bounds refers to common value-added shares across destinations in the one-sector case and common value-added and sector-level expenditure shares across destinations in the multiple-sectors case.

# B Mathematical Derivations

## B.1 Specialized Inputs Meet Armington: Single Sector

### B.1.1 The Gains from Trade

Domestic prices can be written in terms of domestic input expenditure shares as

$$p(j,j) = \left(w(j)\right)^{\beta(j,j)} \left(\frac{a(j\,|j,j)\,(p(j,j)\,\tau(j,j))^{1-\sigma}}{\pi_X(j\,|j,j)}\right)^{\frac{1}{1-\sigma}(1-\beta(j,j))},$$
$$= w(j) \left(\frac{a(j\,|j,j)\,\tau(j,j)^{1-\sigma}}{\pi_X(j\,|j,j)}\right)^{\frac{1}{1-\sigma}\frac{1-\beta(j,j)}{\beta(j,j)}}.$$

Similarly the price index can be written as

$$P(j) = \left(\frac{\varphi(j\,|j)}{\pi_F(j\,|j)}\right)^{\frac{1}{1-\sigma}} p(j,j)\,\tau(j,j),$$
$$= \left(\frac{\varphi(j\,|j)}{\pi_F(j\,|j)}\right)^{\frac{1}{1-\sigma}} w(j) \left(\frac{a(j\,|j,j)}{\pi_X(j\,|j,j)}\right)^{\frac{1}{1-\sigma}\frac{1-\beta(j,j)}{\beta(j,j)}} \tau(j,j)^{\frac{1}{\beta(j,j)}},$$

where I have substituted in the domestic price. Welfare thus equals

$$W(j) = \left[\frac{\pi_F(j\,|j)}{\varphi(j\,|j)} \left(\frac{\pi_X(j\,|j,j)}{a(j\,|j,j)}\right)^{\frac{1-\beta(j,j)}{\beta(j,j)}}\right]^{\frac{1}{1-\sigma}} \tau(j,j)^{-\frac{1}{\beta(j,j)}}.$$

The change with respect to an arbitrary shock to foreign trade costs equals

$$\hat{W}(j) = \left[\hat{\pi}_F(j\,|j)\,\hat{\pi}_X(j\,|j,j)^{\frac{1-\beta(j,j)}{\beta(j,j)}}\right]^{\frac{1}{1-\sigma}}.$$

### B.1.2 The Import Demand System is Not CES

Take $j'' \neq j'$ and $i'' \neq j'$. Using the definition of bilateral input flows

$$\frac{\partial \ln X(j'',j')}{\partial \ln \tau(i'',j')} = \sum_{j\in\mathcal{J}} \frac{\mathcal{X}(j''\,|j',j)}{X(j'',j')} \frac{\partial \ln \mathcal{X}(j''\,|j',j)}{\partial \ln \tau(i'',j')}.$$

From the definition of $\mathcal{X}(j''\,|j',j)$

$$\frac{\partial \ln \mathcal{X}(j''\,|j',j)}{\partial \ln \tau(i'',j')} = \frac{\partial \ln \pi_X(j''\,|j',j)}{\partial \ln \tau(i'',j')} + \frac{\partial \ln (X(j',j)+F(j',j))}{\partial \ln \tau(i'',j')},$$
$$= \frac{\partial \ln \mathcal{X}(j'\,|j',j)}{\partial \ln \tau(i'',j')} + \frac{\partial \ln \pi_X(j''\,|j',j)}{\partial \ln \tau(i'',j')} - \frac{\partial \ln \pi_X(j'\,|j',j)}{\partial \ln \tau(i'',j')}.$$

From the definition of the input expenditure share

$$\frac{\partial \ln \pi_X(j''\,|j',j)}{\partial \ln \tau(i'',j')} = (1-\sigma)\left(1_{[j''=i'']} - \pi_X(i''\,|j',j)\right).$$

Substituting these two equations into the ratio of bilateral imports yields

$$\begin{aligned}
&\frac{\partial \ln X\left(j'',j'\right)/X\left(j',j'\right)}{\partial \ln \tau\left(i'',j'\right)} \\
&\quad=\sum_{j\in\mathcal{J}}\left(\frac{\mathcal{X}\left(j''\left|j',j\right.\right)}{X\left(j'',j'\right)}\frac{\partial \ln \mathcal{X}\left(j''\left|j',j\right.\right)}{\partial \ln \tau\left(i'',j'\right)}-\frac{\mathcal{X}\left(j'\left|j',j\right.\right)}{X\left(j',j'\right)}\frac{\partial \ln \mathcal{X}\left(j'\left|j',j\right.\right)}{\partial \ln \tau\left(i'',j'\right)}\right), \\
&\quad=\sum_{j\in\mathcal{J}}\left(\frac{\mathcal{X}\left(j''\left|j',j\right.\right)}{X\left(j'',j'\right)}\left(\frac{\partial \ln \mathcal{X}\left(j'\left|j',j\right.\right)}{\partial \ln \tau\left(i'',j'\right)}+\left(1-\sigma\right)1_{\left[j''=i''\right]}\right)-\frac{\mathcal{X}\left(j'\left|j',j\right.\right)}{X\left(j',j'\right)}\frac{\partial \ln \mathcal{X}\left(j'\left|j',j\right.\right)}{\partial \ln \tau\left(i'',j'\right)}\right), \\
&\quad=\left(1-\sigma\right)1_{\left[j''=i''\right]}+\sum_{j\in\mathcal{J}}\left(\frac{\mathcal{X}\left(j''\left|j',j\right.\right)}{X\left(j'',j'\right)}-\frac{\mathcal{X}\left(j'\left|j',j\right.\right)}{X\left(j',j'\right)}\right)\frac{\partial \ln \mathcal{X}\left(j'\left|j',j\right.\right)}{\partial \ln \tau\left(i'',j'\right)}.
\end{aligned}$$

### B.1.3 Bounds with Destination-Specific Value-Added Shares

Define the input shares net-of-value-added as $\tilde{\pi}_X\left(j''\left|j',j\right.\right)=\pi_X\left(j''\left|j',j\right.\right)\left(1-\beta\left(j',j\right)\right)$. The lower bound on the gains from trade in equation (5) for country $j'$ is obtained through

$$\begin{aligned}
\underset{\left\{\tilde{\pi}_X\left(j''\left|j',j\right.\right)\right\}_{j'',j\in\mathcal{J}}}{\text{minimize}}\quad & \left(1-\frac{1}{1-\sum_{j''}\tilde{\pi}_X\left(j''\left|j',j'\right.\right)}\right)\ln\left(\frac{\tilde{\pi}_X\left(j'\left|j',j'\right.\right)}{\sum_{j''}\tilde{\pi}_X\left(j''\left|j',j'\right.\right)}\right), \\
\text{subject to}\quad & X\left(j'',j'\right)=\sum_{j\in\mathcal{J}}\tilde{\pi}_X\left(j''\left|j',j\right.\right)\left(X\left(j',j\right)+F\left(j',j\right)\right),\ \forall j'', \\
& \sum_{j''\in\mathcal{J}}\tilde{\pi}_X\left(j''\left|j',j\right.\right)\le 1,\ \forall j, \\
& \tilde{\pi}_X\left(j''\left|j',j\right.\right)\ge 0,\ \forall j'',j,
\end{aligned}\tag{11}$$

As in the case with common value-added shares, the linear constraints restrict the search to domestic production networks consistent with the observable data but are now in terms of the input shares net-of-value-added. The objective function is more complicated since it includes the domestic value-added share in addition to the domestic input share. The original variables can be recovered from the latter as $\pi_X\left(j''\left|j',j\right.\right)=\tilde{\pi}_X\left(j''\left|j',j\right.\right)/\sum_{i''\in\mathcal{J}}\tilde{\pi}_X\left(i''\left|j',j\right.\right)$ and $\beta\left(j',j\right)=1-\sum_{j''\in\mathcal{J}}\tilde{\pi}_X\left(j''\left|j',j\right.\right)$.

### B.1.4 Bounds with Respect to an Arbitrary Shock to Trade Barriers

Let $\hat{\tau}\left(j'',j'\right)$ be a matrix capturing any changes in trade costs. The change in expenditure shares is given by

$$\hat{\pi}_F\left(j''\left|j'\right.\right)=\frac{\left(\hat{p}\left(j'',j'\right)\hat{\tau}\left(j'',j'\right)\right)^{1-\sigma}}{\sum_{i''\in\mathcal{J}}\pi_F\left(i''\left|j'\right.\right)\left(\hat{p}\left(i'',j'\right)\hat{\tau}\left(i'',j'\right)\right)^{1-\sigma}},$$

and

$$\hat{\pi}_X\left(j''\left|j',j\right.\right)=\frac{\left(\hat{p}\left(j'',j'\right)\hat{\tau}\left(j'',j'\right)\right)^{1-\sigma}}{\sum_{i''\in\mathcal{J}}\pi_X\left(i''\left|j',j\right.\right)\left(\hat{p}\left(i'',j'\right)\hat{\tau}\left(i'',j'\right)\right)^{1-\sigma}}.$$

The change in prices is pinned down through

$$\hat{p}\left(j',j\right)=\left(\hat{w}\left(j'\right)\right)^{\beta\left(j',j\right)}\left(\sum_{j''\in\mathcal{J}}\pi_X\left(j''\left|j',j\right.\right)\left(\hat{p}\left(j'',j'\right)\hat{\tau}\left(j'',j'\right)\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}\left(1-\beta\left(j',j\right)\right)},\tag{12}$$

while the change in wages is pinned down through

$$w\left(j'\right)\hat{w}\left(j'\right)=\sum_{j}\left(X\left(j',j\right)\hat{X}\left(j',j\right)+\hat{F}\left(j',j\right)F\left(j',j\right)-\hat{X}\left(j,j'\right)X\left(j,j'\right)\right).\tag{13}$$

Finally, the change in bilateral trade flows is given by

$$\hat{F}(j', j) = \hat{\pi}_F(j' | j) \, \hat{w}(j),$$

and

$$X(j'', j') \hat{X}(j'', j') = \sum_{j \in \mathcal{J}} \pi_X(j'' | j', j) \, \hat{\pi}_X(j'' | j', j) \, (1 - \beta(j')) \left( \hat{X}(j', j) X(j', j) + \hat{F}(j', j) F(j', j) \right). \tag{14}$$

The bounds on the welfare gains from trade are found by minimizing/maximizing the objective function

$$\ln \hat{W}(j') = \frac{1}{1-\sigma} \left[ \ln \hat{\pi}_F(j' | j') + \left( \frac{1 - \beta(j', j)}{\beta(j', j)} \right) \ln \hat{\pi}_X(j' | j', j') \right],$$

subject to the original linear constraints in (8), and to the fixed points for prices in equation (12), for wages in equation (13), and for intermediate input flows in equation (14). The endogenous variables are the benchmark equilibrium expenditure shares $\pi_X(j'' | j', j)$, the change in prices $\hat{p}(j', j)$, the change in wages $\hat{w}(j')$, and the change in intermediate input flows $\hat{X}(j', j)$. The other variables, namely the change in expenditure shares $\hat{\pi}_F(j'' | j')$ and $\hat{\pi}_X(j'' | j', j)$ and the change in final good flows $\hat{F}(j', j)$, can be written in terms of the endogenous variables.

## B.2 Specialized Inputs Meet Armington: Multiple Sectors

### B.2.1 Production and Preferences

There are $\mathcal{J}$ countries and $\mathcal{K}$ sectors per country. Each country-sector, defined as a pair $\{j', k'\} \in \mathcal{J} \times \mathcal{K}$, produces $\mathcal{J}$ differentiated varieties. One for each destination market. The unit price of country-sector $\{j', k'\}$ goods sold to country $j$ is given by

$$p(\{j', k'\}, j) = \left(w(j')\right)^{\beta(\{j',k'\})} \prod_{k'' \in \mathcal{K}} \left( \sum_{j'' \in \mathcal{J}} a(\{j'', k''\} | \{j', k'\}, j) \left( p(\{j'', k''\}, j') \, \tau(\{j'', k''\}, j') \right)^{1-\sigma(k'')} \right)^{\frac{\gamma(k'' \{j', k'\})}{1-\sigma(k'')}}.$$

I have assumed that the aggregate use of inputs from different sectors is Cobb-Douglas with $\beta(\{j', k'\})$ the value-added share and $\gamma(k'', \{j', k'\})$ the expenditure on sector $k''$ inputs such that

$$\beta(\{j', k'\}) + \sum_{k'' \in \mathcal{K}} \gamma(k'', \{j', k'\}) = 1.$$

Further, I have assumed that both of these shares are common across destination markets. Generalizing to destination-specific shares is straightforward but makes the bounds approach very hard to implement numerically and so I ignore these forces; though note that this would deliver even wider bounds. Finally, $\sigma(k'') \geq 1$ is the elasticity of substitution across sector $k''$ inputs from different sources, $\tau(\{j'', k''\}, j')$ is the iceberg trade cost of shipping goods from country-sector $\{j'', k''\}$ to $j'$, and $a(\{j'', k''\} | \{j', k'\}, j)$ disciplines the expenditure share on inputs from each source.

As in production, preferences are Cobb-Douglas in consumption across sectors with $\alpha(k', j)$ the share of final good expenditure of country $j$ on sector $k'$ goods. The price index equals

$$P(j) = \prod_{k' \in \mathcal{K}} \left( \sum_{j' \in \mathcal{J}} \varphi(\{j', k'\} | j) \left( p(\{j', k'\}, j) \, \tau(\{j', k'\}, j) \right)^{1-\sigma(k')} \right)^{\frac{\alpha(k', j)}{1-\sigma(k')}}.$$

The share of sector $k'$ goods purchased from a specific source equals

$$\pi_F(\{j',k'\}|j) = \frac{\varphi(\{j',k'\}|j)\left(p(\{j',k'\},j)\,\tau(\{j',k'\},j)\right)^{1-\sigma(k')}}{\sum_{i'\in\mathcal{J}}\varphi(\{i',k'\}|j)\left(p(\{i',k'\},j)\,\tau(\{i',k'\},j)\right)^{1-\sigma(k')}},$$

with $\sum_{j'\in\mathcal{J}}\pi_F(\{j',k'\}|j) = 1$. Final good flows are given by

$$F(\{j',k'\},j) = \pi_F(\{j',k'\}|j)\,\alpha(k',j)\,w(j)\,L(j).$$

Intermediate input flows can be defined implicitly as in the one-sector model. The expenditure share on sector $k''$ inputs from country $j''$ equals

$$\pi_X(\{j'',k''\}|\{j',k'\},j) = \frac{a(\{j'',k''\}|\{j',k'\},j)\left(p(\{j'',k''\},j')\,\tau(\{j'',k''\},j')\right)^{1-\sigma(k'')}}{\sum_{i''\in\mathcal{J}}a(\{i'',k''\}|\{j',k'\},j)\left(p(\{i'',k''\},j')\,\tau(\{i'',k''\},j')\right)^{1-\sigma(k'')}},$$

with $\sum_{j''\in\mathcal{J}}\pi_X(\{j'',k''\}|\{j',k'\},j) = 1$. Bilateral intermediate input flows satisfy

$$X(\{j'',k''\},\{j',k'\}) = \sum_{j\in\mathcal{J}}\pi_X\left(\{j'',k''\}\middle|\{j',k'\},j\right)\gamma(k'',\{j',k'\})\left(\sum_{k\in\mathcal{K}}X(\{j',k'\},\{j,k\}) + F(\{j',k'\},j)\right).$$

The term on the right traces the value of inputs from $\{j'',k''\}$ used in the production of $\{j',k'\}$. Specifically, the term in parenthesis is gross exports of country-sector $\{j',k'\}$ to market $j$. The term $\gamma(k'',\{j',k'\})$ imputes the aggregate value of inputs from sector $k''$ used for this production. The term $\pi_X\left(\{j'',k''\}\middle|\{j',k'\},j\right)$ imputes the value of inputs from source $j''$. The summation adds the use of inputs in exports to all markets.

### B.2.2 The Gains from Trade

The change in welfare in country $j$ following an arbitrary shock to trade barriers equals

$$\hat{W}(j) = \hat{w}(j)\prod_{k\in\mathcal{K}}\left(\frac{\hat{\pi}_F(\{j,k\}|j)^{\frac{1}{1-\sigma(k)}}}{\hat{p}(\{j,k\},j)}\right)^{\alpha(k,j)},$$

where the change in unit prices is the solution to the following fixed point

$$\hat{p}(\{j,k\},j) = \left(\hat{w}(j)\right)^{\beta(\{j,k\})}\prod_{k'\in\mathcal{K}}\left(\frac{\hat{p}(\{j,k'\},j)}{\hat{\pi}_X(\{j,k'\}|\{j,k\},j)^{\frac{1}{1-\sigma(k')}}}\right)^{\gamma(k',\{j,k\})}.$$

I now solve for the change in unit prices. Since everything is in terms of domestic variables in country $j$, I simplify notation by eliminating the dependence on $j$ and take logs of unit prices to obtain

$$\ln\hat{p}(k) = \beta(k)\ln\hat{w} + \sum_{k'\in\mathcal{K}}\gamma(k',k)\left(\ln\hat{p}(k') - \frac{1}{1-\sigma(k')}\ln\hat{\pi}_X(k'|k)\right).$$

Define the auxiliary variable

$$\ln\hat{\zeta}(k) = \beta(k)\ln\hat{w} - \sum_{k'\in\mathcal{K}}\frac{\gamma(k',k)}{1-\sigma(k')}\ln\hat{\pi}_X(k'|k).$$

The equation for prices can be written using linear algebra as

$$\ln \hat{\boldsymbol{p}} = \boldsymbol{\gamma}^T \ln \hat{\boldsymbol{p}} + \ln \hat{\boldsymbol{\zeta}},$$

which implies that

$$\ln \hat{\boldsymbol{p}} = \left[\mathbb{I} - \boldsymbol{\gamma}^T\right]^{-1} \ln \hat{\boldsymbol{\zeta}}.$$

I now substitute prices into the welfare equation. Start by defining the auxiliary matrix $\boldsymbol{\delta}^T = \left[\mathbb{I} - \boldsymbol{\gamma}^T\right]^{-1}$. Log prices equal

$$\ln \hat{p}(k) = \sum_{k' \in \mathcal{K}} \delta(k', k) \left( \beta(k') \ln \hat{w} - \sum_{k'' \in \mathcal{K}} \frac{\gamma(k'', k')}{1 - \sigma(k'')} \ln \hat{\pi}_X(k'' | k') \right).$$

Log welfare equals

$$\ln \hat{W} = \ln \hat{w} + \sum_{k \in \mathcal{K}} \alpha(k) \left( \frac{1}{1 - \sigma(k)} \ln \hat{\pi}_F(k) - \sum_{k' \in \mathcal{K}} \delta(k', k) \left( \beta(k') \ln \hat{w} - \sum_{k'' \in \mathcal{K}} \frac{\gamma(k'', k')}{1 - \sigma(k'')} \ln \hat{\pi}_X(k'' | k') \right) \right).$$

From the definition of $\boldsymbol{\delta}$, the following relation holds

$$\begin{aligned} \delta(k', k) &= 1_{[k'=k]} + \gamma(k', k) + \sum_{k'' \in \mathcal{K}} \gamma(k', k'') \gamma(k'', k) + \sum_{k'' \in \mathcal{K}} \sum_{k''' \in \mathcal{K}} \gamma(k', k'') \gamma(k'', k''') \gamma(k''', k) + \ldots, \\ &= 1_{[k'=k]} + \sum_{k'' \in \mathcal{K}} \gamma(k', k'') \delta(k'', k). \end{aligned}$$

And since the input and value-added shares sum up to one, i.e. $\beta(k) + \sum_{k' \in \mathcal{K}} \gamma(k', k) = 1$, this implies that

$$\begin{aligned} \sum_{k' \in \mathcal{K}} \beta(k') \delta(k', k) &= \sum_{k' \in \mathcal{K}} \left( 1 - \sum_{k'' \in \mathcal{K}} \gamma(k'', k') \right) \delta(k', k), \\ &= 1. \end{aligned}$$

Finally, since the consumption shares sum up to one, i.e. $\sum_{k \in \mathcal{K}} \alpha(k) = 1$, wages cancel out in the welfare equation. The gains from trade in country $j$ are thus given by

$$\hat{W} = \prod_{k \in \mathcal{K}} \left( \hat{\pi}_F(k)^{\frac{1}{1-\sigma(k)}} \prod_{k' \in \mathcal{K}} \prod_{k'' \in \mathcal{K}} \hat{\pi}_X(k'' | k')^{\frac{\gamma(k'', k') \delta(k', k)}{1 - \sigma(k'')}} \right)^{\alpha(k)}.$$

### B.2.3 Bounds on the Gains from Trade

Going back to the full notation, the bounds are found by solving for the domestic input shares across any pair of sectors while ensuring that the domestic network aggregates to the input-output data. Welfare in country $j'$ is minimized by solving

$$\begin{aligned} \underset{\{\boldsymbol{\pi}_X\}_{j'', k'', k', j}}{\text{minimize}} \quad & \sum_{k' \in \mathcal{K}} \sum_{k'' \in \mathcal{K}} \left( \frac{\gamma(k'', \{k', j'\}) \sum_{k \in \mathcal{K}} \delta(k', \{k, j'\}) \alpha(k, j')}{1 - \sigma(k'')} \right) \ln \pi_X(\{j', k''\} | \{j', k'\}, j'), \\ \text{subject to} \quad & X(\{j'', k''\}, \{j', k'\}) = \sum_{j \in \mathcal{J}} \pi_X\left(\{j'', k''\} \middle| \{j', k'\}, j\right) \gamma(k'', \{j', k'\}) EXP(\{j', k'\}, j), \forall j'', k'', k', \\ & \sum_{j'' \in \mathcal{J}} \pi_X\left(\{j'', k''\} \middle| \{j', k'\}, j\right) = 1, \ \forall k'', k', j, \\ & \pi_X\left(\{j'', k''\} \middle| \{j', k'\}, j\right) \geq 0, \ \forall j'', k'', k', j. \end{aligned}$$

where $EXP(\{j',k'\},j) = \left(\sum_{k\in\mathcal{K}} X(\{j',k'\},\{j,k\}) + F(\{j',k'\},j)\right)$ are aggregate exports of country-sector $\{j',k'\}$ to $j$. As before, the optimization only solves for the specialized inputs linkages *within* country $j'$ so that there are $\mathcal{J}\times\mathcal{K}\times\mathcal{K}\times\mathcal{J}$ endogenous variables. The linear constraints are as in the one-sector model, the first one restricts to parameterizations that fit the data while the latter two impose that the input shares are positive and add up to one across all sources.

Careful inspection reveals that this optimization problem is actually composed of $\mathcal{K}\times\mathcal{K}$ disjoint optimization subproblems. That is, conditional on a pair of sectors $k''$ and $k'$, the input shares determine the destination $j$ of inputs from source each $j''$. Hence, instead of solving the full optimization with $\mathcal{J}\times\mathcal{K}\times\mathcal{K}\times\mathcal{J}$ endogenous variables, the bounds can be found by solving $\mathcal{K}\times\mathcal{K}$ problems of size $\mathcal{J}\times\mathcal{J}$ each

$$\begin{aligned}
\underset{\{\boldsymbol{\pi}_X\}_{j'',j}}{\text{minimize}} \quad & -\ln\pi_X\left(\{j',k''\}\,|\{j',k'\},j'\right),\\
\text{subject to} \quad & X\left(\{j'',k''\},\{j',k'\}\right) = \sum_{j\in\mathcal{J}}\pi_X\left(\{j'',k''\}\,\middle|\{j',k'\},j\right)\gamma\left(k'',\{j',k'\}\right)EXP\left(\{j',k'\},j\right),\forall j'',\\
& \sum_{j''\in\mathcal{J}}\pi_X\left(\{j'',k''\}\,\middle|\{j',k'\},j\right) = 1,\ \forall j,\\
& \pi_X\left(\{j'',k''\}\,\middle|\{j',k'\},j\right) \geq 0,\ \forall j'',j.
\end{aligned}$$

Computationally this trick is extremely useful since the time required to solve this type of optimization problems is exponential in size. Thus, it is much more practical to solve many small problems separately than one big problem at the same time. The solutions to each optimization can then be used together to determine the bounds on the welfare gains from trade.

# C Gravity Regressions in Specialized Inputs Models

I show through simulations that the trade elasticity can be roughly recovered from gravity regressions in specialized inputs models even though structural gravity does not hold. Assume that there are $\mathcal{J} = 10$ countries and a single sector per country. In each simulation I sample parameters from random distributions. Specifically, I take draws $\beta(j', j) \sim$ Uniform$(0,1)$, $a(j'' | j', j) \sim$ Lognormal$(0,1)$, $\varphi(j' | j) \sim$ Lognormal$(0,1)$. I normalize the latter two shares so that $\sum_{j''} a(j'' | j', j) = 1$ and $\sum_{j'} \varphi(j' | j) = 1$. To obtain symmetric trade costs I take $\rho(j', j) \sim$ Uniform $(0, 1/2)$ and define $\tau(j', j) = 1 + \rho(j', j)\rho(j, j')$ for $j' \neq j$ and $\tau(j, j) = 1$. For the roundabout model I run similar simulations while imposing common input shares $a(j'' | j')$.

The only missing parameter is the elasticity of substitution that I set to $\sigma = 6$ so that the roundabout trade elasticity is $1 - \sigma = -5$. In each simulation I construct the input-output table and run the following regression

$$\ln X(j', j) = \alpha_0 + \alpha_{exp}(j') + \alpha_{imp}(j) + \theta \ln \tau(j', j),$$

where $\alpha_0$ is the intercept, and $\alpha_{exp}(j')$ and $\alpha_{imp}(j)$ are exporter and importer fixed effects. The coefficient $\theta$ equals the trade elasticity in the roundabout model if input shares are driven entirely by trade costs, i.e. if $a(j' | j) = 1$ for all pairs. More generally, the coefficient $\theta$ will differ from the trade elasticity since these parameters do vary.

Figure 5 presents the range of estimates for $\theta$ across $10,000$ simulations of the roundabout and specialized inputs models. As discussed, $\theta$ differs from the trade elasticity because of the exogenous input share parameters but note that on average it exactly equals the trade elasticity. Surprisingly, even though structural gravity does not hold in specialized inputs models, the estimates for $\theta$ are not too different from the roundabout model. The average estimate in the roundabout model hits precisely the structural trade elasticity value of $1 - \sigma = -5$, while the average estimate in specialized inputs is only slightly lower at $-4.38$ reflecting the fact that trade costs with third countries affect bilateral trade flows through supply chain linkages. This attenuation can be understood as introducing classical measurement error by consequence of model misspecification.

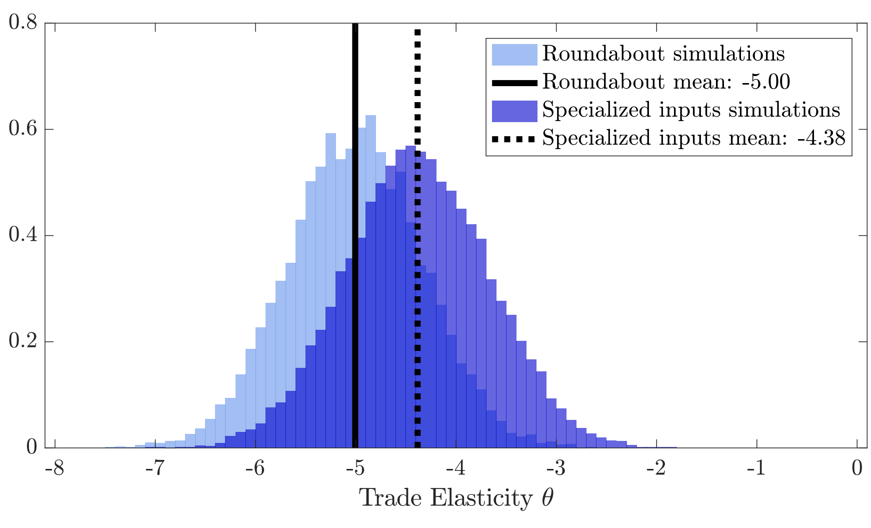


**Figure 5:** Gravity Regressions: The histograms correspond to trade elasticity estimates across $10,000$ simulations of both the roundabout and specialized inputs models. The simulations use a trade elasticity of $1 - \sigma = -5$.