Bank Net Worth and Frustrated Monetary Policy

Alexander K. Zentefis*

Yale School of Management
alexander.zentefis@yale.edu

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Abstract

I present a model of imperfect bank competition and show that monetary transmission depends on bank net worth. In the model, banks are local monopolists for borrowers near them. When they have a lot of equity, banks expand their lending, compete for customers at the edges of their markets, and pass through changes in the policy rate to their loan rates. When they have little equity, banks retreat from rivalry, exploit their monopoly power, and shut off the interest rate channel. The model's predictions are consistent with patterns observed in bank credit markets following the recent financial crisis.

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1 Introduction

Following the global financial crisis of late 2008, central bankers around the world were concerned that the traditional interest rate channel of monetary policy transmission was impaired. Although the Federal Reserve and many other central banks held policy rates at historically low levels, interest rates on bank loans to firms were slow to respond, remaining persistently high. Meanwhile, measures of firm default risk in many bank credit markets had already declined to pre-crisis levels.

The U.S. commercial and industrial (C&I) loan market, which supplies bank credit to small- and medium-sized firms, was an example of the impaired transmission. The spread between the average C&I loan rate and the effective federal funds rate stayed elevated after the crisis, even though delinquency rates, the fraction of nonperforming loans, and net loan charge-offs for new C&I credit had all subsided to levels seen before the crisis. Figures 1(a)-1(b) present the relation. Credit spreads on firm debt issued outside the banking sector, however, such as in the corporate bond market, declined much faster. Figure 2 plots the C&I loan spread in comparison to the corporate bond spread as measured in Gilchrist and Zakrajšek (2012), which captures the difference between corporate bond yields and Treasury rates matched by bond maturity.

After the financial crisis, two other notable phenomena took place in the C&I bank loan market. First, bank concentration dramatically increased, such that the number of banks extending this type of firm credit dropped sharply between 2008 and 2010. Figures 3(a)-3(b) from Berger et al. (2017) illustrate this change. This spike in concentration coincided with the extraordinary wave of bank consolidation that occurred after the financial crisis. From 2007 to 2013, 492 commercial and savings banks were put into FDIC receivership and sold at auction to acquiring banks (Granja et al. (2017)).

Second, some banks expanded their small business loan portfolios across geographic areas, while others contracted. Banks that suffered asset losses from drops in real estate prices shrunk their small business credit across regions. Banks that avoided those losses continued lending and even entered new territories, enlarging the geographic reach of their loans (Bord et al. (2017)).

While these three phenomena—the impairment of the interest rate channel, the rise in bank concentration, and the contraction of lending across localities by damaged banks—may seem unrelated, in this paper I show that all are closely linked. I present a model of imperfect bank competition and demonstrate that all three emerge after net worth in the banking sector declines significantly.

The central mechanism of the model is the following: when banks are flush with equity, their required cost of equity is low, so they compete across different parts of the loan market because doing so is relatively cheap. Competition compels banks to pass through changes in a central bank’s policy interest rate to their lending rates. A severe drop in bank net worth, however, sharply raises the cost of equity across banks, compelling them to exit the loan market.
or consolidate for survival. Competing across each other’s territories is no longer profitable, so instead, banks withdrawal from rivalry, contract their lending, and exploit their local monopoly power in separate parts of the loan market. No longer facing the same competitive pressures, banks do not pass through changes in the policy rate to their lending rates. So long as bank equity remains strained, there is no transmission via the interest rate channel. High bank loan rates in turn have real effects by discouraging investment and reducing production.

A financial crisis is one event that lowers bank capitalization and impairs monetary transmission, but so too does a sudden rise in capital requirements or severe asset write-downs arising from a general recession—a “capital crunch,” for instance, that Bernanke and Lown (1991) document during the 1990 U.S. recession. What matters to my theory is the state of bank equity, rather than what led to that state.

In the model, banks lend money to firms that are managed by entrepreneurs who run projects. These projects create output for consumption and are located around a circle. Locations on the circle represent different geographic areas, sectors, or industries. Banks are local monopolists for borrowers near them. That local monopoly power, however, can always be softened by another bank’s entry.

Banks face a minimum equity capital requirement that is a function of their assets in place, part of which includes their loans to firms. Banks satisfy the requirement using retained earnings or capital raised from a public equity market. This internal and external equity constitute bank net worth.

Banks make loans along arcs of the circle which are centered at their home locations. As they broaden their arcs and increase the scope of their loans, they diversify away risk in their portfolios. Banks that focus on narrow parts of the loan market are more specialized in their lending than are banks that finance projects over broad stretches.

Market power among banks arises out of an entrepreneur’s preference to contract with a bank that specializes more in his or her project (i.e., a bank that is closer to the entrepreneur’s location on the circle). Each bank can carve out a local monopoly market by offering credit at a price that entices entrepreneurs to create firms, borrow, and start their projects rather than pursue some outside option. With more aggressive pricing, a bank can try to lure borrowers away from a neighboring bank, which ignites competition.

When a bank lowers its lending rate to exactly match its neighbor’s, the local monopoly markets of the two banks just touch. At this price of credit, the bank observes a kink in its demand curve for loans. Charging any higher lending rate shrinks the bank’s local market, which the neighbor pays no attention to. Charging any lower lending rate expands the bank’s market into the neighbor’s territory. The amount the bank must offer as a price concession to get new customers increases when the bank switches from a local monopolist to a competitor, which generates the kink. The kink plays a key role in the analysis because a bank that operates there does not adjust its lending rate to small changes in its marginal cost of funding.

This price rigidity at the kink is the reason for an obstructed interest rate pass-through.
Figure 1: Commercial and Industrial Loans, All Commercial Banks

(a) C&I Loan Spread

(b) Delinquency, Nonperforming, and Charge-off Rates

Notes: The C&I loan spread is the difference between the weighted-average effective annual loan rate on all commercial and industrial loans and the effective federal funds rate. Weights are by loan amount. The delinquency rate is the fraction of total C&I loans that are delinquent. Delinquent loans are those past due 30 days or more and still accruing interest, as well as those in nonaccrual status. The nonperforming rate is the fraction of total C&I loans that are nonperforming. Nonperforming loans are those that bank managers classify as 90 days or more past due or nonaccrual. The charge-off rate is the value of C&I loans removed from the books and charged against loss reserves divided by the total value of C&I loans. Charge-off rates are annualized, net of recoveries. Data are quarterly.

Sources: Board of Governors of the Federal Reserve System (C&I loan spread, delinquency rate, charge-off rate). Federal Financial Institutions Examinations Council (nonperforming rate). Data retrieved from FRED, Federal Reserve Bank of St. Louis.

A central result of the model is that all banks in the credit market collectively settle at the kink if aggregate bank equity is sufficiently depleted. A severe drop in net worth tightens the
Notes: The C&I spread is the same as in Figure 1(a). The GZ spread from Gilchrist and Zakrajšek (2012) is an unweighted cross-sectional average of the spreads between the yields of senior unsecured bonds of a sample of U.S. non-financial firms and synthetic risk-free securities that match the cash flows of those bonds. Each time series is normalized so that the value in 2008Q2 equals 100. The dashed line starts from that value and extends to the end of the two series in the figure.


capital requirement, raises the cost of equity, forces bank exit or consolidation, and transitions the loan market to the kink in equilibrium. While there, each surviving bank maintains a local monopoly over a distinct segment of the loan market. As long as equity capital positions stay impaired, no bank finds it optimal to deviate from the kink and trigger competition because doing so would further damage profits. Instead, banks act as if they tacitly collude to keep their lending rates fixed. Efforts by a central bank to get the banking sector to pass through a low policy rate fail.

The absence of transmission leads firm credit spreads to rise after a policy rate cut despite no change to the default risk of loans. High loan rates in turn discourage entrepreneurs from increasing the scale of investment in their projects. Overall firm credit shrinks and aggregate investment and production drop as a result. The state of bank net worth therefore influences
Figure 3: Bank Concentration in the C&I Loan Market

(a) Annual Percent Change in the Number of Banks with at Least 10% C&I Lending

(b) Number of Banks Constituting 50% of C&I Loan Market

Notes: Panel (a) depicts the annual percent change in the number of banks with at least 10% of their loan portfolio consisting of C&I lending. Panel (b) depicts the minimum number of banks required each year to amass 50% of the total market for C&I loans.

Source: Berger et al. (2017).

both the price and quantity of bank credit to firms.

Three conditions in the model reduce the chances the banking sector transitions to the kink where monetary transmission is lost. The first is more internal equity on bank balance sheets in the form of retained earnings. Higher retained earnings allow banks to rely less on issuing external equity, especially during periods of stress. The second is higher monopoly rents, which ironically encourages competition. If banks expect to replenish their internal equity using
greater profits from higher credit spreads on the loans they issue, they have less incentive to retreat from competition if their net worth sharply erodes. The rents allow banks to self-heal. Last is a larger supply of external equity that banks can raise from the market. A heftier investor appetite to supply equity capital to banks reduces the cost of equity and encourages banks to stretch into each others territories, compete, and pass through changes in the policy rate.

These conditions suggest a number of potent policy tools to avoid or open an obstructed interest rate channel when net worth in the banking sector drops severely. Restricting dividends or share buybacks increases retained earnings. Temporarily limiting entry into lending markets or forcing consolidation raises the rents banks can earn to restore net worth internally. And finally, purchasing new equity issuance enlarges the supply of equity for banks, which reduces their funding costs and encourages competition and pass-through.

The model generates a number of testable empirical predictions. A loss of bank net worth (1) worsens the pass-through of changes in the policy rate to bank loan rates; (2) raises firm credit spreads on bank loans after a policy rate cut; (3) increases bank concentration in lending markets; and (4) shrinks both the scope of bank loan portfolios across geographic areas or industries and the scale of bank lending in the quantity of bank credit extended to firms. All four predictions have support empirically. In the main text, I describe the predictions in more detail and cite the empirical studies consistent with each one.

In summary, this paper provides a framework for studying how aggregate bank equity interacts with the industrial organization of the banking sector to affect monetary policy and the real economy. A novelty of the analysis is that bank net worth becomes an indicator for the degree of competition in bank credit markets and the effectiveness of the interest rate channel. Poor health of bank balance sheets can push an economy to an equilibrium where efforts by a central bank to lower the cost of firm credit are repeatedly frustrated.

Literature

This paper combines the insights of several strands of literature to uniquely tie bank net worth to bank competition, monetary policy, and the real economy.

The first strand is the broad body of work exploring the effects of financial frictions on the macroeconomy. The papers most closely related to mine are Bernanke and Gertler (1989), Bernanke et al. (1999), Holmstrom and Tirole (1997), and Gertler and Kiyotaki (2010).

As in Bernanke and Gertler (1989) and Bernanke et al. (1999), frictions in the financial market influence investment and monetary transmission. A key distinction here is banks and the industrial organization of the lending market. Bank net worth will influence bank competition and in turn the interest rate channel.

Net worth in the banking sector plays a major role in the model, as in Gertler and Kiyotaki (2010). Here, the novelty is that net worth influences the degree of competition among banks and the effectiveness of monetary policy.

Holmstrom and Tirole (1997) analyze the effect of changes to the supply of intermediated
financial capital on investment and credit spreads. A difference here is that the market for intermediated financial capital (bank loans) is imperfectly competitive.


A third strand is the industrial organization of banking. Berger et al. (2004) provide a survey. Matutes and Vives (1996) present a model in which banks compete over deposits. Whether a bank is a local monopolist or a competitor depends on the perceptions of that bank’s likelihood of failure. I fix depositor beliefs (banks cannot fail) to emphasize how bank net worth alters bank lending competition.

Matutes and Vives (2000) analyze how imperfect competition affects bank portfolio choice and whether deposit regulation intensifies or weakens risk-taking. Loans in my model all carry the same risk, so I can focus on the choice of pass-through rather than the choice of portfolio risk.

This paper owes a large debt to Salop (1979), whose structure of monopolistic competition on a circle I adopt. Other papers have also used the Salop framework to explore a variety of issues in banking:

Besanko and Thakor (1992) present a spatial model in which banks differentiate in loans and deposits and study the welfare implications of relaxed barriers to entry. To focus on bank lending, I have banks competing only in the credit market.

Chiappori et al. (1995) study the effects of deposit regulation on bank lending rates. In their model, the interest rate channel can also be hampered, but only when deposit rates are capped and deposits are bundled with credit services; otherwise, full transmission occurs. The deposit rate in my model is unregulated to put attention on bank lending.

Sussman and Zeira (1995) look at financial development across U.S. states and present a macroeconomic model in which costs of intermediation increase with the distance between the borrower and the bank. Their focus is not on monetary transmission or the role of bank net worth.

Andrés and Arce (2012) and Andrés et al. (2013) examine monetary policy within dynamic macroeconomies that feature imperfect competition among banks in the loan market. However, net worth in the banking system has no role in those economies and hence, interest rate pass-through is never impaired after a deterioration in bank balance sheets.

Hauswald and Marquez (2006) feature bank-screening technology whose signal quality
declines with the borrower's distance from the bank. Their focus is on banks strategically screening borrowers to carve out different segments of the loan market and soften competition. In my model, all borrowers are identical prior to obtaining a loan, and my focus is on aggregate bank equity and the effects of bank competition on monetary policy and the real economy.

The final strand is the empirical and theoretical work on interest rate pass-through and its relation to banks. One of the most robust empirical findings on impediments to interest rate pass-through is increased bank concentration (Cottarelli and Kourelis (1994); Borio and Fritz (1995); Mojon (2000); Sørensen and Werner (2006); van Leuvensteijn et al. (2008); Gigineishvili (2011)), which occurs in the model. Early work by Hannan and Berger (1991) and Neumark and Sharpe (1992) find a similar relation in deposit rates, as does Drechsler et al. (2017) in more recent work. Aristei and Gallo (2014) and Hristov et al. (2014) provide evidence that pass-through deteriorated in the Euro area during the financial crisis. Scharfstein and Sunderam (2016) find that higher mortgage lender concentration reduces the pass-through of declines in RMBS yields to mortgage rates.

Models of interest rate pass-through in the banking sector typically assign market power to banks, but they treat incomplete pass-through using either sticky prices (Hülsewig et al. (2009)) or menu costs (Hannan and Berger (1991); Cottarelli and Kourelis (1994); Scharler (2008); Hülsewig et al. (2009); Gerali et al. (2010); Günter (2011)). I micro-found the pass-through impairment from the kink in the demand curve for bank credit. The kink arises endogenously from the competitive market structure of the banking sector.

2 Economy

A real economy exists for two periods and the environment is a circle. Lining the circle are technologies called “projects” that may initiate in the first period. Projects return a random value of output for consumption in the second period. Banks are also located on the circle and finance these projects with loan contracts. Banks retain market power over borrowers but engage in monopolistic competition à la Salop (1979). In this section, projects are limited to a single unit of investment. In Section 5, the scale of investment can vary. Variable investment scale will allow credit in the economy to fluctuate with the health and competitiveness of the banking sector.

2.1 Projects

Firms operate projects, and a project and a firm are synonymous in the model. A continuum of projects are uniformly distributed around the circle. The circumference of the circle is normalized to one, and a project is identified by its unique location \( s \in [0, 1) \) on the circle. Projects on different parts of the circle can be interpreted as belonging to different geographic areas, industries, sectors, or subsets of any of these.

A project is indivisible and risky. A firm's owner is an entrepreneur who invests one unit of a homogeneous good to initiate a project in the first period. In the second period, the project produces one of two possible values of output: high or low. Denote the high return on the project
The returns are arranged

\[ 0 < \kappa < 1 < \bar{\kappa}, \]

making the low return a strict loss on investment (a failure), and the high return a strict gain (a success). The random returns on projects are identically distributed and pairwise uncorrelated. Let the probability of success for each project be \( \sigma \in (0, 1) \). The distributional assumptions imply a weak law of large numbers (see Uhlig (1996)). Hence, there is zero aggregate risk in the economy. If all projects on the circle initiate, total output will always equal \( \sigma\bar{\kappa} + (1 - \sigma)\kappa \).

### 2.2 Entrepreneurs

Entrepreneurs are endowed with a project and an outside option worth \( w \) in utility. They are risk neutral and lack any wealth. An entrepreneur decides either to choose the outside option or create a firm, start the project, and consume the net return. Projects are non-tradeable and non-transferable between entrepreneurs. From the absence of personal wealth, entrepreneurs must rely entirely on bank financing to initiate their projects. Because bank loans are the only source of funding available to entrepreneurs, their firms should be considered small to medium in size. Banks issue take-it-or-leave-it loan offers to entrepreneurs with the project posted as collateral. If the entrepreneur defaults, control rights of the project transfer to the bank, and the entrepreneur receives nothing.

Entrepreneurs are identified by the locations of their projects \( s \in [0, 1) \). Consider an entrepreneur positioned at location \( s \) who thinks about taking out a loan from a bank at location \( i \) on the circle. Let \( r_{L,i} \) be the gross lending rate bank \( i \) charges on the loan. The expected utility of entrepreneur \( s \) who undertakes the project and borrows from bank \( i \) is

\[
U_{s(i)} = \sigma(\bar{\kappa} - r_{L,i}) - \tau|s - i|.
\]

The expected return of the project to the entrepreneur is the high output net of the loan repayment, weighted by the probability of success. Entrepreneurs also have preference to borrow from a bank “nearby.” This preference for proximity is represented by the “distance” cost \( \tau|s - i| \), where \( \tau \) captures the strength of the preference and \( |s - i| \) is the shortest arc length between \( s \) and \( i \).

There are many interpretations of the proximity preference. An entrepreneur could prefer a closer bank because the bankers there speak the same language, or demand less paperwork, or has a branch physically nearest the entrepreneur’s shop, or because the bank has a reputation for specializing in lending to the industry or area the entrepreneur’s project is in. Paravisini et al. (2017) provide empirical evidence that firms that export have a greater likelihood of borrowing from a bank that specializes in that firm’s exporting country.
2.3 Banks

There are \( n \geq 2 \) banks located equidistantly about the circle. The ability to evaluate projects involves expertise in writing loan contracts with entrepreneurs, collecting payments, and liquidating projects in the event of non-payment. Banks are unique in the economy in having this ability. Therefore, banks alone finance projects and they do so with loan contracts. In default, a project’s productivity after output is zero, so the bank’s only recourse is to liquidate the project and recover the full value of the project \( \kappa \).\(^1\)

A bank can only finance projects that are positioned along arcs centered at its home location (headquarters). Let \( \Delta_i \in [0, 1] \) denote the arc length of the projects financed by a bank that is headquartered at position \( i \) on the circle. Because each project requires a single unit of financing, the size of the total loan portfolio of bank \( i \) is then \( \Delta_i \). This length can also be considered the “scope” of the bank’s portfolio because it represents the range of areas or industries the bank lends. A visual depiction of a bank \( i \)'s portfolio is given in Figure 4. Successfully extending credit to a specific set of industries or locations requires expertise in those markets. The arc length \( \Delta_i \) can be interpreted as an indicator of a bank’s expertise or specialization.

Figure 4: Bank Loan Portfolio

![Bank Loan Portfolio Diagram](image)

Notes: Projects are uniformly distributed around the circle. Bank \( i \) is headquartered at the bottom dot. The bank’s loan portfolio scope \( \Delta_i \) is the length of the arc centered at bank \( i \)'s headquarters. The three remaining dots represent other banks in the loan market.

Banks know entrepreneurs have a preference for proximity and they are aware of the distance cost function \( \tau |s - i| \). However, the location of an entrepreneur is unobservable to any

\(^1\)The reason control rights transfer to the bank and the entrepreneur receives nothing in default can be justified by a variety of agency reasons. One is a costly state verification assumption similar to Townsend (1979) in which the entrepreneur alone observes the project return; a debt contract compels the entrepreneur to report the outcome of the project truthfully.
bank at the time the loan is contracted. Also at that time, all projects share identical potential
returns per unit of investment and bear the same risk. For these reasons, a bank does not
engage in any degree of price discrimination, but instead posts a single lending rate $r_{L,i}$, while
taking into account the rates of all other banks on the circle. Entrepreneurs will self-select into
banks according to their preferences and the posted lending rates. The results of the model go
through even if banks can price discriminate. (See Internet Appendix 9.2.)

Finally, banks must pay a fixed cost $f$ to enter and operate in the commercial loan market.
(A few examples of possible fixed costs are complying with regulations, constructing a new
branch, and building the organizational structure.) Market power without a fixed cost would
encourage an unlimited number of banks to enter the circular lending market. This fixed cost
is capitalized into the assets of the bank, making the total assets of a typical bank $\Delta_i + f$.

2.4 Bank capital structure

Banks finance themselves using deposits and equity capital. The weak law of large numbers
that applied to the entire circle also applies to individual bank portfolios. Any incremental loan
portfolio is fully diversified and will earn a certain amount. A fraction $\sigma$ of the entrepreneurs
will succeed and repay the bank, whereas the other fraction will fail, after which the bank will
recover the low output.

Upon entering the circular loan market, a bank has a positive amount $e_{0,i}$ of retained
earnings from some previous operations. These initial earnings are the ex ante internal equity
of the bank. The bank can also raise an amount $e_i$ of external equity from the capital market.
Banks face an equity capital requirement: they are required to finance a fraction $\lambda \in (0, 1)$ of
their assets with equity. That equity capital constraint is

$$e_{0,i} + e_i \geq \lambda (\Delta_i + f).$$  \hspace{1cm} (2)

Banks issue equity from the public capital market, but the aggregate supply of external
equity from which banks draw upon is limited to a fixed amount $\xi$. Unlike with equity capital,
the supply of deposits is deep, so banks can issue them without limit. Because perfect diversi-
fication will render the loan portfolio riskless, deposits earn the riskless interest rate $r$. The
central bank has direct control over this real rate as a monetary policy tool in the model. Some
form of sticky prices or another friction transpires in the background to let that happen.

To avoid unnecessary complication, I assume that the cost of both internal and external
equity capital is the same. In other words, banks face a single cost of equity capital $r_e$, and that
required return will be determined endogenously through equilibrium in the equity market.

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2Gârleanu et al. (2015) also use a circular economy with diversification along arcs. Their setting is one with
investor portfolios and asset pricing, rather than with banks and competition, and diversification in their economy
is increasingly expensive.

3The certainty in the bank’s loan portfolio returns implies that a bank could finance lending entirely using
risk-free deposits. Bank diversification that shrinks bank portfolio risk is similar in spirit to the benefits of bank
diversification identified in Diamond (1984).

4Internet Appendix 9.1 presents a model with an endogenous equity capital requirement.
This assumption implies that either the initial equity holders of the bank (who financed the bank during some earlier period in which the retained earnings were made) are the same suppliers of capital as those in the public equity market, or at minimum, the initial and subsequent equity holders require the same return on capital. Implicitly, I assume there are no agency frictions between internal managers of the bank and external equity holders.

2.5 Bank decision

A typical bank $i$ chooses a lending rate $r_{L,i}$ and amount of external equity capital $e_i$ to maximize profits. In doing so, it perfectly knows and takes as given the demand curve for bank credit (described below), the lending rates of other banks, the number of banks $n$ on the circle, and the costs of debt and equity capital.

Let $FC(\Delta_i)$ denote the financing cost function of the bank, capturing payments to depositors and equity holders. Profits of a typical bank are

$$\pi_i = \sigma r_{L,i} \Delta_i + (1 - \sigma) \kappa \Delta_i - FC(\Delta_i).$$

The first term in (3) are repayments from the fraction of projects that succeed. The second term is the proceeds from the fraction that fail. The financing cost function $FC(\Delta_i)$ is

$$FC(\Delta_i) = r(\Delta_i + f - (e_{0,i} + e_i)) + r_e(e_{0,i} + e_i).$$

The bank requires an amount $\Delta_i + f$ in financing. It will use its starting retained earnings $e_{0,i}$ and issued amount $e_i$ in equity capital and finance the rest of its assets from deposits. The bank maximizes (3) subject to (2).

2.6 Demand curve for bank credit

The purpose of this section is to construct the demand curve for bank credit from the perspective of a typical bank. In doing so, I describe the industrial organization of the banking sector.

An entrepreneur chooses which bank to finance his or her project in order to maximize utility presented in (1). For an entrepreneur to borrow from a bank and undertake the project at all, the expected return from the project less the distance cost must exceed the outside option $w$. Because of a preference for proximity, an entrepreneur will choose between only three alternatives: the outside option, a loan from the first bank to his or her clockwise direction on the circle, and a loan from the first bank to his or her counterclockwise direction. An entrepreneur would never borrow from a more distant bank.

Each bank is located a distance $\frac{1}{n}$ from its neighbors. Let $i + 1$ be the location of the neighboring bank in the clockwise direction of bank $i$. Similarly, let $i - 1$ be the location of the counterclockwise neighbor. Because I will focus on symmetric equilibria, there is no harm in having the two neighboring banks share a common price of credit $r_L$ when bank $i$ is making its decision. The demand curve for credit for bank $i$ will consist of a monopoly, competitive, and kinked component.
**Monopoly**

The monopoly portion of bank $i$’s demand curve consists of the set of lending rates the bank can charge and face no competition from its neighboring banks. To begin, if bank $i$ sets $r_{L,i} > \kappa - \frac{w}{\sigma}$, no entrepreneur on the circle would find it worthwhile to borrow from the bank. The price of the bank’s loan would be so high that even the entrepreneur located at the bank’s headquarters would rather pursue the outside option. As bank $i$ lowers the price of credit, however, it will start attracting entrepreneurs whose surplus from the project exceeds the outside option value. Denote by $x_{i,i+1}$ the distance from the bank’s headquarters in the clockwise direction such that the entrepreneur located at that distance is indifferent between pursuing the project and the outside option. Formally, $x_{i,i+1}$ satisfies

$$\sigma (\kappa - r_{L,i}) - \tau x_{i,i+1} = w.$$  

Solving for $x_{i,i+1}$ yields

$$x_{i,i+1} = \frac{\sigma (\kappa - r_{L,i}) - w}{\tau}.$$  

A typical bank will fund projects on either side of it, so the monopoly demand curve for bank $i$, denoted $\Delta_{i,m} = x_{i,i-1} + x_{i,i+1}$, is

$$\Delta_{i,m} = \frac{\sigma (\kappa - r_{L,i}) - w}{\tau / 2}.$$  

This quantity defines the potential local monopoly market of the typical bank. The monopoly demand curve is increasing in the high output return $\kappa$. It is declining in the lending rate $r_{L,i}$, the outside option value $w$, and distance cost $\tau$. Although a bank faces no competition from other banks in its local monopoly market, the bank implicitly competes with the outside option of entrepreneurs.

**Competitive**

The competitive part of bank $i$’s demand curve consists of the set of lending rates that would expand bank $i$’s loan portfolio into the lending market of a neighboring bank, igniting competition between the two. If an entrepreneur is choosing between two banks, it must mean the expected return on the project exceeds the outside option value. The entrepreneur will borrow from the bank offering the lower financing and distance cost.

Let $x^\prime_{i,i+1}$ be the distance between bank $i$ and the entrepreneur who is indifferent between bank $i$ and bank $i + 1$. The entrepreneur’s utility function in (1) implies that $x^\prime_{i,i+1}$ must satisfy

$$x^\prime_{i,i+1} = \frac{\sigma}{2\tau} \left( r_L - r_{L,i} + \frac{\tau}{\sigma n} \right).$$  

Because the typical bank competes against two neighbors, its competitive demand curve
Δ_{i,c} = x'_{i,i-1} + x'_{i,i+1}, making
\[ \Delta_{i,c} = \frac{\sigma}{\tau} (r_L - r_{L,i}) + \frac{1}{n}. \] (6)

Bank \( i \)'s competitive credit market shrinks the more its lending rate exceeds the rates of the neighbors. Additionally, the more banks on the circle, the closer every entrepreneur is to a potential bank, which narrows the competitive market of any one bank.

Finally, one can see that each entrepreneur will prefer funding the project using a single bank. Petersen and Rajan (1994) document that small U.S. firms tend to concentrate their bank borrowing from one source. The marginal entrepreneur flips a fair coin and picks the bank according to the result.\(^5\)

Kinked

When bank \( i \) reduces its lending rate to exactly match the neighboring rate \( r_L \), its local monopoly market will just touch the monopoly markets of its two neighbors, and a “kinked” market arises. Denote this kinked lending rate \( r_{L,i,k} \).

The kinked market gets its name from the kink in the demand curve at the lending rate \( r_{L,i,k} \). If bank \( i \) set its lending rate just above \( r_{L,i,k} \), its local monopoly market would be segregated from that of its neighbors. The bank would lend according to the monopoly demand function in (5). The slope of the corresponding monopoly inverse demand function is \( \frac{dr_{L,i,m}}{d\Delta} = -\frac{\tau}{2\sigma} \). Alternatively, if the bank set a lending rate just below \( r_{L,i,k} \), its local monopoly market would cross the markets of the two neighboring banks, setting off competition. Bank \( i \) would collect demand according to the competitive demand curve in (6). The slope of the corresponding competitive inverse demand function is \( \frac{dr_{L,i,c}}{d\Delta} = -\frac{\tau}{\sigma} \). The difference in the slopes of the monopoly and competitive portions generates a kink in the demand for bank loans.\(^6\)

The kink in the demand curve for bank credit is a key feature of the lending market and critical for the main results. In the theory of kinked demand curves,\(^7\) prices under oligopoly may “stick” around a focal price. That price is sustainable in equilibrium out of each firm’s belief that undercutting will trigger a price war, but charging more leads no other firm to follow. The demand curve an individual firm faces will have a kink at the focal price.

The same economic reasoning applies here. If a bank reduced its lending rate below \( r_{L,i,k} \),

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\(^5\)When competing with neighboring banks, bank \( i \) could reduce \( r_{L,i} \) enough to capture even those entrepreneurs residing at the neighbors’ locations. Such a pricing strategy would drive the neighboring banks out of the market, and create a jump discontinuity in the demand curve for bank \( i \)'s credit. I forbid this predatory pricing in order for an equilibrium to exist.

\(^6\)The slope of the competitive portion is twice that of the monopoly portion for the following reason: When a typical bank is a local monopolist that seeks to expand its loan portfolio scope \( \Delta \), by an increment, it must offer a price concession of the amount \( \tau \) in order to entice the marginal entrepreneur to borrow from a more distant, less specialized bank. But when the bank tries to expand in a competitive market, it must offer the same price concession as before plus an additional amount \( \tau \) because the marginal entrepreneur is now closer to a neighboring bank that is more specialized in the entrepreneur’s industry. The extra concession is meant to lure the entrepreneur away from the competitor.

it would expand its segment of the loan market into the territories of the neighboring banks, sparking competition and hurting profits. Alternatively, raising the lending rate simply reduces the scope of that bank’s local monopoly market, which neighboring banks can safely ignore. The novelty here is that net worth in the banking sector will determine whether banks settle at that focal price.

Figure 5 illustrates bank $i$’s demand curve for loans, stitching together the monopoly, kinked, and competitive lending markets.

Figure 5: Demand Curve for Bank Credit from a Typical Bank $i$.

### 2.7 Equilibrium

I study symmetric, pure-strategy, Nash equilibria. An equilibrium is characterized by a tuple $E \equiv \{r_L, n, r_e\}$, where $r_L$ is the single lending rate charged by all banks, $n$ is the positive integer number of equally spaced banks on the circle, and $r_e$ is the cost of bank equity capital. The definition of an equilibrium is provided below.

**Definition.** *(Equilibrium)* An equilibrium of the economy is a tuple $E \equiv \{r_L, n, r_e\}$ such that

1. Every bank’s choice of lending rate $r_L$ is profit-maximizing,
2. This choice of lending rate earns an amount $v \geq 0$ in expected profits per bank,
3. The circle contains no gaps ($\Delta_i = \frac{1}{n}, \forall i$),
4. The bank equity capital market clears,
5. The market clearing cost of bank equity $r_e \geq r$.

The market for bank credit will be characterized by monopolistic competition, as in Cham-
berlin (1933), Robinson (1969), and Salop (1979). Entrepreneur preference for proximity will be a source of differentiation among banks that gives them market power in loan pricing, even when competing with one another to fund projects. Banks perfectly compete for deposits and bank equity.

I assume the supply of aggregate bank equity capital $\xi > 0$. Because the cost of equity $r_e$ is at least as high as the cost of debt $r$, banks will issue the minimum equity capital necessary to meet their constraints. Hence, the equity capital requirement in (2) always binds.

I allow potentially positive profits ($v \geq 0$) in equilibrium in order to permit banks to recapitalize themselves after a drop net worth. By collecting rents from the loan market, they do not have to rely as heavily on raising capital from the external market when distressed. They can self-heal.

Permitting positive equilibrium profits, however, will put up a strict barrier to entry and influence the equilibrium number of banks $n$. I interpret this barrier as arising from the actions of the central bank of the economy. Beyond control of the interest rate $r$ and the required equity capital share $\lambda$, an important power of the central bank in the model is influence over profits (rents) $v$ earned in the banking sector. In reality, this kind of power can be implemented by outright restricting entry or forcing bank consolidation.

Some restrictions need to be put on the required equity capital share $\lambda$ in order for the equity market to clear. The first is that the share does not exceed the total supply of external equity available. The second is that the requirement is high enough so that banks cannot rely entirely on initial retained earnings to gain entry into the loan market. The third puts a bound on the relative restrictions.

Assumption 1. The required bank equity capital share $\lambda$ satisfies

1. $\xi - \lambda > 0$,
2. $f\lambda - e_0 > 0$,
3. $\frac{\xi - \lambda}{f\lambda - e_0} \geq 2$.

Competitive and kinked equilibria

Three types of equilibria are possible in the economy: monopoly, kinked, and competitive. These types correspond to the three parts of the demand curve for bank credit.

A convenient way to visualize the equilibrium of the economy is to plot the average revenue and average cost curves of banks, given a price $r_e$ that clears the equity market. When equilibrium profits are zero, the point of tangency between the two curves indicates the equilibrium. Tangency ensures all $n$ banks in the loan market jointly earn zero expected profits at the profit-maximizing lending rate $r_L$.\(^8\)

\(^8\)When equilibrium profits $v > 0$, the equilibrium is pinned down by the unique point at which the slopes of the average revenue and cost curves match. But there, the average revenue curve lies above the average cost curve.
The average revenue curve is a simple affine transformation of the demand curve for bank loans. The part of the average revenue curve at which the average cost curve lies tangent indicates the equilibrium as monopoly, competitive, or kinked. If the average cost curve touches the kink in the average revenue curve, the first-order condition of the bank’s problem will hold as a strict inequality.

I focus on kinked and competitive equilibria rather than monopoly. The monopoly equilibrium does not add to the main results, so I ignore it. Dividing the bank profit function in (3) by $\Delta_i$ gives the average revenue and average cost functions. To simplify notation, I define the weighted average cost of financial capital to be $r_\lambda = (1 - \lambda)r + \lambda r_e$. The average revenue and average cost functions $AR(\Delta_i)$ and $AC(\Delta_i)$, respectively, are

\[
AR(\Delta_i) = \sigma r_{L,i} + (1 - \sigma) \kappa_i, \\
AC(\Delta_i) = r_\lambda \left(1 + \frac{f}{\Delta_i}\right).
\]

An illustration of a kinked and competitive equilibrium when $v = 0$ is presented in Figure 6.

**Figure 6: Kinked and Competitive Equilibria**

$Notes$: The equilibrium lending rate $r_L$ and loan portfolio scope $\Delta$ are determined at the point where the average revenue ($AR$) curve and average cost ($AC$) curve are tangent (competitive) or just touch (kinked). Kinked average revenue and average cost curves are solid; competitive are dashed. Equilibrium profits $v = 0$ in the figure.

The average cost curve is downward sloping and convex because of the fixed cost of bank entry $f$. In both equilibria, banks specialize their lending over non-overlapping segments of the
credit market. In the kinked case, *monopoly* markets just touch and competition is threatened, whereas in the competitive case, *competitive* markets just touch and competition is active.\(^\text{9}\)

**Market clearing**

The condition of no gaps on the circle ensures the market for bank credit clears: the aggregate demand for project financial capital (1) will match the aggregate supply of bank loans \((n \times \frac{1}{n})\) at the equilibrium lending rate \(r_L\).

The cost of bank equity \(r_e\) is pinned down by market clearing in the equity market. Supply of external equity is fixed at \(\xi\). The aggregate demand for equity is the sum of the individual minimum equity capital requirements in (2). Formally the clearing condition is

\[
\xi = \lambda (1 + nf) - ne_0. \quad (7)
\]

Bank demand for external equity is increasing in the equity capital requirement and aggregate fixed cost of entry, but decreasing in the total earnings banks retain on their balance sheets before entering the loan market.

**3 Monetary transmission**

This section presents the results on interest rate pass-through for the unit investment case. The subscripts in the propositions signify different values of the endogenous objects across the two equilibria. For better clarity, I present the results of this section in partial equilibrium before the equity market clearing condition is satisfied. In the next section, I discuss what changes in general equilibrium when the equity market clears.

**Lemma 1. (Lending rates)** The bank lending rate in a competitive equilibrium is

\[
r_{L,c} = \frac{1}{\sigma} \left( r_{\lambda,c} - (1 - \sigma) \kappa + \frac{\tau}{n_c} \right), \quad (8)
\]

where the weighted average cost of capital \(r_{\lambda,c} = (1 - \lambda) r + \lambda r_{e,c}\).

The kinked equilibrium lending rate is

\[
r_{L,k} = \kappa - \frac{w}{\sigma} - \frac{\tau}{2\sigma n_k}. \quad (9)
\]

\(^9\)Another way to distinguish the two types of equilibria is to consider a deviation by a bank thinking to raise its lending rate. In a kinked equilibrium, if a bank were to raise the rate, its customers would elect to take the outside option rather than borrow. In a competitive equilibrium, its customers would still borrow, but from the neighboring bank. A positive deviation in the kinked equilibrium kicks entrepreneurs out of the credit market; in the competitive equilibrium, it relinquishes them to a competitor.
3.1 Perfect pass-through

Consider first the competitive lending rate. The multiplier $\frac{1}{\sigma}$ is a risk-adjustment for the probability of loan repayment. A greater probability of default (lower $\sigma$) increases the loan rate. The first term after is the marginal cost of bank financing passed onto entrepreneurs. The second term is the expected marginal recovery value from a project in default. The more a bank can get from liquidation in the low state, the less it can charge the entrepreneur in the high state. These first two terms represent the cost of borrowing if entrepreneurs picked banks solely on price with no preference for those nearby ($\tau = 0$). In that case, perfect competition would drive banks to charge exactly the marginal cost of extending credit.

When banks can differentiate themselves by specialization ($\tau > 0$), they charge a markup over marginal cost—the last term in (8). The slope of the competitive demand curve and the number of banks determine the size of this markup. The costlier it is for entrepreneurs to contract with banks at a distance from their industries or areas (large $\tau$), the faster their demand for loans from remote banks drops off. Banks nearby exploit this feature of demand and charge a larger markup against those entrepreneurs who pick them. More banks competing in the market for loans (large $n_c$) lowers individual market power and shrinks the markup. Equation (8) demonstrates that the price of firm credit (and by extension the credit spread $r_{L,c} - r$) combines firm-specific components ($\sigma, \kappa, \tau$), a bank-specific component ($r_{\lambda,c}$) and a market-specific component ($n_c$).

Monetary transmission in the competitive case is perfect: variation in the interest rate passes through fully to bank lending rates. Competition is the reason: if a bank observes its cost of funding drop and does not lower its lending rate, it would lose customers to a neighboring bank. To remain in business, the bank must pass through any changes in the interest rate to its lending rate.

3.2 No pass-through

In a kinked equilibrium, banks operate off the kink in the demand curve for loans, so the profit optimality condition will not determine the equilibrium lending rate. The lending rate instead is taken off the monopoly portion of the demand curve for bank credit and presented in (9).

Banks in a kinked equilibrium are local monopolists in segmented industries of the credit market in which they specialize. They compete against the outside option of entrepreneurs rather than with each other. As seen in (9), a bank charges a higher lending rate if a project yields a higher successful output $\pi$ or if the probability of success $\sigma$ is higher. The bank exploits the attraction of pursuing a project by increasing the loan rate. On the other hand, the bank cuts the lending rate if the outside option $w$ is worth more. To entice an entrepreneur to take out a loan, the bank has to lower the price of a loan.

The kink in the demand curve generates a jump discontinuity in the marginal revenue curve of a bank. For this reason, if banks operate at the kink in equilibrium, small changes in the cost of providing a loan do not affect the cost of obtaining a loan. No part of the marginal
cost of a bank enters (9).

This result is critical to understanding why monetary policy is ineffective if the economy is in a kinked equilibrium. Any changes in the policy rate has no direct impact on the lending rate. Competition is missing to compel banks to adjust their loan rates after changes to their cost of funding. Banks act as if they tacitly collude to keep prices fixed. Each bank knows every other bank will not deviate from the kinked lending rate $r_{L,k}$. So no bank does. There is no transmission. Indeed, a decline in the interest rate will lead the firm credit spread $r_{L,k} - r$ to increase because the loan rate stays fixed.  

Finally, a perverse feature of the kinked lending rate is that more banks in the credit market leads all to raise their loan rates. Here is why: in a kinked credit market, banks are local monopolists. More banks on the circle means that an entrepreneur can find one that specializes in an industry or area “closer” to the entrepreneur’s. A bank takes advantage of its greater local monopoly power by charging a higher lending rate.

### 3.3 Number of banks

Banks will enter the circle until the minimum allowable profits per bank $v$ is met. This profit condition determines the number of banks in equilibrium. That condition is

$$\frac{1}{n} \left( \sigma r_L + (1 - \sigma) \kappa \right) - r_\lambda \left( \frac{1}{n} + f \right) = v. \tag{10}$$

The number of banks that operate in the lending market must be a positive integer. There is no guarantee, however, that the solution to (10) is an integer. Therefore, the equilibrium number of banks will be the largest previous integer to the solution of the profit condition.  

Lemma 2 presents the equilibrium number of banks for the competitive and kinked case.

**Lemma 2.** *(Entry)* The number of banks in a competitive equilibrium is

$$n_c = \sqrt{\frac{\tau}{v + fr_\lambda,c}}. \tag{11}$$

The number of banks in a kinked equilibrium is

$$n_k = \frac{\phi - r_\lambda,k + \sqrt{\left( \phi - r_\lambda,k \right)^2 - 2\tau \left( v + fr_\lambda,k \right)}}{2 \left( v + fr_\lambda,k \right)}. \tag{12}$$

---

*10* The kink generates a sharp prediction of zero pass-through. Generally, a region of a demand curve that features higher concavity leads a monopolist to pass through less of any changes to marginal cost. The kink creates a sharp concavity in the demand curve for loans. In Internet Appendix 9.3, I provide one way of “smoothing” the kink and show that the limited monetary pass-through is preserved.

*11* There is also no guarantee that the solution to (10) is unique. If there are multiple positive solutions, however, the unique equilibrium number of banks will be the largest one. The reason is the following. Pick two adjacent positive solutions $n_1 < n_2$. Both positive roots signify an equilibrium number of banks such that profits equal $v$. If profits equal $v$ for both a smaller number of banks $n_1$ and a larger number of banks $n_2$, it must be that profits exceeding $v$ can be made for some number of banks $n_1 < n < n_2$. Why else would more banks enter?
where \( \phi \equiv \sigma \pi + (1 - \sigma) \xi - w \) is the expected return of the project net of the entrepreneur’s outside option value.

In the competitive case, a higher weighted average cost of capital \( r_{\lambda,c} \) or fixed cost of entry \( f \) reduces the number of banks in the loan market. Allowing positive profits \( v \) in the loan market also reduces entry. A rise in the distance cost \( \tau \), on the other hand, increases differentiation among banks—entrepreneurs would prefer a closer, more specialized bank—which entices more banks to enter.

In the kinked case, a higher marginal (\( r_{\lambda,k} \)) or fixed (\( f \)) cost of extending a loan decreases the number of banks. So too does higher allowable profits \( v \), just like the competitive case. A greater return to the entrepreneur \( \pi \) increases entry because it increases the price of credit, whereas a larger outside option \( w \) discourages entry because it lowers the price of credit.

Unlike the competitive case, a rise in the distance cost \( \tau \) leads to exit. Recall that banks in the kinked equilibrium are local monopolists. If entrepreneurs strongly prefer to contract with a close bank, banks must reduce their lending rates to convince entrepreneurs to borrow rather than pursue the outside options. From (9), a lower distance cost reduces the price of credit, which reduces profit margins and curtails entry.\(^{12}\)

### 4 Bank net worth

In this section, I discuss the general equilibrium of an economy with unit investment. Here, the market for bank equity clears, which pins down the cost of bank equity. I demonstrate how bank net worth determines the kind of equilibrium in the credit market. I also discuss some central bank policy tools that can influence the status of monetary transmission.

#### 4.1 Cost of bank equity

Substituting the number of banks in the competitive and kinked equilibria from Lemma 2 into the market clearing condition in (7) delivers the cost of equity capital \( r_e \) in each equilibrium, which I present in the next lemma.

**Lemma 3.** (Cost of bank equity) The cost of bank equity capital in a competitive equilibrium is

\[
r_{e,c} = \frac{\tau}{f\lambda} \left( \frac{f\lambda - e_0}{\xi - \lambda} \right)^2 - \frac{v}{f\lambda} - r \left( \frac{1 - \lambda}{\lambda} \right),
\]

whereas the cost of equity in a kinked equilibrium is

\[
r_{e,k} = \frac{\phi}{\lambda} \left( \frac{f\lambda - e_0}{f\xi - e_0} \right) - \frac{v}{\lambda} \left( \frac{\xi - \lambda}{f\xi - e_0} \right) - \frac{\tau}{2\lambda} \left( \frac{(f\lambda - e_0)^2}{(\xi - \lambda)(f\xi - e_0)} \right) - r \left( \frac{1 - \lambda}{\lambda} \right).
\]

Comparative statics of the cost of equity across the two equilibria with respect to notable

\(^{12}\)It is important to note that an increase in \( n \) does not necessarily imply new bank creation. Entry can simply be existing banks lending to a new market that is a subset of a geographic area, industry, or sector represented by the circle. Exit can be either bank failure, merger or consolidation, or withdrawal from the circular loan market.
parameters are presented in Lemma 4. If a condition is provided in the lemma, it is sufficient for the comparative static, but not necessary.

**Lemma 4.** (Comparative statics, cost of bank equity)

A larger distance cost $\tau$ raises the cost of equity in the competitive equilibrium $\left( \frac{dr_{e,c}}{d\tau} > 0 \right)$, but lowers it in the kinked $\left( \frac{dr_{e,k}}{d\tau} < 0 \right)$.

In both equilibria, higher loan market rents $v$ reduces the cost of equity $\left( \frac{dr_{e}}{dv} < 0 \right)$. Higher initial retained earnings $e_0$ lowers the cost of equity in the competitive equilibrium $\left( \frac{dr_{e,c}}{de_0} < 0 \right)$ and does the same in the kinked equilibrium $\left( \frac{dr_{e,k}}{de_0} < 0 \right)$ if $\tau/\phi < 2$ and $\phi/v < 2$.

An expansion in the supply of external bank equity capital $\xi$ reduces the competitive cost of equity $\left( \frac{dr_{e,c}}{d\xi} < 0 \right)$. It does the same in the kinked equilibrium $\left( \frac{dr_{e,k}}{d\xi} < 0 \right)$ if $\phi/v < 2$ and $2(f\lambda - e_0)\sqrt{\phi} > (\xi - \lambda)\sqrt{2v + \tau/4}$.

A larger distance cost $\tau$ raises market power in the competitive equilibrium, which leads banks to charge a higher lending rate. Higher expected profits in turn encourage entry and raises $r_{e,c}$. Conversely, a higher distance cost in the kinked case lowers the lending rate, discourages entry, and lowers the cost of equity capital.

A greater supply of external equity $\xi$ lowers the cost of equity in both equilibria. Higher permissible rents $v$ in the loan market limits entry and decreases demand for external equity $\xi$, which lowers the cost of equity capital in both equilibria. The same is true of greater initial retained earnings $e_0$. Hence banks that can rely more heavily on internal equity via retained earnings or ex post rents can reduce their overall cost of funding, similar to models that feature an external finance premium such as those in Bernanke and Gertler (1989) and Bernanke et al. (1999).

### 4.2 Determining the equilibrium

Bank net worth comprises three types of bank equity: initial retained earnings $e_0$, earned profits $v$, and raised capital market equity $\xi$. The initial retained earnings and the earned profits are the internal equity of the bank, whereas the public market equity is the external equity. Sufficient changes in any one component can transition an economy between competitive and kinked equilibria.

Indeed, a sudden drop in net worth can move the credit market away from competition. All banks in that situation shrink their loan portfolios, each exploits its local monopoly power over firms, and all settle at the focal kinked lending rate. No bank thinks any other will deviate in pricing, so all act as if they tacitly agree to refrain from competing. Consequently, the interest rate channel of monetary transmission closes.

Using average revenue and cost curves, Figure 7 illustrates an economy that switches equilibria. The economy begins competitive and features a cost of bank equity $r_{e,c}$. After a large
decline in net worth, the cost of equity spikes to \( \bar{r} \), which raises the average cost curve. Facing higher funding costs, banks exit or consolidate, which reduces the number of banks \( n \) remaining in the loan market. Fewer banks raises the revenue per bank, which lets the survivors remain profitable. The average revenue curve moves out as a result until it meets the higher average cost curve, which occurs at the kink. The equilibrium switches from competitive to kinked.

Figure 7: Transition to Kinked Equilibrium after Decline in Bank Net Worth

Notes: The economy starts in a competitive equilibrium with cost of equity \( r_e \). The average revenue and cost curves of the competitive equilibrium are dotted. A drop in bank net worth raises the cost of equity to \( \bar{r} \). The average cost curve increases, banks exit or consolidate, the average revenue curve moves out, and the economy moves to a kinked equilibrium. The curves of the kinked equilibrium are solid.

The equilibrium of an economy is determined by a set of conditions. A necessary and sufficient condition for a competitive equilibrium is that the optimal competitive loan rate is below the monopoly portion of the demand curve. Mathematically,

\[
\tau_{L,c} \leq \bar{r} - \frac{w}{\sigma} - \frac{\tau}{2\sigma n_c}.
\]

The intuition for this condition is in Figure 6: in a competitive equilibrium, the point of tangency is below the monopoly part of the demand curve.

In a kinked equilibrium, expected profits exceed \( v \) when the loan rate is the monopoly demand curve from (5) instead of the optimal rate that is generated from a monopolist bank’s
first order condition. Mathematically,

\[ \pi |_{r_{L,m}} = \pi - \frac{w}{\sigma} \tau \nu^m > v, \]

where \( \pi \) is the bank profit function in (3) and the vertical line represents “conditional on.” The intuition here is the following: in both monopoly and kinked equilibria, local monopolist banks compete against the outside option of entrepreneurs rather than with each other. In a monopoly equilibrium, a bank will charge a loan rate that makes the marginal entrepreneur strictly prefer borrowing over the outside option. However, a kinked equilibrium is characterized by an entrepreneur being indifferent between borrowing and the outside option—hence the reason the kinked loan rate is taken directly off the monopoly portion of the demand curve rather than the bank optimality condition. Therefore, holding fixed the number of lenders, a monopolist bank can earn excess rents above \( v \) by raising its loan rate until the marginal entrepreneur becomes indifferent. 

This second inequality is necessary, but not sufficient for a kinked equilibrium. The inequality also holds under a competitive equilibrium. Therefore, a necessary and sufficient condition for a kinked equilibrium is satisfaction of the second inequality but failure of the first.

Substituting the appropriate endogenous objects into the two inequalities above yields two quadratic polynomials in external equity \( \xi \). The larger roots of these polynomials are functions of the two types of internal equity (\( v \) and \( e_0 \)) and the required equity capital share \( \lambda \). Conditions on the value of \( \xi \) relative to the values of the polynomial roots dictate whether the equilibrium is kinked or competitive. Proposition 1 explains.

**Proposition 1.** (Determining the equilibrium) Let \( h_1 (v, e_0, \lambda) \) and \( h_2 (v, e_0, \lambda) \) be the two larger roots of quadratic polynomials \( H_1 (\xi) \) and \( H_2 (\xi) \) that are defined over aggregate external bank equity \( \xi > \lambda \). Those roots are

\[
\begin{align*}
h_1 &= \lambda + \frac{1}{2} \left( \frac{f \lambda - e_0}{v + f \phi} \right) \left( \tau f + \sqrt{\tau (\tau f^2 + 2 (v + f \phi))} \right), \\
h_2 &= \lambda + \frac{1}{2} \left( \frac{f \lambda - e_0}{v + f \phi} \right) \left( \frac{3}{2} \tau f + \sqrt{\tau \left( \frac{9}{4} \tau f^2 + 4 (v + f \phi) \right)} \right).
\end{align*}
\]

They are arranged \( h_2 > h_1 \). The economy is in a competitive equilibrium when \( \xi \geq h_2 \); the equilibrium is kinked when \( \xi \in (h_1, h_2) \).

Figure 8 illustrates the two quadratic polynomials \( H_1 \) and \( H_2 \) over the domain \( \xi > \lambda \). Values of \( \xi \) for which \( H_2 \) is zero or positive (shaded in blue) constitute a competitive equilibrium, whereas values of external equity for which \( H_2 \) is strictly negative and \( H_1 \) is strictly positive (shaded in red) constitute a kinked equilibrium.

When the supply of aggregate external equity \( \xi \) is high, the cost of equity is low and banks

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13Eyeing these excess rents, other banks will enter the lending market until profits earned by all banks in the kinked equilibrium equal \( v \).
actively compete in the lending market. Following a severe enough drop in $\xi$, the banking sector becomes impaired as the cost of equity spikes, and the economy shifts into the red kinked region. Here, banks constrict their loan portfolios, refrain from competing, and close the interest rate channel. If equity capital positions improve ($\xi$ increases), banks have incentive to break their fixed pricing, expand the scopes of their loan portfolios, enter the markets of other banks, and resume competition. The interest rate channel then opens.

Changes in either type of internal equity $v$ and $e_0$ or changes in the equity share constraint $\lambda$ adjust the positions of the roots $h_1$ and $h_2$. A convenient way to learn how the roots shift is to study the interval between them. The length of that interval is the “size” of the kinked region. Let $\eta \equiv h_2 - h_1$ be that length. A wider interval implies that smaller declines in external equity $\xi$ can push the economy into a kinked equilibrium. In this sense, an economy featuring a larger $\eta$ is more susceptible to losing bank competition and monetary transmission after net worth positions in the banking sector deteriorate. Lemma 5 describes how the size of the kinked region changes with these parameters.

**Lemma 5.** (Size of the kinked region) The size of the kinked region $\eta$ is increasing in the equity share constraint $\left( \frac{d\eta}{d\lambda} > 0 \right)$ and decreasing in both starting retained earnings $\left( \frac{d\eta}{de_0} < 0 \right)$ and ex post rents $\left( \frac{d\eta}{dv} < 0 \right)$.

A tighter equity capital share constraint expands the kinked region, making the banking sector more susceptible to losing competition and monetary transmission after net worth positions in the banking sector deteriorate.
system more susceptible to entering a kinked equilibria. On the other hand, better capitalized banks via greater retained earnings ex ante or higher loan market rents ex post shrinks the kinked region. If banks can self-heal after a deterioration in net worth using profits from the larger credit spread $r_L - r$ they charge on firm loans, the banking system is less likely to enter the kinked equilibrium. Ironically, the more rents banks expect to earn with their local monopoly power, the more likely they will continue competing after drops in their net worth.

4.3 Central bank policy

Lemma 5 reveals that changes in either external equity $\xi$, internal equity $e_0$ and $v$, or the capital requirement $\lambda$ influence the equilibrium of an economy. Higher internal equity from $e_0$ or $v$ shrinks the region $\eta$ and reduces the likelihood an economy enters a kinked equilibrium for any fixed quantity of external equity $\xi$. More external equity $\xi$ helps an economy avoid the kinked region for any fixed internal equity $e_0$ and $v$. And finally, a looser capital requirement $\lambda$ reduces the kinked region holding fixed all types of equity.

Therefore, changes in any of these parameters can be an effective means for a central bank to prevent an economy from falling into a kinked equilibrium or to push an economy out of one. Easing capital requirements after a significant decline in net worth would reduce $\lambda$. Purchasing newly issued bank equity increases the supply of external equity $\xi$, which reduces the cost of equity, encourages competition and opens the interest rate channel. Discouraging dividend payments or share buybacks ex ante is a way to increase $e_0$, while permitting larger ex post rents from lending increases $v$.

Not all tools need to be used at once. Sufficient expansions in $\xi$ through central bank purchases of bank equity can help transition an economy away from the kinked region without easing capital requirements or boosting internal equity, for example. Or adequate restrictions on dividends or share buybacks could be enough for an economy to remain within the competitive region after a decline in net worth without the central bank purchasing equity or permitting banks to earn market rents.

Although higher $v$ shrinks the kinked region, Lemma 2 demonstrates that greater ex post profits restricts entry into the loan market (or forces consolidation) and grants greater market power to incumbent banks. Fewer banks means firms must travel a greater “distance” on average to obtain credit, which entrepreneurs prefer less. A central bank would have to consider this welfare loss in the design of policy.

Restricting entry is a tool that could be implemented on a limited basis after the banking sector has already suffered severe losses in net worth. Banks could be allowed to re-capitalize themselves until the public markets restore a greater supply of external equity $\xi$ at a lower cost $r_c$. Allowing banks temporary monopoly rents would be a means for an economy to avoid entering the kinked region and losing monetary transmission entirely or a way to move the economy out of the region and restore transmission quicker.
4.4 Discussion

It is now useful to revisit monetary transmission when the equity market is allowed to clear. The results on interest rate pass-through from the previous section were derived in partial equilibrium before equity market clearing. There I discussed how the transmission channel fully opens in the competitive case but entirely closes in the kinked one.

Clearing in the equity market influences both the cost of equity $r_e$ and the number of banks $n$. From Lemma 2, the number of banks in the two equilibria are functions of the weighted average cost of capital $r_\lambda$. From Lemma 3, any change in the interest rate $r$ is exactly offset by a commensurate change to the cost of equity $r_e$, which leaves $r_\lambda$ unchanged. Therefore, both $n$ and $r_\lambda$ are immune to variation in the interest rate in both the kinked and competitive equilibria once the equity market clears.

Monetary transmission was absent from the kinked lending rate in partial equilibrium and is still absent in general equilibrium. Once the equity market clears and banks are allowed to enter or exit, monetary pass-through is even lost in a competitive equilibrium. Indeed, the competitive number of banks and the loan rate after the equity market clears are

$$n_c = \frac{\xi - \lambda}{f\lambda - e_0},$$

$$r_{L,c} = \frac{\tau}{\sigma f} \left( \frac{f\lambda - e_0 \left( f\xi - e_0 \right)}{\left( \xi - \lambda \right)^2} \right) - \frac{v}{\sigma f} \left( \frac{1 - \sigma}{\sigma} \right) \varepsilon.$$ 

Higher bank external equity $\xi$ and retained earnings $e_0$ encourage entry and lowers the cost of bank capital $r_\lambda$. Both effects push down the competitive loan rate. Higher ex post rents $v$ also ease the cost of capital and lower the loan rate. A tightening of the capital requirement $\lambda$, on the other hand, discourages entry and raises the loan rate.

Notably missing from either the number of banks or the competitive loan rate is the interest rate $r$. Bank entry or exit is the reason. As an illustration, suppose a central bank cuts the policy rate $r$. Doing so initially lowers the cost of capital $r_\lambda$. A drop in the marginal cost of bank funding increases bank profits above $v$, enticing other banks to enter. These entrants require external equity capital to finance firm lending and this entry shifts out the demand curve for bank equity. The demand curve increases enough so that the cost of equity $r_e$ rises to exactly offset the initial drop in $r_\lambda$.

If other institutions were restricted from entering the loan market, incumbent banks would earn abnormal profits in the meantime and the interest rate channel would stay open. One barrier to entry is time: it takes time for a bank to initiate operations in a new geographic or industrial loan market. So one can then consider the number of banks in the loan market "fixed" in the short run, but "variable" in the long run. In that case, a central bank can lower the price of firm credit in this economy in the short run via the interest rate channel so long as
banks are competing, but any effect is eventually erased in the long run.  

From the equity market clearing condition in (7), one can see that adjustments to the cost of equity \( r_e \) come entirely via changes in the number of banks \( n \). However, if project investment were allowed to scale, total lending in the economy would be another margin for \( r_e \) to vary. I turn there next.

5 Variable investment scale

An advantage of an economy with unit investment is that the piece-wise linear demand curve for bank credit makes the model easy to solve. A shortcoming of that economy is that aggregate lending and investment remains constant at one. Variation in bank net worth only affects the price of credit. There is no way to study in that economy how the quantity of credit fluctuates with bank net worth.

One way to allow variable loan quantity is to insert a downward sloping demand curve for investment. That way aggregate investment, lending, and output in the economy will fluctuate with changes in bank net worth. Another advantage of inserting variable investment is that total lending of a bank can be decomposed into two parts: the scale (the quantity lent at individual locations) and the scope (the space of locations). Each will be affected by bank equity differently.

5.1 Demand curve for loans

Isoelastic investment demand and linear production technology generate tractable results. So let \( t_s(i) \) denote the investment demand of entrepreneur \( s \) who borrows from bank \( i \). The quantity \( t_s(i) \) is the scale of investment in the project. Because projects are financed entirely using bank credit, \( t_s(i) \) also is the amount lent to entrepreneur \( s \). Let \( y(t_s(i)) \) denote the production output of the project. The investment demand and production functions are

\[

t_s(i) = \frac{b}{r_{L,i}} \tag{15}
\]

\[
y_s(i) = \kappa t_s(i) \tag{16}
\]

where \( b > 0 \) is a constant, \( \kappa = \pi \) if the project is successful, and \( \kappa = \kappa \) if the project fails. Let \( \sigma \) be the probability of success, just as in the case with unit investment. The value of the project to the entrepreneur is the expected proceeds from production, net of the principal and interest payments to the bank. Again, the entrepreneur expects to receive nothing should the project fail.

The expected utility of entrepreneur \( s \) is

\[
U_s(i) = \sigma (\kappa - r_{L,i}) t_s(i) - \tau \| s - i \|
\]

---

\(^{14}\)One method that would have the policy rate enter the loan rate even after entry or exit is to let banks charge a price of firm credit that is by design a spread over the interest rate: \( r_L \equiv r + m \), where \( m \) is the spread banks choose optimally. Pass-through would be absent in the kinked equilibrium again, but the competitive equilibrium would always feature an open interest rate channel by construction.
and he or she can also choose an outside option worth $w$. To conserve notation, let $v_{s(i)} \equiv \sigma (\kappa - r_{L,i}) \iota_{s(i)}$ be the expected net return of the project for the entrepreneur. Using exactly the same procedure as before, one can construct monopoly and competitive demand curves for bank $i$. They are

$$
\Delta_{i,m} = \frac{v_{s(i)} - w}{\tau/2},
$$
$$
\Delta_{i,c} = \frac{v_{s(i)} - v_s}{\tau} + \frac{1}{n},
$$

where $v_s \equiv \sigma (\kappa - r_L) \iota_s$ is the expected net return to the entrepreneur when borrowing from a neighboring bank that charges loan rate $r_L$. These two demand curves are analogous to their counterparts in (5) and (6) when investment was fixed.

5.2 Bank problem

The bank profit function will consist of loan repayments from successful entrepreneurs, recovery values from projects that fail, and financial payments to creditors and equity holders. For simplicity, suppose $\kappa = 0$, making recovery valueless.

Banks still issue loans along arcs $\Delta_i$ centered at their headquarters, but now, each loan has size $\iota_i$. Because of these two dimensions of lending, I refer to $\Delta_i$ as the scope of bank $i$'s loan portfolio and $\iota_i$ as the scale of the portfolio. Bank $i$'s total lending is then $\Delta_i \iota_i$.

Like before, a bank will face a minimum equity capital requirement: $e_{0,i} + e_i \geq \lambda (\Delta_i \iota_i + f)$, which now accounts for the variable scale of lending. The constraint will bind. With this, the profit function of a bank is

$$
\pi_i = \sigma r_{L,i} \Delta_i \iota_i - r_\lambda (\Delta_i \iota_i + f),
$$

where again $r_\lambda = (1 - \lambda) r + \lambda r_e$ is the weighted average cost of bank financial capital.

5.3 Equilibrium

The definition of an equilibrium is identical to Definition 2.7. Like before, the cost of bank equity $r_e$ will be pinned down by the clearing of the equity market. Rather than (7), however, the equity market clearing condition is now

$$
\xi = \lambda (\iota + nf) - n e_0.
$$

Also, instead of a covered circle, the equilibrium will be represented by a covered cylinder, as depicted in Figure 9. Each bank will have loan portfolio scope $\Delta = \frac{1}{n^*}$ and scale $\iota = \frac{b}{r_L^*}$ for an equilibrium number of banks $n^*$ and lending rate $r_L^*$.

I focus again on the kinked and competitive equilibria. Like before, the kinked lending rate is taken off the monopoly portion of a bank’s demand curve, whereas the competitive lending rate arises from the first order condition of a bank’s problem. Lemma 6 presents the two lending rates.
Lemma 6. (Lending rates, variable investment scale) When the scale of project investment can vary, the bank lending rate in a competitive equilibrium is

\[ r_{L,c} = \frac{1}{\sigma} r_{\lambda,c} - \frac{\tau}{b\sigma n e}. \]  

The kinked equilibrium lending rate is

\[ r_{L,k} = \frac{\kappa}{1 + \frac{1}{b\sigma} \left( w + \frac{\tau}{2n e} \right)}. \]  

The comparative statics of both lending rates are the same as in the unit investment case. Less risky projects (higher \( \sigma \)) lead to a lower competitive loan rate but a higher kinked rate. A larger distance cost \( \tau \) increases the competitive rate but decreases the kinked. More competition (higher \( n \)) decreases the competitive rate but perversely increases the kinked for the same reasons given in the previous case. Again, in the kinked equilibrium the interest rate channel is closed, with \( r_{\lambda,k} \) not entering the price of firm credit.

What is different now is the investment demand parameter \( b \). Locally monopolist banks in a kinked equilibrium exploit higher demand by charging a higher loan rate. In contrast, competitive banks charge a lower rate when \( b \) is higher. The lower rate is a consequence of the unitary elasticity for investment demand. Aggregate loan repayments (principal plus interest) \( r_{L,t} = b \) is constant. A positive increase to investment demand will compels competitive banks to lower rates or face losing customers. Each bank expects to issue a larger loan per borrower, so it must charge a lower rate on the amount lent.

Using the equity market clearing condition in (18), one can solve for the cost of financial
capital $r_{\lambda,c}$ in the competitive equilibrium when the number of banks is held fixed at $n_c$. In the kinked case, one cannot perform such an exercise because the cost of capital $r_{\lambda,k}$ does not directly enter the equity market clearing condition. It only enters indirectly through $n_k$.

The cost of bank financial capital in the competitive case with variable investment is

$$r_{\lambda,c} = \frac{\lambda b \sigma}{\xi - n_c (f \lambda - e_0) + \frac{\lambda \tau}{K \sigma n_c}}.$$

A disadvantage of the variable investment model is that the solution for $r_{\lambda,c}$ that accounts for endogenous $n_c$ is not available in closed-form. Nevertheless, much can still be learned while holding the number of banks constant, putting attention on the short-run, during which adjustments to the number of banks is less likely.

Expansions in aggregate external equity $\xi$ or retained earnings $e_0$ lower the cost of equity $r_{e,c}$ and by extension the cost of overall financial capital $r_{\lambda,c}$. Tightening of the capital constraint parameter $\lambda$ raises the cost of equity. A larger demand for investment funds from higher $b$ raises the cost of financial capital because it raises the amount of external funding banks require in order to supply the loans and satisfy demand.

Substituting $r_{\lambda,c}$ into the price of firm credit in (19) gives the competitive lending rate and aggregate investment as functions of the number of banks and bank net worth:

$$r_{L,c} = \frac{b \lambda}{\xi - n_c (f \lambda - e_0)},$$

$$\iota_c = \frac{\xi - n_c (f \lambda - e_0)}{\lambda}.$$

Equations (21) and (22) reveal how not only the price of credit, but also the quantity of credit, aggregate investment, and by extension, real production are influenced by net worth in the banking sector. Better capitalized banks reduces the price of firm credit, encourages firm borrowing, and enlarges investment and output. Here too one can see that the scale $\iota_c$ of a typical bank's loan portfolio also expands with its net worth.

6 Empirical predictions with supporting evidence

A number of testable empirical predictions are generated from the model where investment demand and the scale of bank loan portfolios can vary as well as from the version featuring unit demand. The predictions I describe next relate to the interest rate channel of monetary transmission, small firm credit spreads, the spatial variation of bank loan portfolios, and bank concentration. All four are directly affected by the state of bank net worth.

6.1 Interest rate pass-through

A central prediction of the model with unit demand is the impairment of the interest rate channel when bank net worth is low. The low net worth pushes banks into a kinked equilibrium in which the loan rate is (9). A monetary easing that cuts the policy rate will not reduce the
price of credit for firms borrowing from banks. In any test, it is important that the market value of bank equity is used rather than the book value.

The model predicts precisely zero pass-through in a kinked equilibrium because of the sharp kink in the demand curve for credit; but more realistically, the prediction is diminished pass-through during periods of low bank net worth. (The model extension in Internet Appendix 9.3 predicts a weakened interest rate channel when banks are at the “smoothed” kink.)

The findings of Acharya et al. (2017) support this prediction. Studying the Euro area from 2006–2010, they document diminished pass-through to borrower loan spreads following ECB liquidity provisions among unhealthy, low net-worth banks. Healthier, high net-worth banks decreased loan spreads after the monetary easing. Small- and medium-sized firms were particularly affected by the limited pass-through because of their reliance on bank debt. These firms cut investment and employment. Aristei and Gallo (2014) and Hristov et al. (2014) also provide evidence that pass-through deteriorated in the Euro area during the financial crisis.

6.2 Credit spreads

A consequence of the impaired pass-through is heightened firm credit spreads \( r_L - r \) when net worth is lower in the banking sector. A policy rate cut that is not joined by a commensurate decline in loan rates will increase the difference between the price of firm credit and the policy rate. Some evidence for this prediction is presented in Figure 2, which gives the commercial and industrial (C&I) loan spread for small and medium size firms and the corporate bond spread generated in Gilchrist and Zakrajšek (2012). Spreads in the corporate bond market fell much faster than the C&I loan spread to pre-crisis levels, despite declining measures of default risk in the C&I loan market. An explanation for the difference is that the C&I loan market is subject to forces of imperfect competition among banks, as implied by the model, which a more competitive bond market featuring tradeable securities among investors is free from.

6.3 Spatial variation

In equilibrium after entry and exit has taken place, the scope of all banks \( \Delta = \frac{1}{n} \) in both the kinked and competitive cases because of symmetry. Nevertheless, one can study a typical bank’s deviation from an equilibrium. Do so by fixing the number of banks \( n \) in the loan market and the loan prices of the other banks. Then see how a typical bank’s loan portfolio scope varies with net worth. In the competitive case, the scope of a typical bank’s loan portfolio \( \Delta_{i,c} \) is

\[
\Delta_{i,c} = \frac{v_s(i) - v_s}{\tau} + \frac{1}{n_c},
\]

where \( v_s(i) = \sigma (\kappa - r_{L,i}) \tau s(i) \) and \( v_s = \sigma (\kappa - r_L) \tau s \) for the neighboring bank charging loan rate \( r_L \) and carrying loan scale \( \tau_s \). Inserting the competitive rate (21) and loan scale (22) into the loan scope gives

\[
\Delta_{i,c} = \frac{\sigma \kappa b}{\tau} \left( \frac{\xi - n (f\lambda - e_0)}{b\lambda} - \frac{1}{r_L} \right) + \frac{1}{n_c}, \tag{23}
\]
Equation (23) reveals another prediction of the model: bank lending adjusts spatially (across geographic areas or industries) in a positive way with net worth. Hence both loan scale (from (22)) and scope vary with the health of bank balance sheets.

Bord et al. (2017) document this variation in small business loan portfolios across geographic areas in the U.S. Furthermore, one can interpret $\Delta_{i,c}$ as a measure of bank specialization. The model predicts that as banks become more capitalized, they reach out into new geographic territories or industries, enhancing competition with existing banks in those areas. Acharya et al. (2006) and Berger et al. (2017) document significant variation in bank specialization across industries in the cross-section and through time. Paravisini et al. (2017) does the same for specialization across exporting countries.

6.4 Bank concentration

A last prediction of the model is higher concentration in the bank loan market after a drop in net worth. In the version of the model with unit demand, the number of banks is expressed analytically in (11) for the competitive equilibrium and (12) for the kinked. A drop in net worth raises the cost of equity, pushing out the average cost curve in Figure 7. A severe enough decline can transition the economy from the competitive to kinked case. But even within either type of equilibrium, a drop in net worth raises the cost of managing a bank. For lending to remain profitable, some banks must exit the loan market, including via consolidation, which reduces the number of banks $n$ that continue lending.

Supporting this prediction, Berger et al. (2017) find a significant increase in bank concentration in the U.S. C&I loan market after the financial crisis, which coincides with the high number of banks that failed and were sold at auction to healthier banks during that period, which Granja et al. (2017) document.

7 Conclusion

This paper presents a model in which the industrial organization of the bank credit market affects the real economy and monetary policy. A driving force of that competitive structure is the net worth of banks. A sufficient drop in aggregate bank equity transitions the economy to an equilibrium where banks consolidate for survival, retreat to local monopoly markets, and act as if they tacitly collude not to compete. A wide commercial loan spread and impaired interest rate pass-through transpire as a consequence.

An important contribution of this paper is to tie bank net worth to the degree of competition and specialization in the banking sector, as well as to the functioning of the interest rate channel of monetary policy. Lower bank net worth raises the cost of equity and encourages local monopolies. This development obstructs the interest rate channel.

Although the economic mechanism differs, the model in this paper relates to the literature on intermediary asset pricing (e.g., He and Krishnamurthy (2013); Brunnermeier and Sannikov (2014); Moreira and Savov (2017)). It does so in the following ways: (1) scarcity of specialist (bank) equity makes the spread of risky assets (loans) over safe assets (deposits) rise after
negative shocks to bank equity positions; (2) this elevated spread can persist as long as bank equity does not increase; (3) banks can profit from high loan spreads to internally recapitalize and lower their cost of capital; and (4) purchasing new bank equity issuance is especially effective at reducing the risky asset spread.

Key novelties here are that (1) the model can generate stickiness in the loan spread despite improvements in equity positions, as long as banks continue to refrain from competing; (2) monetary easing via a drop in the policy rate worsens the spread when banks are not competing; (3) banks shrink and expand the spatial scope of their portfolios in accordance with their net worth; (4) the spatial overlap of portfolios increases with competition and net worth; and (5) the model features entry into the lending market, whereas models in the intermediary asset pricing literature typically do not.

The effects of the industrial organization of banks on the real economy is a topic ripe for future research. Recent empirical work has further explored the issue. Scharfstein and Sunderam (2016) document diminished sensitivity of mortgage interest rates and home refinancing to MBS yields in counties with high bank concentration. Paravisini et al. (2017) find credit-supply shocks to a specialized bank have a disproportionate effect on exports to that bank’s country of expertise. Drechsler et al. (2017) find that after a policy rate hike, banks in more concentrated deposit markets cut back their lending more than banks in less concentrated markets. Bord et al. (2017) study how the variety in the health of bank balance sheets after the financial crisis changed the market share and aggregate supply of credit to small businesses. Much still remains to be studied.
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8 Internet Appendix A: Proofs

This appendix has the proofs of the paper and is supplemental.

8.1 Proof of Lemma 1

In a competitive equilibrium, the first order condition for optimality using the profit function from (3) and (4) is

$$\sigma \left( r_L + \Delta \frac{dr_L}{d\Delta} \right) + (1 - \sigma) \kappa = r_{\lambda,c}.$$

Substituting the slope of the competitive demand curve $\frac{dr_L}{d\Delta} = -\frac{\tau}{\sigma}$ and using the equilibrium condition $\Delta = \frac{1}{nc}$ gives the competitive equilibrium lending rate in (8).

In a kinked equilibrium, the first order condition for optimality does not hold with equality, so the lending rate is instead the monopoly demand curve. Solving (5) for the lending rate and setting $\Delta = \frac{1}{nk}$ gives (9).

8.2 Proof of Lemma 2

In both the competitive and kinked cases, the number of banks in equilibrium is determined by the profit condition in (10), reprinted here:

$$\frac{1}{n} \left( \sigma r_L + (1 - \sigma) \kappa \right) - r_{\lambda} \left( \frac{1}{n} + f \right) = v.$$

Substitute the competitive lending rate from (8) to get

$$\frac{\tau}{n} - nr_{\lambda,c} = nv.$$

Solving for $n$ gives (11).

In the kinked case, substitute the kinked lending rate from (9) into the profit condition to get

$$\sigma \kappa + (1 - \sigma) \kappa - (w + r_{\lambda,k}) - \frac{\tau}{2n} = n \left( v + fr_{\lambda,k} \right).$$

Solving for $n$ gives (12), which is real-valued so long as

$$\sigma \kappa + (1 - \sigma) \kappa - (w + r_{\lambda,k}) \geq \sqrt{2\tau \left( v + fr_{\lambda,k} \right)}.$$

There are two roots to the profit condition in the kinked case. The larger one is the only equilibrium number of banks by the reason given in the main text.

8.3 Proof of Lemma 3

The equilibrium cost of equity is found by inserting the number of banks given in Lemma 2 into the equity market clearing condition in (7) and solving for $r_e$. Re-arranging (7) gives

$$n = \frac{\xi - \lambda}{f \lambda - \varepsilon_0}.$$
Substitute the competitive number of banks from (11) into the above expression and re-arrange terms to get the competitive equilibrium cost of equity in (13). In the kinked case, substituting the number of banks from (12) into the equity market clearing condition and re-arranging gives

\[ r_{\lambda,k} = \frac{\phi - bv - \frac{\tau}{2}}{1 + bf}, \]

where \( \phi \equiv \sigma \pi + (1 - \sigma) \kappa - w \) and \( b \equiv \frac{\xi - \lambda}{f\lambda - e_0} \). Simplifying and isolating \( r_e \) delivers the kinked equilibrium cost of equity in (14).

### 8.4 Proof of Lemma 4

The comparative statics for \( v \) and \( \tau \) across the two equilibria are unambiguous. The derivative of the competitive cost of equity with respect to initial retained earnings \( e_0 \) is

\[ \frac{dr_{e,c}}{de_0} = -\frac{2\tau (f\lambda - e_0)}{f\lambda (\xi - \lambda)^2} < 0. \]

In the kinked case the derivative is

\[ \frac{dr_{e,k}}{de_0} = \frac{\tau (f\lambda - e_0) - \phi (\xi - \lambda)}{\lambda (f\xi - e_0) (\xi - \lambda)} - \frac{\tau (f\lambda - e_0)^2}{2\lambda (f\xi - e_0)^2 (\xi - \lambda)} + \frac{\phi (\xi - \lambda) (f\lambda - e_0) - v (\xi - \lambda)^2}{\lambda (f\xi - e_0)^2 (\xi - \lambda)}. \]

The second term is negative. The first term is negative if

\[ \frac{\tau}{\phi} < \frac{\xi - \lambda}{f\lambda - e_0}, \]

which holds under Assumption 1 provided \( \frac{\tau}{\phi} < 2 \). The third term is negative if

\[ \frac{\phi}{v} < \frac{\xi - \lambda}{f\lambda - e_0}, \]

which holds under the same assumption provided \( \frac{\phi}{v} < 2 \). Hence if both those conditions hold, \( \frac{dr_{e,k}}{de_0} < 0 \).

Regarding \( \xi \), the derivative of the competitive equity rate is

\[ \frac{dr_{e,c}}{d\xi} = -\frac{2\tau (f\lambda - e_0)^2}{f\lambda (\xi - \lambda)^3} < 0. \]

The derivative of the kinked equity rate is

\[ \frac{dr_{e,k}}{d\xi} = \frac{\phi (f\lambda - e_0) - 2v (\xi - \lambda)}{\lambda (f\xi - e_0) (\xi - \lambda)} + \frac{\tau (f\lambda - e_0)^2}{\lambda (f\xi - e_0)^2 (\xi - \lambda)} - \frac{\phi (\xi - \lambda) (f\lambda - e_0)}{\lambda (f\xi - e_0)^2 (\xi - \lambda)} \]

\[ + f \left[ \frac{\tau}{2} (f\lambda - e_0)^2 + v (\xi - \lambda)^2 - \phi (\xi - \lambda) (f\lambda - e_0) \right]. \]
The first term is negative if
\[
\frac{\phi}{v} < 2 \left( \frac{\xi - \lambda}{f\lambda - e_0} \right).
\]

By Assumption 1, this condition is satisfied if \( \frac{\phi}{v} < 4 \). A stricter condition that matches the previous condition for \( e_0 \) is \( \frac{\phi}{v} < 2 \). The other two terms are negative if
\[
\phi (\xi - \lambda) (f\lambda - e_0) > v (\xi - \lambda)^2 + \frac{\tau}{2} (f\lambda - e_0)^2.
\]
Since \( \xi - \lambda \geq 2 (f\lambda - e_0) \) by the assumption, this inequality is satisfied if
\[
2\phi (f\lambda - e_0)^2 > \left( v + \frac{\tau}{8} \right) (\xi - \lambda)^2
\]
Multiply both sides by 2 and take square roots to get
\[
2 (f\lambda - e_0) \sqrt{\phi} > (\xi - \lambda) \sqrt{2v + \frac{\tau}{4}}.
\]

### 8.5 Proof of Proposition 1

I present the conditions that separate equilibria. Those conditions are expressible as two quadratic polynomials defined over aggregate bank equity \( \xi > \lambda \). The larger roots of these polynomials will then define intervals that separate equilibria because the smaller roots will each be less than \( \lambda \).

Given a parameter vector \( \theta \), a necessary and sufficient condition for a monopoly equilibrium is that the optimal lending rate \( r_{L,m} \) is on the monopoly demand curve. Mathematically,
\[
r_{L,m} = \kappa - \frac{w}{\sigma} - \frac{\tau}{2\sigma n_m}.
\] (24)

A necessary and sufficient condition for a competitive equilibrium is that the optimal competitive lending rate is below the monopoly demand curve, so that
\[
r_{L,c} \leq \kappa - \frac{w}{\sigma} - \frac{\tau}{2\sigma n_c}.
\] (25)

Finally, for a kinked equilibrium, it must be that expected monopoly profits when the monopoly lending rate is replaced with the monopoly demand curve in (24) exceed \( v \). This condition distinguishes a kinked from monopoly equilibrium. Mathematically,
\[
\pi \bigg|_{r_{L,m}} = \pi - \frac{w}{\sigma} - \frac{\tau}{2\sigma n_m} > v,
\] (26)
where \( \pi \) is the bank profit function in (3) and the vertical line represents “conditional on.” The inequality in (26) is necessary, but not sufficient for a kinked equilibrium. The inequality also holds under a competitive equilibrium. Therefore, a necessary and sufficient condition for a kinked equilibrium is (26) and the failure of (25). If all three cases are unsatisfied, no
equilibrium exists under the parameter vector $\theta$.

The slope of the monopoly demand curve is one-half that of the competitive demand curve. Therefore, to obtain the monopoly lending rate, number of banks, and cost of capital, simply substitute $\tau$ in the competitive versions with $\frac{\tau}{2}$. Doing so gives the three monopoly objects:

$$r_{L,m} = \frac{1}{\sigma} \left( r_{\lambda,m} - (1 - \sigma) \kappa + \frac{\tau}{2n_m} \right),$$

$$n_m = \sqrt{\frac{\tau/2}{v + fr_{\lambda,m}}},$$

$$r_{\lambda,m} = \frac{\tau/2}{f} \left( \frac{f\lambda - e_0}{\xi - \lambda} \right)^2 - \frac{v}{f}.$$

Working with the three conditions, start with the bank expected profit function

$$\pi = \sigma r_L \Delta + (1 - \sigma) \kappa \Delta - r_{\lambda}(\Delta + f).$$

Now substitute $r_L$ with the monopoly demand curve $r_{L,m} = \kappa - \frac{w}{\sigma} - \frac{\tau}{2\sigma} \Delta$ to get

$$\pi = (\phi - r_{\lambda,m}) \Delta - \frac{\tau}{2} \Delta^2 - fr_{\lambda,m},$$

where again $\phi \equiv \sigma \kappa + (1 - \sigma) \kappa - w$. Next solve for the optimal $\Delta$ to get

$$\Delta^* = \frac{\phi - r_{\lambda,m}}{\tau}.$$

Substitute $\Delta^*$ back into the profit function to obtain the value function

$$V = \frac{(\phi - r_{\lambda,m})^2}{2\tau} - fr_{\lambda,m}.$$

Condition (26) implies that the value function $V$ must exceed the equilibrium profits $v$. Substitute the monopoly equilibrium cost of capital $r_{\lambda,m}$ given above to get

$$(v + f\phi) \xi^2 - [2\lambda (v + f\phi) + f\tau (f\lambda - e_0)] \xi + \left[ \lambda^2 (v + f\phi) + \frac{\tau}{2} (f^2 \lambda^2 - e_0^2) \right] > 0.$$

Let $H_1$ denote the left-hand side of this inequality. The function $H_1$ is a quadratic polynomial defined over the domain $\xi > \lambda$. The value $H_1(0) > 0$, $H_1'(0) < 0$, and the leading coefficient is positive. These properties imply the quadratic is concave up and cuts the y-axis from above.

The discriminant $D$ of the quadratic is

$$D = \tau (f\lambda - e_0)^2 \left( \tau f^2 + 2 (v + f\phi) \right) > 0,$$

meaning both roots are real. Let $r_1$ and $r_2$ denote the two roots with $r_1 < r_2$. The product of the
two roots is
\[
r_{1,2} = \frac{\lambda^2 (v + f\phi) + \frac{\tau}{2} (f^2\lambda^2 - e_0^2)}{v + f\phi} > 0,
\]
meaning the roots share the same sign. The sum of the two roots is
\[
r_1 + r_2 = \frac{2\lambda (v + f\phi) + f\tau (f\lambda - e_0)}{v + f\phi} > 0,
\]
meaning both are positive. Apply the quadratic formula to get
\[
r_1, r_2 = \lambda + \frac{1}{2} \left( \frac{f\lambda - e_0}{v + f\phi} \right) \left( \tau f \pm \sqrt{\tau^2 f^2 + 2\tau (v + f\phi)} \right).
\]
The larger root \(r_2 > \lambda\). However, the term
\[
\tau f - \sqrt{\tau^2 f^2 + 2\tau (v + f\phi)} < 0,
\]
implying \(r_1 < \lambda\). Because a kinked equilibrium is defined in part by values of \(\xi\) such that \(H_1(\xi) > 0\) and \(\xi > \lambda\), only root \(r_2\) is viable.

Reclassify the root \(h_1 \equiv r_2\). I reprint it below:
\[
h_1 = \lambda + \frac{1}{2} \left( \frac{f\lambda - e_0}{v + f\phi} \right) \left( \tau f + \sqrt{\tau^2 f^2 + 2\tau (v + f\phi)} \right).
\]
One condition on external bank equity that defines the kinked equilibrium is \(\xi > h_1\) because values of \(\xi\) in that interval satisfy \(\xi > \lambda\) and (26).

The condition for the competitive equilibrium can be determined by substituting the equilibrium competitive lending rate, cost of equity, and number of banks into (25). Doing so gives
\[
(v + f\phi)\xi^2 - \left[ 2\lambda (v + f\phi) + \frac{3}{2} \tau f (f\lambda - e_0) \right] \xi + \left[ \lambda^2 (v + f\phi) + \frac{\tau}{2} (f^2\lambda^2 + f\lambda e_0 - 2e_0^2) \right] \geq 0.
\]
Define \(H_2\) as the quadratic polynomial on the left-hand side of the inequality above. By Assumption 1, the constant term is positive because
\[
f^2\lambda^2 + f\lambda e_0 - 2e_0^2 > e_0^2 + f\lambda e_0 - 2e_0^2
= e_0 (f\lambda - e_0)
> 0.
\]
Like \(H_1\), this quadratic is defined over the positive interval \(\xi > \lambda\). Also like for \(H_1\), one can show that both roots of \(H_2\) are real and positive, but the smaller root is less than \(\lambda\) so can be
ruled out. The larger root, denoted $h_2$, is

$$h_2 = \lambda + \frac{1}{2} \left( \frac{f\lambda - e_0}{v + f\phi} \right) \left( \frac{3}{2} \frac{\tau f}{s_1} + \sqrt{\tau \left( \frac{9}{4} \tau f^2 + 4(v + f\phi) \right)} \right).$$

Because $H_2(\xi) \geq 0$ for values of $\xi \geq h_2$, the equilibrium is competitive over this interval of external equity.

One can see that $h_2 > h_1$. Therefore, if the value of external bank equity $\xi \in (h_1, h_2)$, then (26) is satisfied and (25) fails, making the equilibrium kinked.

### 8.6 Proof of Lemma 5

Take the difference $\eta \equiv h_2 - h_1$ to get

$$\eta = \frac{1}{2} \left( \frac{f\lambda - e_0}{v + f\phi} \right) \left( \frac{1}{2} \frac{\tau f}{s_1} + \sqrt{\tau \left( \frac{9}{4} \tau f^2 + 4(v + f\phi) \right)} - \sqrt{\tau (\tau f^2 + 2(v + f\phi))} \right).$$

The interval is clearly increasing in $\lambda$ and decreasing in $e_0$. The comparative static with respect to $v$ is

$$\frac{\partial \eta}{\partial v} = -\left( \frac{f\lambda - e_0}{v + f\phi} \right) \left[ \tau f s_2 (\tau f + 2s_1) + 4\tau (2s_2 - s_1) (\tau f^2 + v + f\phi) \right] \frac{8(v + f\phi)^2 s_1 s_2}{s_1 s_2},$$

where

$$s_1 = \sqrt{\tau \left( \frac{9}{4} \tau f^2 + 4(v + f\phi) \right)},$$

$$s_2 = \sqrt{\tau (\tau f^2 + 2(v + f\phi))}.$$

Because $2s_2 - s_1 > 0$, the comparative static is unambiguously negative.

### 8.7 Proof of Lemma 6

For the kinked lending rate, substitute the expected net return of the project to the entrepreneur

$$v_s(i) = \sigma b \left( \frac{\pi}{r_{L,i}} - 1 \right)$$

into the monopoly demand curve $\Delta_{i,m} = \frac{v_s(i) - w}{\sigma r_{L,i}}$. Solving for the lending rate and using the equilibrium condition $\Delta_{i,m} = \frac{1}{n_k}$ in a kinked equilibrium delivers (20).

For the competitive lending rate, start with the first derivative of the profit function with respect to the lending rate $r_L$

$$\pi' = \sigma r_L (\Delta' + \Delta_0') + \sigma \Delta - r_\lambda (\Delta' + \Delta_0').$$
This makes the first order condition in the competitive case

\[(\sigma r_{L,c} - r_\lambda) (\Delta' + \Delta'') = -\sigma \Delta'.\]

In the competitive equilibrium, \(\Delta_{i,c} = \frac{1}{n_c}\). Furthermore, \(\iota' = -\frac{b}{r_{L,c}}\) and \(\Delta'_{i,c} = -\frac{\sigma b}{r_{L,c}}\). Making these substitutions and solving for \(r_{L,c}\) gives (19).
9 Internet Appendix B: Extensions

This appendix has extensions of the main model and is supplemental.

9.1 An endogenous equity capital requirement

I provide an extension of the model that has an endogenous equity capital requirement. The setting features project outcomes that are correlated. The size of a bank’s loan portfolio $\Delta_i$ will now directly influence its level of diversification and the amount of external equity capital it must issue. To simplify, I set the probability of success $\sigma = \frac{1}{2}$ and starting retained earnings $e_{0,i} = 0$. I study interest rate pass-through in kinked and competitive equilibria in this new environment.

Projects still produce one of two possible returns $\pi$ or $\kappa$ at the end of the second period. However, the probability that a project produces the high return now takes a special form. This form allows all projects to bear the same expected probability of success prior to financing, but different probabilities after initiation. At the beginning of the first period, the probability that project $j$ on the circle reaches the high state is random. This random probability takes the form

$$\tilde{\Pr} (H|j, \tilde{u}) = \frac{1}{2} \left( 1 + \cos \left( 2\pi \left( j + \tilde{u} \right) \right) \right),$$

(27)

where $\tilde{u} \sim U [0, 1]$. The object in (27) is a random measure that maps a realization of the uniform shock $\tilde{u}$ to a probability distribution over the two states at location $j$. I call (27) the success probability of a project.

The periodicity of the cosine function guarantees $\tilde{\Pr} : [0, 1] \mapsto [0, 1]$. The shock $\tilde{u}$ is realized at the end of the first period. Properties of a project’s success probability are presented in Lemma 7.

Lemma 7. The success probability in (27) satisfies the following properties:

1. (Distributional symmetry) The probability density function of a project’s success probability is the same at all locations.
2. (Mean and variance) Each project is expected to succeed half the time with variance $\frac{1}{8}$.
3. (Distance-dependent covariance) The covariance between projects $j$ and $k$ in their success probabilities is $\frac{1}{8} \cos \left( 2\pi \left( j - k \right) \right)$.

The form in (27) is a way to make a project’s probability of reaching the high state invariant to its location. Prior to the realization of $\tilde{u}$, a project’s outcome distribution cannot be distinguished from its neighbors’ because all projects bear the same uncertainty of success. As a result, projects at every location share the same expected probability of generating the high return $\pi$—namely, $\frac{1}{2}$.

Another important feature of production is that the covariance of success probabilities between projects depends exclusively on the distance between those projects rather than on their locations. From the lemma, the correlation between the success probabilities of projects...
located at positions $j$ and $k$ on the circle is

$$\text{corr} \left( \tilde{\Pr}(H|j, \tilde{u}), \tilde{\Pr}(H|k, \tilde{u}) \right) = \cos (2\pi (j - k)).$$

The above expression implies projects located near one another on the circle have more positively correlated probabilities of success than those located farther apart. Projects positioned opposite one another on the circle have the lowest correlated probability. This correlation structure is meant to capture the notion of integrated industries (e.g., metals and automobiles) or nearby geographic areas (e.g., neighboring cities) sharing more correlated production outcomes than more “distant” ones.

Figures 10(a)-10(b) present an illustration of project uncertainty. At the start of the first period, each project around the circle bears the same uncertainty of project success, having one-half chance of yielding the high return. Once the shock $\tilde{u}$ is drawn at the end of the first period, projects bear different probabilities of success according to their locations. Those close to one another on the circle share similar likelihoods of yielding the high return.

Figure 10: Example of Success Probabilities across Projects

Notes: At the beginning of the first period, all projects share the same expected success probability of one-half. This common probability of success in expectation is represented in Figure 10(a) by the color yellow along the entire circle. In the middle of the period, the shock $\tilde{u}$ is realized. The example in the figure has a realized value of $u = 0$. At that moment, projects differ in their success probabilities according to (27). In Figure 10(b), arcs of the circle with projects having high success probability are colored green. Arcs with projects having low success probability are colored red. The four numbers positioned around the circle are the success probabilities of the projects located at those positions.

A project's life follows this sequence: at the beginning of the first period, the project is
financed and the entrepreneur’s investment is made. At the end of the first period (which is
also the beginning of the second period), \( \tilde{u} \) is realized, which determines the project’s actual
probability of the high return, denoted \( \Pr (H|j,u) \). No action related to project financing or the
project itself can be made at that time. Finally, at the end of the second period, the project
produces either the high or low output.

9.1.1 Bank diversification

The new form of project uncertainty prevents banks from perfectly diversifying their portfolios
with infinitesimal lending arcs. With correlated project returns, the degree of bank diversifica-
tion will depend on the size of the arc. For a given \( \Delta_i \), a bank’s average probability per project
of receiving payment \( r_{L,i} \) prior to the realization of \( \tilde{u} \) is

\[
\tilde{\Pr} (H|j,\tilde{u}) = \frac{1}{\Delta_i} \int_{-(\Delta_i)/2}^{(\Delta_i)/2} \frac{1}{2} \left( 1 + \cos \left( 2\pi (i + j + \tilde{u}) \right) \right) dj,
\]

\[
= \frac{1}{2} + \frac{\sin (\pi \Delta_i)}{\Delta_i} \frac{\cos \left( 2\pi (i + \tilde{u}) \right)}{2\pi}.
\]

I call (28) the repayment rate of bank \( i \)’s loan portfolio, as it is the fraction of projects whose
owners can repay the bank.

Two components comprise the repayment rate of a portfolio: diversification and residual
uncertainty. The diversification component captures the reduction in the uncertainty of a loan
portfolio’s payoff from the bank choosing a larger arc length around the circle. The residual
uncertainty component reflects the risk that remains in a loan portfolio that is imperfectly
diversified.

Important properties of the repayment rate are presented in Lemma 8.

Lemma 8. The repayment rate of bank \( i \)’s portfolio satisfies the following properties:

1. (Common mean) The expected repayment rate is always \( \frac{1}{2} \), no matter the choice of \( \Delta_i \).
2. (No diversification) As \( \Delta_i \downarrow 0 \), the bank’s repayment rate approaches the same probability
   of a single project succeeding.
3. (Declining variance) As \( \Delta_i \) increases, the variance of the repayment rate declines.
4. (Perfect diversification) As \( \Delta_i \uparrow 1 \), the repayment rate approaches \( \frac{1}{2} \), no matter the realiza-
   tion of \( \tilde{u} \).

The repayment rate of an imperfectly diversified bank is a random variable prior to the
realization of \( \tilde{u} \). Regardless of a bank’s arc length, though, the expected repayment rate on its
portfolio is always \( \frac{1}{2} \) : the bank expects half its loan portfolio to repay and half to default.

As a bank lends to more and more entrepreneurs around the circle, it reduces the variability
of its repayment rate by diversifying its loan portfolio. Eventually, if a bank lends the circum-
ference of the circle, its portfolio becomes risk-free and immune to the random realization of $\bar{u}$. In this case, half the portfolio will succeed and half will fail.

### 9.1.2 Bank capital structure

A bank’s capital structure will consist of equity and deposits again. Bank equity holders perfectly observe the uniform shock $\bar{u}$ and thus the realized profits of their bank’s loan portfolio. Depositors, on the other hand, do not. Depositors know the loan repayment rate function in (28), however, and hence they are certain of their bank’s minimum possible repayment rate. For this reason, the minimum profit of the loan portfolio is the maximum amount depositors can prove and recover from the bank in bankruptcy court. Depositors are only willing to finance their bank up to this amount.

Depositor preference for safe assets can also generate this contract, as bank debt, unlike bank equity, will bear no risk. Deposits can be considered fully collateralized by the minimum loan portfolio return of the bank.

The minimum of (28) over the shock $\bar{u}$ is

$$\Pr_{\min} (r_{L,i} \mid \Delta_i) = \frac{1}{2} \left( 1 - \frac{\sin (\pi \Delta_i)}{\pi \Delta_i} \right).$$

Denote the minimum loan profits for bank $i$ as $\pi_{i,\min}$. Depositors are willing to lend an amount up to the discounted face value of $\pi_{i,\min}$. Because deposits are safe, the deposit rate is the risk-free interest rate $r$. Let $d_i$ be the amount of deposits a bank chooses. Banks can raise deposits up to the amount

$$d_i \leq \frac{\pi_{i,\min}}{r}. \tag{29}$$

I call the maximum amount a bank can raise in deposits the *debt capacity* of the bank. Whatever additional outside financial capital a bank requires to finance its operations it obtains from the equity market at the required expected equity return $r_e$. Like in the main model, equity will be at least as expensive as debt ($r_e \geq r$) in equilibrium.

The minimum repayment rate of a bank determines its debt capacity, which influences its leverage. As a bank increases $\Delta_i$, it expands lending operations to more and more industries or areas across the circle and diversifies its portfolio. Depositors, in turn, are willing to lend more to the bank. The minimum repayment rate of the bank is increasing in $\Delta_i$ and so too will its debt capacity and leverage. Liang and Rhoades (1991), McAllister and McManus (1993) and Demsetz and Strahan (1997) document a positive correlation between bank diversification and leverage.

The debt capacity condition in (29) can equivalently be considered a minimum equity capital requirement that is imposed by the market rather than an outside rule as in the model of the main text. Denote the total assets of the bank by $a_i = \Delta_i + f$. Substituting the balance

---

$^{15}$Bank $i$ suffers its minimum repayment rate if the shock lands at location $|\frac{1}{2} - i|$ if $i \in \left[0, \frac{1}{2}\right]$ or location $1 - |\frac{1}{2} - i|$ if $i \in \left(\frac{1}{2}, 1\right)$. If the shock lands at some other location for all $i \in \mathcal{N}$, where $\mathcal{N}$ is the set of bank locations, every bank’s realized repayment rate exceeds the minimum.
sheet identity $a_i = d_i + e_i$, the constraint can be written as $e_i \geq \Delta_i + f - \frac{\pi_{i, \text{min}}}{r}$. So rather than choosing an amount $d_i$ in deposits, the bank instead chooses an amount $e_i$ in equity, provided its choice satisfies a minimum amount.

### 9.1.3 Bank decision

With this set-up, expected profits of a typical bank are

$$\pi_i = \frac{1}{2} r_{L,i} \Delta_i + \frac{1}{2} \kappa \Delta_i - FC(\Delta_i).$$

(30)

The financing cost function $FC(\Delta_i)$ consists of the payments to depositors and equity holders. The function is

$$FC(\Delta_i) = rd_i + r_e(\Delta_i + f - d_i).$$

(31)

The minimum loan profits that determine the constraint of (29) are

$$\pi_{i, \text{min}} = \text{Pr}_{\text{min}}(r_{L,i}|\Delta_i) r_{L,i} \Delta_i \quad + \quad (1 - \text{Pr}_{\text{min}}(r_{L,i}|\Delta_i)) \kappa \Delta_i.$$

The bank maximizes (30) subject to (29).

### 9.1.4 Monetary transmission

Entrepreneur preferences are identical to those in the main text and so are the demand curves for bank credit. Because $r_e \geq r$ in equilibrium, (29) will bind. Solving a bank’s problem gives the competitive and kinked lending rates. Proposition 2 presents the rates.

**Proposition 2.** (Lending rates) The bank lending rate in a competitive equilibrium with an endogenous equity capital requirement is

$$r_{L,c} = \frac{r_{e,c} + \frac{1}{2} \left( \frac{2 \tau}{n_c} - \kappa \right) + \left( \frac{r_{e,c}}{r} - 1 \right) \phi \left( \frac{1}{n_c} \right)}{\frac{1}{2} + \left( \frac{r_{e,c}}{r} - 1 \right) \psi \left( \frac{1}{n_c} \right)},$$

(32)

where the functions $\phi$ and $\psi$ are defined in Appendix 9.1.10 by equations (38) and (39), respectively.

The kinked equilibrium lending rate is

$$r_{L,k} = \bar{r} - 2w - \frac{T}{n_k}.$$

(33)

### 9.1.5 Perfect pass-through

Consider first the competitive lending rate. Suppose the supply of equity were so large the equity market cleared at the lower-bound price $r_e = r$. In this situation, deposits and equity are perfect substitutes, so the bank faces a single cost of financial capital $r$. Because the bank
could finance itself entirely with equity, the constraint (29) would be slack. The functions $\phi$ and $\psi$ in (32) reflect a bank’s debt capacity. They enter the lending rate if the constraint binds. Hence, they are set to zero for now.

The lending rate in such a competitive equilibrium would be

$$r_{L,c} = 2r - \kappa + \frac{2\tau}{n_c}.$$  \hspace{1cm} (34)

The competitive lending rate is analogous to the one in the main text given in (8) and consists of the marginal cost of financing plus a markup. The interest rate enters the bank lending rate linearly, so perfect pass-through occurs again because of competition.

9.1.6 Imperfect pass-through

Now suppose equity were scarce, so that the equity market cleared at price $r_e > r$. A bank would have strict preference for cheaper deposit financing, so the credit constraint would bind. The debt capacity of the bank now becomes important when the bank chooses the competitive loan rate.

The loan rate is now (32). The rate reflects the blend of debt and equity in the bank’s capital structure. The functions $\phi$ and $\psi$ capture this blend. They adjust the marginal cost of financing after incremental growth in the loan portfolio. This fact can be seen clearly from Lemma 9, which presents a typical bank’s marginal financing cost function $FC' (\Delta_i)$ as it increases its portfolio size $\Delta_i$.

Lemma 9. The marginal financing cost function of a typical bank $i$ is

$$\frac{dFC (\Delta_i)}{d\Delta_i} = r_e + \left(\frac{r_e}{r} - 1\right) (\phi (\Delta_i) - \psi (\Delta_i) r_{L,i}).$$  \hspace{1cm} (35)

A marginal increase in a bank’s loan portfolio has two effects on its cost of funding. The first effect is a higher financing cost from the need for more equity to fund the portfolio at price $r_e$. The second effect is a decrease in the cost of funding as the bank tilts its capital structure to cheaper debt financing because of greater diversification.

The second term in (35) is the cost savings from diversification. It reflects changes in the minimum possible profit $\pi_{i,\text{min}}$, which influence a bank’s debt capacity. For a fixed lending rate $r_{L,i}$, an expansion in $\Delta_i$ increases the debt capacity of the bank at rate $\psi$. The higher debt capacity generates financing cost savings at rate $\left(\frac{r_e}{r} - 1\right)$, which are passed onto entrepreneurs via a lower lending rate.

Greater diversification decreases a bank’s minimum failure rate on its portfolio $(1 - Pr_{\text{min}})$. While a lower failure rate means the bank will receive payment $r_{L,i}$ on more of its loans, it also means the bank will retrieve the low output $\kappa$ on less of its loans, as fewer will default. Less recovery values from fewer defaults reduces the minimum loan profit of the bank and its debt capacity. The function $\phi$ is the rate at which debt capacity decreases with $\Delta_i$. Lower debt capacity increases marginal financing costs, which raises the competitive lending rate.
When the market’s constraint on bank financing binds, the degree of pass-through now relies on bank capital structure. The relation is non-linear and imperfect. Interest rate pass-through depends on the functions $\phi$ and $\psi$, which reflect a bank’s debt capacity and its diversification. The pass-through also depends on the cost of equity capital $r_e$, which will be a function of aggregate bank net worth.

### 9.1.7 Negative pass-through

In a kinked equilibrium, the interest rate channel is closed completely in the “short run” when the number of banks $n_k$ is held fixed for the same reasons as in given in the main model. The only effect a lower interest rate has on bank lending rates in a kinked equilibrium will be indirect through adjustments in the number of banks $n_k$ over the “long run.”

A lower interest rate will reduce the average cost of operating a bank, increase profits, and encourage entry into the lending market. An important perverse feature of the kinked lending rate in (9), however, is that more banks in the credit market actually leads all of them to raise their loan rates.

In a kinked credit market, banks are local monopolists. More banks on the circle means that an entrepreneur can find one that specializes in an industry or area “closer” to the entrepreneur’s. A bank takes advantage of its greater local monopoly power by charging a higher lending rate.

Conversely, fewer banks lead all of them to reduce their loan rates. When a bank exits the lending market, the average entrepreneur needs to “travel” a longer distance on the circle and contract with a bank that is less specialized in his or her particular industry or location than before. Undertaking the project becomes less attractive to the entrepreneur relative to the outside option $w$. Because a typical bank in a kinked market is competing against its borrowers’ outside options, it needs to lower the lending rate to encourage the entrepreneur to borrow instead.

By encouraging bank entry, an accommodative monetary policy has the unintended effect of increasing the cost of bank credit to firms and worsening the commercial loan spread. I call a decrease to the policy rate that leads to an increase in the bank loan rate (and vice versa) “negative pass-through.”

To study the effects of a policy rate cut, consider again the expected profit function of a bank:

$$\pi_i = \frac{1}{2} \left(r_{L,i} + \kappa\right) \Delta_i - r_e \left(\Delta_i + f\right) + \left(r_e - r\right) \frac{\pi_{i,\min}}{r}.$$

A direct effect of a decline in $r$ is to encourage bank entry from higher expected profits because of lower funding costs. Another first order effect works in the same direction: a lower policy rate increases the debt capacity $\frac{\pi_{i,\min}}{r}$ of a bank, allowing it to raise cheaper debt financing over equity capital. This effect can be considered an “asset pricing” channel of monetary policy in this extension because the rate cut increases the value of the collateral banks post to raise cheaper debt, which increases expected bank profits. These first order effects lead to more
entry and negative pass-through.

One second order general equilibrium effect works in the same direction: higher debt capacities reduce demand for equity capital, which lowers the cost of equity $r_e$ and further increases expected profits and entry. Another second order effect comes from the reaction of the equity market and works in the opposing direction: more banks in the lending market increases the demand for equity capital, which puts upward pressure on $r_e$ and dampens entry. A final second order effect also works in the opposite direction: more banks on the circle reduces the arc length of each one and the minimum possible profits $\pi_{i,\text{min}}$, which limits the capacity to raise cheaper debt and reduces entry.

Provided the positive forces for entry dominate, a kinked equilibrium features negative pass-through once accounting for entry. Figure 11 illustrates the effect of a lower interest rate on the average cost curve that encourages bank entry and negative pass-through. The average cost curve shifts inward from the lower funding costs, which raises profits of existing banks and gives reason for other banks to enter the lending market.

Figure 11: Bank Entry from Lower Interest Rate, Kinked Equilibrium

Notes: The policy rate declines from $r$ to $\bar{r}$, which shifts the red average cost curve inward from the dotted to solid line. This change leads the number of banks to increase from $n$ to $\bar{n}$, which pushes the blue average revenue curve inward from the dotted to solid line. In the example, ex post profits $v = 0$. 

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9.1.8 Proof of Lemma 7

The success probability of a project at location $j$ from (27) is

$$\tilde{\Pr} (H|j, \tilde{u}) = \frac{1}{2} \left( 1 + \cos \left( 2\pi (j + \tilde{u}) \right) \right)$$

The success probability can be treated as a transformation of a uniform random variable $x_j = 2\pi (j + \tilde{u})$, which has support $[2\pi j, 2\pi (j + 1)]$. For ease of notation, define

$$Y_j \equiv \frac{1}{2} \left( 1 + \cos (x_j) \right).$$

For $y \in [0, 1]$, the equation

$$y = \frac{1}{2} \left( 1 + \cos (x_j) \right)$$

has two solutions in $[2\pi j, 2\pi (j + 1)]$. Therefore, the transformed density is

$$f_{Y_j}(y) = 2 \times \frac{2}{\sqrt{1 - y^2}} \times \frac{1}{2\pi} = \frac{2}{\pi \sqrt{1 - y^2}},$$

for $y \in [0, 1]$ and zero otherwise for all $j$. The leading factor of 2 accounts for the two solutions in the support. Thus, the density of the success probability is the same at all locations.

The expected probability of success for a single project is $\frac{1}{2}$. Because $\tilde{u} \sim U[0, 1]$, integrate the success probability over the unit interval to get

$$E \left[ \tilde{\Pr} (H|j, \tilde{u}) \right] = \int_0^1 \frac{1}{2} \left( 1 + \cos \left( 2\pi (j + \tilde{u}) \right) \right) d\tilde{u},$$

$$= \frac{1}{2} + \frac{1}{2\pi} \left[ \sin \left( 2\pi (j + 1) \right) - \sin \left( 2\pi j \right) \right],$$

$$= \frac{1}{2} + \frac{1}{2\pi} \left[ \sin \left( 2\pi j \right) \cos (2\pi) + \cos \left( 2\pi j \right) \sin (2\pi) - \sin \left( 2\pi j \right) \right],$$

$$= \frac{1}{2} + \frac{1}{2\pi} \left[ \sin \left( 2\pi j \right) - \sin \left( 2\pi j \right) \right],$$

$$= \frac{1}{2},$$

where the third equality follows from the sum-difference formula for sine.
The variance of the success probability is

\[
\sigma^2 \left[ \tilde{Pr}(H|j, \tilde{u}) \right] = E \left[ \tilde{Pr}(H|j, \tilde{u})^2 \right] - E \left[ \tilde{Pr}(H|j, \tilde{u}) \right]^2 ,
\]

\[
= \frac{1}{4} E \left[ \left( 1 + \cos \left( 2\pi (j + \tilde{u}) \right) \right)^2 \right] - \frac{1}{4},
\]

\[
= \frac{1}{4} \int_0^1 \cos^2 \left( 2\pi (j + \tilde{u}) \right) d\tilde{u},
\]

Using the half-angle trigonometric formula \( \cos^2 u = \frac{1 + \cos(2u)}{2} \), the variance can be written as

\[
\sigma^2 \left[ \tilde{Pr}(H|j, \tilde{u}) \right] = \frac{1}{8} \int_0^1 \left[ 1 + \cos \left( 4\pi (j + \tilde{u}) \right) \right] d\tilde{u},
\]

\[
= \frac{1}{8} \left[ 1 + \frac{1}{4\pi} \left[ \sin \left( 4\pi (j + 1) \right) - \sin \left( 4\pi j \right) \right] \right],
\]

\[
= \frac{1}{8}.
\]

Finally, the covariance of success probabilities between projects located at \( j \) and \( k \) on the circle is

\[
cov \left( \tilde{Pr}(H|j, \tilde{u}), \tilde{Pr}(H|k, \tilde{u}) \right) = E \left[ \tilde{Pr}(H|j, \tilde{u}) \tilde{Pr}(H|k, \tilde{u}) \right] - \frac{1}{4},
\]

\[
= \frac{1}{4} E \left[ \cos \left( 2\pi (j + \tilde{u}) \right) \cos \left( 2\pi (k + \tilde{u}) \right) \right].
\]

Using the cosine product formula \( \cos(a) \cos(b) = \frac{1}{2} \left[ \cos(a + b) + \cos(a - b) \right] \) gives

\[
cov \left( \tilde{Pr}(H|j, \tilde{u}), \tilde{Pr}(H|j, \tilde{u}) \right) = \frac{1}{8} \cos \left( 2\pi (j - k) \right) + \frac{1}{8} E \left[ \cos \left( 2\pi (j + k + 2\tilde{u}) \right) \right],
\]

\[
= \frac{1}{8} \cos \left( 2\pi (j - k) \right) + \frac{1}{8} \int_0^1 \cos \left( 2\pi (j + k + 2\tilde{u}) \right) d\tilde{u},
\]

\[
= \frac{1}{8} \cos \left( 2\pi (j - k) \right).
\]

9.1.9 Proof of Lemma 8

The repayment rate of bank \( i \) conditional on its chosen arc length \( \Delta_i \) is

\[
\tilde{Pr} \left( r_{L,i}|\Delta_i, \tilde{u} \right) = \frac{1}{2} + \frac{\sin (\pi \Delta_i) \cos \left( 2\pi (i + \tilde{u}) \right)}{\Delta_i \cdot 2\pi}.
\]
As $\Delta_i \downarrow 0$, the bank’s lending arc reduces to its home location. Apply L’Hôpital’s rule to get

$$
\lim_{\Delta_i \downarrow 0} \tilde{\Pr} (r_{L,i}|\Delta_i, \tilde{u}) = \lim_{\Delta_i \downarrow 0} \left\{ \frac{1}{2} + \pi \cos (\pi \Delta_i) \frac{\cos \left( \frac{2\pi (i + \tilde{u})}{2\pi} \right)}{2\pi} \right\},
$$

$$
= \frac{1}{2} \left( 1 + \cos \left( \frac{2\pi (i + \tilde{u})}{2\pi} \right) \right),
$$

$$
= \tilde{\Pr} (H|j, \tilde{u}).
$$

Next, to see that the expected repayment rate of a bank’s portfolio is always $\frac{1}{2}$, no matter its arc length $\Delta_i$, integrate the repayment rate over the unit interval:

$$
E \left[ \tilde{\Pr} (r_{L,i}|\Delta_i, \tilde{u}) \right] = \int_0^1 \left[ \frac{1}{2} + \frac{\sin (\pi \Delta_i) \cos \left( \frac{2\pi (i + \tilde{u})}{2\pi} \right)}{\Delta_i} \right] d\tilde{u},
$$

$$
= \frac{1}{2} + \frac{\sin (\pi \Delta_i)}{(2\pi)^2 \Delta_i} \left[ \sin (2\pi (i + 1)) - \sin (2\pi i) \right],
$$

$$
= \frac{1}{2}.
$$

The variance of the repayment rate is

$$
\sigma^2 \left[ \tilde{\Pr} (r_{L,i}|\Delta_i, \tilde{u}) \right] = \int_0^1 \left( \tilde{\Pr} (r_{L,i}|\Delta_i, \tilde{u}) - E \left[ \tilde{\Pr} (r_{L,i}|\Delta_i, \tilde{u}) \right] \right)^2 d\tilde{u},
$$

$$
= \int_0^1 \left( \tilde{\Pr} (r_{L,i}|\Delta_i, \tilde{u}) - \frac{1}{2} \right)^2 d\tilde{u},
$$

$$
= \int_0^1 \left( \frac{\sin (\pi \Delta_i) \cos \left( \frac{2\pi (i + \tilde{u})}{2\pi} \right)}{\Delta_i} \right)^2 d\tilde{u},
$$

$$
= \left[ \frac{\sin (\pi \Delta_i)}{2\pi \Delta_i} \right]^2 \int_0^1 \cos^2 \left( \frac{2\pi (i + \tilde{u})}{2\pi} \right) d\tilde{u}.
$$

Using the half-angle formula again, the variance can be written as

$$
\sigma^2 \left[ \tilde{\Pr} (r_{L,i}|\Delta_i, \tilde{u}) \right] = \frac{1}{2} \left[ \frac{\sin (\pi \Delta_i)}{2\pi \Delta_i} \right]^2 \int_0^1 \left[ 1 + \cos (4\pi (i + \tilde{u})) \right] d\tilde{u},
$$

$$
= \frac{1}{2} \left[ \frac{\sin (\pi \Delta_i)}{2\pi \Delta_i} \right]^2 + \frac{1}{8\pi} \left[ \frac{\sin (\pi \Delta_i)}{2\pi \Delta_i} \right]^2 \left[ \sin (4\pi (i + 1)) - \sin (4\pi i) \right],
$$

$$
= \frac{1}{2} \left[ \frac{\sin (\pi \Delta_i)}{2\pi \Delta_i} \right]^2.
$$

The variance of the repayment rate is strictly decreasing for $\Delta_i \in (0, 1)$. To see why, take
the first derivative of (36) with respect to $\Delta_i$:

$$
\frac{\partial \sigma^2}{\partial \Delta_i} \left[ \Pr (r_{L,i} | \Delta_i, \tilde{u}) \right] = \left[ \frac{\sin (\pi \Delta_i)}{2\pi \Delta_i} \right] \frac{\partial \sin(\pi \Delta_i)}{\partial \Delta_i} \\
= \left[ \frac{\sin (\pi \Delta_i)}{2\pi \Delta_i} \right] \left[ \frac{\pi \Delta_i \cos (\pi \Delta_i) - \sin (\pi \Delta_i)}{2\pi (\Delta_i)^2} \right],
$$

$$
= \frac{\pi \Delta_i \cos (\pi \Delta_i) \sin (\pi \Delta_i) - \sin^2 (\pi \Delta_i)}{4\pi^2 (\Delta_i)^3}.
$$

(37)

The sign of $\frac{\partial \sigma^2}{\partial \Delta_i} \left[ \Pr (r_{L,i} | \Delta_i, \tilde{u}) \right]$ is determined by the numerator of (37), as the denominator is always positive. The variance is non-increasing in $\Delta_i$ if

$$
\pi \Delta_i \cos (\pi \Delta_i) \sin (\pi \Delta_i) - \sin^2 (\pi \Delta_i) \leq 0.
$$

For $\Delta_i = 0$ or $\Delta_i = 1$, the numerator is zero, so the above inequality holds. For $\Delta_i \in (0, 1)$, $\sin (\pi \Delta_i) \neq 0$, so the expression can be written as

$$
\pi \Delta_i \cos (\pi \Delta_i) \leq \sin (\pi \Delta_i).
$$

Perform a change of variable $\theta = \pi \Delta_i$. The aim is to show

$$
\theta \cos \theta < \sin \theta
$$

over the domain $\theta \in (0, \pi)$. Because $\cos \theta \leq 0$ and $\sin \theta > 0$ over the interval $\theta \in \left[ \frac{\pi}{2}, \pi \right)$, the relation holds there. Now define the function

$$
f (\theta) \equiv \sin \theta - \theta \cos \theta.
$$

Note that $\lim_{\theta \to 0} f (\theta) = 0$ and $f' (\theta) = \theta \sin \theta > 0$ for $\theta \in (0, \frac{\pi}{2})$. Therefore, $f (\theta) > 0$ over the lower half of the interval. This proves the numerator of (37) is negative for $\Delta_i \in (0, 1)$ and that the variance of the repayment rate is non-increasing for $\Delta_i \in [0, 1]$ and strictly decreasing over the open unit interval.

Finally, as $\Delta_i \uparrow 1$, the repayment rate has the following limit:

$$
\lim_{\Delta_i \uparrow 1} \Pr (r_{L,i} | \Delta_i, \tilde{u}) = \frac{1}{2} + \sin (\pi) \frac{\cos \left( 2\pi (i + \tilde{u}) \right)}{2\pi},
$$

$$
= \frac{1}{2}.
$$

Thus, the repayment rate becomes a constant $\frac{1}{2}$, no matter the realization of the random variable $\tilde{u}$.
9.1.10 Proof of Proposition 2 and Lemma 9

In a competitive equilibrium, the first order condition for optimality is

\[
\frac{1}{2} (r_{L,i} + \kappa) + \frac{1}{2} \Delta_i \left( \frac{d r_{L,i}}{d \Delta_i} \right) = \frac{d FC \left( \Delta_i \right)}{d \Delta_i}.
\]

From (31) and a binding constraint (29), the marginal financing cost function is

\[
\frac{d FC \left( \Delta_i \right)}{d \Delta_i} = r_{e,c} + \left( 1 - \frac{r_{e,c}}{r} \right) \frac{d \pi_{i,\min}}{d \Delta_i}.
\]

Computing \( \frac{d \pi_{i,\min}}{d \Delta_i} \) gives for the marginal financing cost function:

\[
\frac{d FC \left( \Delta_i \right)}{d \Delta_i} = \left( 1 - \frac{r_{e,c}}{r} \right) \left( 1 - Pr_{\min} \left( \Delta_i \right) \right) \frac{d \pi_{i,\min}}{d \Delta_i} + \left( 1 - \frac{r_{e,c}}{r} \right) \left( r_{L,i} + \Delta_i \left( \frac{d r_{L,i}}{d \Delta_i} \right) \right) Pr_{\min} \left( \Delta_i \right),
\]

\[
+ \left( 1 - \frac{r_{e,c}}{r} \right) \left( r_{L,i} - \frac{d \pi_{i,\min}}{d \Delta_i} \right) \frac{d Pr_{\min} \left( \Delta_i \right)}{d \Delta_i},
\]

\[
+ r_{e,c}.
\]

Substituting the slope of the competitive demand curve \( \frac{d r_{L,i}}{d r_i} = -2\tau \) and re-arranging terms gives

\[
\frac{d FC \left( \Delta_i \right)}{d \Delta_i} = \left( 1 - \frac{r_{e,c}}{r} \right) \left[ Pr_{\min} \left( \Delta_i \right) \left( r_{L,i} - 2\tau \Delta_i \right) + \left( 1 - Pr_{\min} \left( \Delta_i \right) \right) \kappa \right],
\]

\[
+ \left( 1 - \frac{r_{e,c}}{r} \right) \left( r_{L,i} - \kappa \right) \Delta_i \frac{d Pr_{\min} \left( \Delta_i \right)}{d \Delta_i} + r_{e,c}.
\]

The first line in the above expression is the marginal financing cost savings from greater diversification and larger debt capacity, holding fixed the minimum repayment rate. The first term of the second line is the cost savings from a higher minimum repayment rate after a marginal increase in the loan portfolio scale \( \Delta_i \). The bank gets repaid on more of its loans and recovers less of the low project returns. The final term \( r_{e,c} \) of the second line is the marginal cost of equity financing.

The marginal financing cost function can be conveniently represented by separating terms involving \( r_{L,i} \). Doing so gives
\[
\frac{dFC(\Delta_i)}{d\Delta_i} = -\left(\frac{r_{e,c}}{r} - 1\right) \left[ Pr_{\min}(\Delta_i) + \frac{dPr_{\min}(\Delta_i)}{d\Delta_i} \right] r_{L,i},
\]
\[
+ \left(\frac{r_{e,c}}{r} - 1\right) \left[ 2\tau \Delta_i Pr_{\min}(\Delta_i) - \kappa (1 - Pr_{\min}(\Delta_i)) \right],
\]
\[
+ \left(\frac{r_{e,c}}{r} - 1\right) \kappa \Delta_i \frac{dPr_{\min}(\Delta_i)}{d\Delta_i},
\]
\[
+ r_{e,c}
\]

Next, define the functions
\[
\phi(\Delta_i) \equiv 2\tau \Delta_i Pr_{\min}(\Delta_i) - \kappa (1 - Pr_{\min}(\Delta_i)),
\]
\[
+ \frac{\kappa}{2} \Delta_i \frac{dPr_{\min}(\Delta_i)}{d\Delta_i},
\]
\[
\psi(\Delta_i) \equiv Pr_{\min}(\Delta_i) + \frac{dPr_{\min}(\Delta_i)}{d\Delta_i} \Delta_i,
\]
and re-write the marginal financing cost function as
\[
\frac{dFC(\Delta_i)}{d\Delta_i} = r_{e,c} + \left(\frac{r_{e,c}}{r} - 1\right) (\phi(\Delta_i) - \psi(\Delta_i) r_{L,i}).
\]

The function \(\psi > 0\). Provided \(\kappa\) is not too large, then also \(\phi > 0\). The terms in \(\phi\) are those that increase the marginal cost of financing for the bank, whereas those in \(\psi\) decrease the marginal cost of financing.

Using the representation of \(\frac{dFC(\Delta_i)}{d\Delta_i}\) in the last expression, the optimality condition becomes
\[
\frac{1}{2} (r_{L,i} + \kappa - 2\tau \Delta_i) = r_{e,c} + \left(\frac{r_{e,c}}{r} - 1\right) (\phi(\Delta_i) - \psi(\Delta_i) r_{L,i}).
\]

Solving for \(r_{L,i}\) and using the equilibrium condition \(\Delta_i = \frac{1}{n_c}\) gives the competitive equilibrium lending rate:
\[
r_{L,c} = \frac{r_{e,c} + \frac{1}{2} \left(\frac{2\kappa}{n_c} - \kappa\right) + \left(\frac{r_{e,c}}{r} - 1\right) \phi\left(\frac{1}{n_c}\right)}{\frac{1}{2} + \left(\frac{r_{e,c}}{r} - 1\right) \psi\left(\frac{1}{n_c}\right)}.
\]

In a kinked equilibrium, the first order condition for optimality does not hold with equality, so the lending rate is instead the monopoly demand curve. Just as in the main text, solving (5) for the lending rate and setting \(\Delta_i = \frac{1}{n_k}\) gives (33).

### 9.2 Price discriminating banks

In this section, I assume that banks can identify the location of any prospective borrower. They are free to offer a loan rate that depends on the borrower’s location. I do so to demonstrate the robustness of the interest rate pass-through results to price discrimination. For simplicity let a project’s recovery value \(\kappa = 0\).
I consider first-degree price discrimination in that a bank can capture the entire consumer surplus. A simple way to insert price discrimination is to allow banks to charge a personalized fixed premium to each entrepreneur for taking out a loan. The fixed premium could be a loan application or closing fee.

The premium would need to depend on the borrower’s distance from the bank. It would be highest for those closest to the bank, because these borrowers retain the largest surplus under uniform pricing, which the main model uses. The personalized fixed premium is a two-part tariff or affine pricing schedule. It is equivalent to a system of personalized prices in which each borrower pays a sum equal to his or her willingness to pay.

Under both the kinked and competitive cases, let $S_i(x)$ be the net surplus for an entrepreneur located a distance $x$ from bank $i$, which is charging lending rate $r_{L,i}$. The total amount of money $T_i(x)$ the entrepreneur pays for a loan from bank $i$ then is

$$T_i(x) = S_i(x) + r_{L,i}.$$ 

### 9.2.1 Kinked case

In the kinked case, the indifference condition for the entrepreneur located a distance $x$ from bank $i$ is

$$\sigma (\kappa - r_{L,i}) - \tau x = w.$$ 

Without price discrimination, the equilibrium kinked lending rate is $r_{L,k} = \kappa - \frac{w}{\sigma} - \frac{\tau}{2\sigma k}$. For an entrepreneur to have surplus, it must mean that

$$\sigma (\kappa - r_{L,i}) - \tau x - w \geq 0.$$ 

The surplus is the left-hand-side of this inequality. Substituting the kinked lending rate for $r_{L,i}$ gives

$$S_i(x) = \tau \left( \frac{1}{2nk} - x \right).$$ 

The surplus is positive for $x \leq \frac{1}{2nk}$. The upper bound is the edge of bank $i$’s local monopoly market. The personalized premium is decreasing in the borrower's distance from bank $i$. The bank has the most market power over those borrowers nearest to it. Those entrepreneurs can be charged the largest premium and yet still take out the loan.

Since the kinked interest rate with price discrimination does not depend on the interest rate (holding fixed the number of banks), the absence of interest rate pass-through in a kinked equilibrium holds again.
9.2.2 Competitive case

For the competitive case, it is easiest to assume that the supply of equity is so large that \( r_{e,c} = r \), which makes the equity capital requirement moot. The expected profit function is then

\[
\pi = \sigma r_{L,i} \Delta_i - r (\Delta_i + f).
\]

The equilibrium competitive lending rate is

\[
r_{L,c} = \frac{1}{\sigma} \left( r + \frac{\tau}{n_c} \right).
\]

The indifference condition for a borrower located a distance \( x \) from bank \( i \) is

\[
\sigma r_{L,i} + \tau x = \sigma r_L + \tau \left( \frac{1}{n_c} - x \right).
\]

The entrepreneur has surplus when borrowing from bank \( i \) so long as

\[
\sigma r_L + \tau \left( \frac{1}{n_c} - x \right) - (\sigma r_{L,i} + \tau x) \geq 0.
\]

Simplifying gives the surplus when borrowing from bank \( i \):

\[
S_i(x) = \sigma (r_L - r_{L,i}) + \tau \left( \frac{1}{n_c} - 2x \right).
\]

The lower bank \( i \) sets the lending rate, the more surplus goes to the entrepreneur; the closer the entrepreneur is to bank \( i \), the more surplus he or she receives.

In equilibrium, the lending rates match, so the surplus is

\[
S(x) = \tau \left( \frac{1}{n_c} - 2x \right).
\]

This surplus is positive for \( x \leq \frac{1}{2n_c} \). An entrepreneur located the distance \( x = \frac{1}{2n_c} \) is in between the two banks. That entrepreneur is marginal, so he or she will not be charged a personalized premium. Everyone else will be charged the fixed premium according to their distances from the bank. Compared to the kinked case, the surplus in the competitive case declines twice as fast as \( x \) increases due to the entrepreneur’s credible alternative of contracting with a competitor. Like before, the competitive lending rate features perfect pass-through.

9.3 Smoothing the kink

In this section, I provide one way to “smooth” the kink in the demand curve for bank credit (make the region differentiable). I do so in order to demonstrate that the limited pass-through at the kink is robust even after the kink is smoothed.
9.3.1 General Case

Generally, the pass-through of marginal costs to prices is lower at points on a downward sloping demand curve that feature greater concavity. In the model, the kink is a sharp way of creating concavity in the demand curve for loans when consumer preferences would otherwise imply a linear demand curve (and perfect pass-through). As long as the smoothing procedure preserves the highest concavity at the smoothed kink, the pass-through will be lowest there, just as when the kink is sharp.

One way to smooth the kink is to assume that banks are unsure about which demand curve they face when setting a loan price. Since banks set prices at the margin, they may equivalently be unsure about the slope of the demand curve at a given price.

The two possible slopes a bank faces is either \(-\tau\) or \(-2\tau\). I assume the bank assigns probability \(h\) that the slope is \(-2\tau\), which is to say that it believes with probability \(h\) that it is competing with a neighboring bank. The probability with which the bank believes it is a local monopolist is \(1 - h\).

I assume that \(h\) is an increasing function of the bank’s market share \(\Delta\), is continuous, and is three times differentiable over the domain I specify below. This kind of uncertainty can be rationalized by a bank not knowing the precise boundary of its neighbor’s market. The bank knows, however, that as it expands market share, it has more and more likely penetrated that boundary. I assume the bank knows the number of banks operating in the lending market \(n\).

For simplicity, I also assume the bank cannot recover any value from a loan in default (\(\kappa = 0\)) and that the probability of a loan’s repayment \(\sigma = \frac{1}{2}\). I also assume the bank is flush with equity so that its cost of financial capital is the interest rate \(r\). Under these assumptions, the expected profit function of a bank is
\[
\pi = \frac{1}{2} r_L \Delta - r (\Delta + f).
\]

The bank will chose a market share \(\Delta\) that satisfies the first order condition
\[
\Omega \equiv r_L (\Delta) + \Delta r'_L (\Delta) - 2r = 0.
\]

By the implicit function theorem, the quantity pass-through \(\frac{\partial \Delta}{\partial r}\) is
\[
\frac{\partial \Delta}{\partial r} = -\frac{\partial \Omega}{\partial \Delta} \cdot \frac{\partial \Delta}{\partial r} = \frac{2}{r'_L + \Delta r''_L (\Delta)}.
\]
By the chain rule, the interest rate pass-through is

\[
\frac{\partial r_L}{\partial r} = r'_L \frac{\partial \Delta}{\partial r}.
\]

Substituting the quantity pass-through gives

\[
\frac{\partial r_L}{\partial r} = r'_L \left( \frac{2}{2r'_L + \Delta r''_L} \right).
\]

By symmetry in market shares and a completely covered circle, \( \Delta = \frac{1}{n} \). Substituting this relation into the interest rate pass-through above and re-arranging terms gives

\[
\frac{\partial r_L}{\partial r} = \frac{2r'_L}{2r'_L + \frac{1}{n} r''_L} = \frac{2}{2 + \frac{1}{n} r''_L}
\]

Define the concavity of the demand curve as

\[
\omega(\Delta) \equiv \frac{r''_L(\Delta)}{r'_L(\Delta)}.
\]

The interest rate pass-through is then

\[
\frac{\partial r_L}{\partial r} = \frac{2}{2 + \frac{\omega(\Delta)}{n}}.
\]

With a downward sloping demand curve, a larger concavity implies a lower interest rate pass-through. Also, as the number of banks tends to infinity \( n \to \infty \), the market reaches perfect competition and features perfect pass-through.

Returning to a bank's uncertainty over the slope of its demand curve, one has

\[
r'_L(\Delta) = -2\tau \times h(\Delta) - \tau \times (1 - h(\Delta)) = -\tau - \tau h(\Delta) = -\tau (1 + h(\Delta))
\]

This object is the slope of the bank's subjective demand curve given its beliefs about being a competitor or local monopolist. The second derivative is

\[
r''_L(\Delta) = -\tau h'(\Delta).
\]
The concavity of the demand curve for loans is then

\[ \omega (\Delta) = \frac{h'(\Delta)}{1 + h(\Delta)}. \]

Because \( h \) is a probability, I require \( h(\Delta) \geq 0 \) for all \( \Delta \). I also assume that \( h' \geq 0 \), making the bank increasingly believe it is competing as it expands. These assumptions make \( \omega (\Delta) \geq 0 \). Furthermore, a bank knows that its headquarters is located at \( \Delta = 0 \). It also knows that its loan portfolio reaches the neighboring bank’s headquarters when \( \Delta = \frac{2}{n} \). Therefore, it is reasonable to assume that \( \lim_{\Delta \downarrow 0} h(\Delta) = 0 \) and \( \lim_{\Delta \uparrow \frac{2}{n}} h(\Delta) = 1 \). I define the domain of \( h \) to be the closed interval \([0, \frac{2}{n}]\).

If the concavity is uniquely highest at the point midway between headquarters of banks, then the lowest pass-through would occur at the location of the kink in the main model. The function \( \omega \) should be globally maximized at \( \frac{1}{n} \) and hence satisfy

\[ \begin{align*}
\omega' \left( \frac{1}{n} \right) &= 0, \quad (40) \\
\omega'' \left( \frac{1}{n} \right) &< 0. \quad (41)
\end{align*} \]

A simple way to ensure the global maximum is uniquely reached at \( \Delta = \frac{1}{n} \) is to assume that the function \( \omega (\Delta) \) is strictly concave along the entire interval. A sufficient but not necessary condition for the global maximum is

\[ \omega'' (\Delta) < 0, \quad \forall \Delta \in \left[0, \frac{2}{n}\right]. \] (42)

The restrictions on the belief function \( h \) imposed by the conditions (40)-(41) will smooth the kink in the demand curve in a way to minimize the interest rate pass-through at the kink. If condition (42) is also imposed, then it is guaranteed that that point will uniquely minimize interest rate pass-through.

### 9.3.2 An example \( h \)

Given the assumptions for \( h \), a cumulative distribution function with non-negative bounded support that satisfies (40)-(41) and where the maximum is unique would deliver an appropriate example.

I use the beta distribution \( h (x) = \text{Beta} (x; \alpha, \beta) \), where I make the linear transformation \( x = \frac{\Delta}{\frac{2}{n}} \) that adjusts the support to \([0, 1]\). Thus, a bank that has a portfolio size \( x = 1 \) believes its loan portfolio has reached the headquarters of the two neighboring banks.

Choosing parameters \( \alpha \) and \( \beta \) that deliver a global maximum for \( \omega \) at \( x = \frac{1}{2} \) would give what is needed, which is the highest curvature at the kink in the baseline model. I find those parameters computationally. I search for the parameter vector \((\alpha, \beta)\) that minimizes the function \( |\text{argmax}_{x(\alpha, \beta)} \omega (x (\alpha, \beta)) - \frac{1}{2}| \) over the unit interval and confirm graphically that
the solution is the parameter vector that delivers the global maximum of $\omega(x)$. I use 10,000 starting points for the optimization routine.

The optimal solution from the search is $\alpha = 1059$ and $\beta = 1046$. The differences between the smooth and original demand curves under these parameters, however, are difficult to see. Therefore, in the figures below, I use the parameters $\alpha = 10.59$ and $\beta = 10.46$, which make the differences clearer and deliver a concavity function $\omega$ that approximately is maximized at $x = \frac{1}{2}$.

Figures 12(a)-12(b) plot the cumulative distribution function $h(\Delta)$ and the concavity function $\omega(\Delta)$. 
Figure 12: Uncertainty Over the Demand Curve for Loans

(a) Cumulative distribution function $h(\Delta)$

(b) Concavity function $\omega(\Delta)$

**Notes:** The function $h(\Delta)$ represents the probability a bank believes it is competing with a neighboring bank. This uncertainty affects the weight a bank places on the two possible slopes in the demand curve for loans. For $h$, I use a Beta ($x; \alpha, \beta$) distribution. The function $\omega(\Delta)$ is the concavity of the subjective demand curve implied by the bank’s beliefs. The support of the beta distribution has been transformed from the unit interval to $[0, \frac{2}{n}]$ in the figures. The scale parameters for the beta distribution are $\alpha = 10.59$ and $\beta = 10.46$.

Figure 12(b) reveals that the concavity of the subjective demand curve is maximized at $\frac{1}{n}$.
which is the location of the kink. A plot of the probability density function associated with \( h \) would show that the uncertainty about the slope of the loan demand curve is concentrated around \( \frac{1}{n} \), which is the location a bank could easily believe is still part of its local monopoly market or the beginning of its neighbor’s market.

Figure 13 illustrates the original sharp kink featuring no uncertainty about the demand curve and the corresponding smoothed kink in which there is uncertainty. For \( \Delta < \frac{1}{n} \), the smooth curve begins to deviate from the original curve once the bank starts assigning positive probability to competing with a neighbor. The bank reduces its lending rate faster in order to attract the marginal borrower because the bank believes it might now be competing for that customer. As \( \Delta \) approaches \( \frac{1}{n} \), the original kink is entirely “rounded out” and the smooth demand curve displays the most concavity. Interest rate pass-through will be lowest in that region, though not zero.

As the bank extends its loan portfolio further past \( \frac{1}{n} \), it becomes more confident that it is competing with the neighbor, so the bank puts more weight on the slope of the demand curve being \(-2\tau\). The original and smooth demand curves converge.

Figure 13: Smooth and Original Demand Curves

Notes: The dashed curve is the original demand curve that features a kink at \( \Delta = \frac{1}{n} \). The solid curve is the smooth demand curve that is derived from a bank being uncertain about the slope of the demand curve it faces. The uncertainty is captured by a beta distribution with scale parameters \( \alpha = 10.59 \) and \( \beta = 10.46 \).